Hindawi Publishing Corporation
Journal of Electrical and Computer Engineering
Volume 2013, Article ID 973813, 6 pages
http://dx.doi.org/10.1155/2013/973813

Research Article

Approximation for Transient of Nonlinear Circuits Using RHPM and BPES Methods

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Received 20 December 2012; Accepted 19 January 2013

Academic Editor: Esteban Tlelo-Cuautle

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The microelectronics area constantly demands better and improved circuit simulation tools. Therefore, in this paper, rational homotopy perturbation method and Boubaker Polynomials Expansion Scheme are applied to a differential equation from a nonlinear circuit. Comparing the results obtained by both techniques revealed that they are effective and convenient.

1. Introduction

Industrial competition constantly pushes the area of electronic circuit design to the limits of technology. This has caused a rapid growth in the levels of integration for integrated circuits and the emergence of novel devices such as single-electron transistors and memristors. Because of this, the development and improvement of mathematical and numerical tools, applied to circuit simulation for the transient domain, are important. In the dynamic domain (transient), the circuit analysis is carried out only numerically because the resulting differential equations are highly nonlinear. Nevertheless, several methods are focused to find approximate solutions to nonlinear differential equations like homotopy perturbation method (HPM) [1–12], rational homotopy perturbation method (RHPM) [5, 6], variational iteration method (VIM) [13–16], and Boubaker Polynomials Expansion Scheme (BPES) [17–36], among many others. Therefore, we propose the comparison between RHPM and BPES methods by solving the nonlinear differential equation that represents the dynamics of a nonlinear circuit. The results should be a meaningful supply for monitoring complex nonlinear circuits behaviours and responses. In fact, the used protocols try to embed boundary conditions instead of direct solving, as preceded in spectral or limit-cycle bifurcations approaches.

This paper is arranged as follows. In Section 2, we present the differential equation of a nonlinear circuit. Sections 3 and 4 present the fundamentals of RHPM and BPES methods, respectively. The solutions obtained using both methods are explained in Section 5. Comparisons between the two methods and some other results presented in the recent literature have been illustrated in Section 6. Conclusions will be discussed in Section 7.

2. Nonlinear Circuit

The rapid increase in the number of transistors by integrated circuit and the increase of complexity for the models (a result of lowering the dimension of the components) results in a complex calculation for the transient. Furthermore, the task of tracing the transient for nonlinear circuits is a critical and difficult task. In fact, commercial circuit simulators do not provide any symbolic/analytic solution for the transient of any given circuit. Instead, the simulator provides only

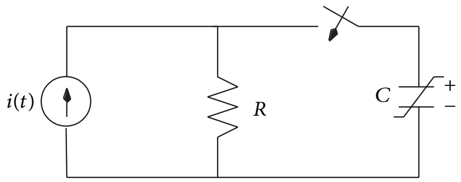

Figure 1: Nonlinear RC circuit.

numerical data that allows circuit designers to explore a limited range of dynamics of nonlinear circuits.

Consider the analysis of the nonlinear circuit depicted in Figure 1 as a case study [37]. Let the branch relationship of the nonlinear capacitor to be

$$v(q) = \alpha q^3, \tag{1}$$

where v is the voltage, q is the charge of the capacitor, and α is a parameter of the capacitor.

By applying Kirchhoff Laws, we obtain the equation for the transient

$$\frac{dq(t)}{dt} + \frac{1}{R} v(q) = i(t). \tag{2}$$

Therefore,

$$\frac{dq(t)}{dt} + \frac{\alpha}{R} q^3(t) = i(t). \tag{3}$$

If we consider the case for DC excitation, then $i(t) = I$, resulting in

$$\frac{dq(t)}{dt} + \frac{\alpha}{R} q^3(t) = I, \qquad q(0) = 0. \tag{4}$$

3. Fundamentals of the Rational Homotopy Perturbation Method

The rational homotopy perturbation method RHPM [5, 6] can be considered as a combination of the classical perturbation technique [38, 39] and the homotopy (whose origin is in the topology) [40–42] but not restricted to a small parameter like traditional perturbation methods. For example, RHPM requires neither small parameter nor linearization, but only few iterations to obtain accurate solutions.

To figure out how RHPM method works, consider a general nonlinear equation in the form:

$$A(u) - f(r) = 0, \quad r \in \Omega \tag{5}$$

with the following boundary conditions:

$$B\left(u, \frac{\partial u}{\partial u}\right) = 0, \quad r \in \Gamma, \tag{6}$$

where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytical function, and Γ is the domain boundary for Ω. A can be divided into two operators L and N, where L is linear and N nonlinear; from this last statement, (5) can be rewritten as

$$L(u) + N(u) - f(r) = 0. \tag{7}$$

Generally, a homotopy can be constructed in the form [1–3]:

$$\begin{aligned} H(v,p) = {} & (1-p)\left[L(v) - L(u_0)\right] \\ & + p\left[L(v) + N(v) - f(r)\right] = 0, \\ & p \in [0,1], \; r \in \Omega, \end{aligned} \tag{8}$$

where p is a homotopy parameter, whose values are within the range of 0 and 1, u_0 is the first approximation for the solution of (6) that satisfies the boundary conditions.

When $p \to 0$, (8) is reduced to

$$L(v) - L(u_0) = 0, \tag{9}$$

where operator L possesses trivial solution.

For $p \to 1$, (8) is reduced to the original problem

$$N(v) + L(v) - f(r) = 0. \tag{10}$$

Assuming that the solution for (8) can be written as a power series of p:

$$v = \frac{v_0 + p v_1 + p^2 v_2 + \cdots}{w_0 + p w_1 + p^2 w_2 + \cdots}, \tag{11}$$

where $v_0, v_1, v_2, \ldots$ are unknown functions to be determined by the RHPM, and $w_0, w_1, w_2, \ldots$ are known analytic functions of the independent variable.

Substituting (11) into (8) and equating identical powers of p terms, it is possible to obtain values for the sequence $v_0, v_1, v_2, \ldots$.

When $p \to 1$ in (11), it yields in the approximate solution for (5) in the form:

$$u = \lim_{p \to 1}(v) = \frac{v_0 + v_1 + v_2 + \cdots}{w_0 + w_1 + w_2 + \cdots}. \tag{12}$$

Convergence of RHPM method is studied in [5, 6].

4. Fundamentals of the Boubaker Polynomials Expansion Scheme BPES

The Boubaker Polynomials Expansion Scheme BPES [17–36] is a resolution protocol, which has been successfully applied to several applied-physics and mathematical problems. The BPES protocol ensures the validity of the related boundary conditions regardless of main equation features. The Boubaker Polynomials Expansion Scheme BPES is based on the Boubaker polynomials first derivatives properties:

$$\begin{aligned} \sum_{q=1}^{N} B_{4q}(x)\Big|_{x=0} &= -2N \neq 0, \\ \sum_{q=1}^{N} B_{4q}(x)\Big|_{x=r_q} &= 0, \\ \sum_{q=1}^{N} \frac{dB_{4q}(x)}{dx}\Big|_{x=0} &= 0, \\ \sum_{q=1}^{N} \frac{dB_{4q}(x)}{dx}\Big|_{x=r_q} &= \sum_{q=1}^{N} H_q, \end{aligned} \tag{13}$$

with

$$H_n = B'_{4n}(r_n) = \left(\frac{4r_n \left[2 - r_n^2\right] \times \sum_{q=1}^{n} B_{4q}^2 (r_n)}{B_{4(n+1)}(r_n)} + 4r_n^3 \right). \tag{14}$$

Several solutions have been proposed through the BPES in many fields like numerical analysis [17–20], theoretical physics [21–24], mathematical algorithms [25], heat transfer [26], homodynamic [27, 28], material characterization [29], fuzzy systems modelling [30–34], and biology [35, 36].

5. Application of RHPM and BPES

5.1. Solution Using RHPM Method. Using (8), we establish the following RHPM homotopy map:

$$(1-p)\left(v' - u'_0\right) + p\left(v' + \frac{\alpha}{R}v^3 - I\right) = 0, \tag{15}$$

where the trial function $u_0 = 0$.

Using (11), we propose the following rational solution:

$$\begin{aligned} v = &\left(v_0 + pv_1 + p^2 v_2 + p^3 v_3 + p^4 v_4 \right.\\ &\left. + p^5 v_5 + p^6 v_6 + p^7 v_7\right) \\ &\times (1 + pk_1 x^3 + p^2 k_2 x^6)^{-1}, \end{aligned} \tag{16}$$

where $w_0 = 1$, $w_1 = k_1 x^3$, and $w_2 = k_2 x^6$.

We substitute (16) into (15), regroup, and equate terms with identical powers of p. In order to fulfil boundary condition of (16), it follows that $v_0(0) = 0, v_1(0), \ldots$ for the homotopy map.

The results are recast in the following systems of differential equations:

$$\begin{array}{lll} p^0: & v'_0 = 0, & v_0(0) = 0, \\ p^1: & v'_1 - I + \frac{\alpha}{R}v^3 + 2v'_0 K_1 X^3 - 3X^2 v_0 K_1 = 0, & v_1(0) = 0, \\ \vdots & \vdots & \vdots \end{array} \tag{17}$$

Solving (17) yields

$$\begin{aligned} v_0 &= u_0 = 0, \\ v_1 &= Ix, \\ v_2 &= Ik_1 x^4 \\ &\vdots \end{aligned} \tag{18}$$

Substituting (18) into (16) and calculating the limit when $p \to 1$, we obtain the seventh-order approximation:

$$\begin{aligned} q(t) &= \lim_{p\to 1}(v) \\ &= \left(Ix + \left(Ik_1 - (1/4)\left(\alpha I^3/R\right)\right)x^4 \right.\\ &\quad + \left(Ik_2 - (1/4)\left(\alpha k_1 I^3/R\right) + (3/28)\left(\alpha^2 I^5/R^2\right)\right)x^7 \\ &\quad \left. - (1/4)\left(\alpha k_2 I^3/R\right)x^{10}\right) \\ &\quad \times \left(1 + k_1 x^3 + k_2 x^6\right)^{-1}. \end{aligned} \tag{19}$$

If we consider $R = 1/20$, $I = 10$, and $\alpha = 40$ as reported in [37], it is possible to obtain the adjustment parameters using the procedure reported in [6, 12], resulting in $k_1 = 36289$ and $k_2 = 4471843$.

5.2. Solution Using the Boubaker Polynomials Expansion Scheme BPES. The Boubaker Polynomials Expansion Scheme BPES is applied to (4) using the setting expression:

$$q(t) = \frac{1}{2N_0}\sum_{k=1}^{N_0} \lambda_k \times \frac{dB_{4k}(r_k t)}{dt}. \tag{20}$$

Using the properties provided by (13), boundary conditions are verified in advance of the resolution process. The system in (16) is reduced to

$$\frac{1}{2N_0}\sum_{k=1}^{N_0}\lambda_k r_k \frac{d^2 B_{4k}(r_k t)}{d^2 t} + \frac{\alpha}{8N_0^3 R}\left[\sum_{k=1}^{N_0}\lambda_k \frac{dB_{4k}(r_k t)}{dt}\right]^3 = I. \tag{21}$$

Boundary conditions become redundant since they are already verified by the proposed expansion, consecutively, and thus, majoring and integrating along the given interval for the time variable t transform the problem in a linear system with unknown real variables: $\lambda_k|_{k=1\ldots N_0}$. Calculations are reduced to approximately $(8N_0)^3$arithmetical operations. Solutions are obtained by using the Householder [39, 40] algorithm detailed elsewhere and are denoted by $\lambda_k^{(\mathrm{sol.})}|_{k=1\ldots N_0}$. The final solution is given as

$$q(t) = \frac{1}{2N_0}\sum_{k=1}^{N_0}\lambda_k^{(\mathrm{sol.})} \times \frac{dB_{4k}(r_k t)}{dt}. \tag{22}$$

6. Results and Discussion

From Table 1, we can observe that RHPM solution (19) and BPES solution (22) are in good agreement with the numerical results obtained using Fehlberg fourth-fifth-order Runge-Kutta method with degree four interpolant (RKF45) [43, 44] (built-in function of Maple software). In order to guarantee a good numerical reference, RKF45 is configured using an absolute error of $10^{-7}$and a relative error of 10^{-6}. The power

Table 1: Numerical comparison of proposed solutions and RKF45 solution of (4).

t	$q(t)$ (RKF45)	RHPM	BPES $N_0 = 37$	BPES $N_0 = 107$	BPES $N_0 = 113$
0.00	0.00000	0.00000	0.00000	0.00000	0.00000
0.01	0.098066	0.098070	0.09436	0.09456	0.09827
0.02	.17475	0.17481	0.17123	0.17395	0.17513
0.03	.21314	0.21388	0.21051	0.21266	0.21394
0.04	0.22656	0.22802	0.22669	0.22557	0.22775
0.05	0.23054	0.23005	0.22678	0.23009	0.23076
0.06	0.23165	0.23235	0.22346	0.23056	0.23246

of RHPM method is based on the capability of rational expressions containing a huge amount of information of dynamics from asymptotic problems.

Moreover, convergence of the BPES algorithm has been obtained for moderate values of N_0 ($N_0 < 120$), since, as mentioned above, boundary conditions were verified in advance of the resolution process. Both methods generated analytical expressions useful for other analysis like circuit power consumption; such analytical expressions can provide more information about the nature and behaviour of circuits than numerical integration schemes with variable step size [41, 44–46]. Nonetheless, semianalytical techniques like RHPM and BPES may be combined with numerical methods [43–46] to improve the simulation tools of VLSI circuits.

7. Conclusion

In this paper, powerful analytical methods like rational homotopy perturbation method (RHPM) and Boubaker Polynomials Expansion Scheme (BPES) are presented to construct semianalytical solutions for the transient of a nonlinear circuit. The results exhibited that both techniques are powerful, obtaining highly accurate analytical expressions for the transient of a simple test circuit. While RHPM yielded accurate and reliable results, BPES exhibited the advantage of ensuring the validity of boundary conditions regardless of main equation features. This feature made the protocol yielding faster and provided more convergent solutions than many numerical integration schemes with variable step size. Further work is necessary to extend the use of both methods for larger circuits.

Acknowledgments

The authors gratefully acknowledge the financial support provided by the National Council for Science and Technology of Mexico (CONACyT) through Grant CB-2010-01 no. 157024. The authors would like to express their gratitude to Rogelio-Alejandro Callejas-Molina and Roberto Ruiz-Gomez for their contribution to this project.

References

[1] J. H. He, “Homotopy perturbation technique,” *Computer Methods in Applied Mechanics and Engineering*, vol. 178, no. 3-4, pp. 257–262, 1999.

[2] J. H. He, “A coupling method of a homotopy technique and a perturbation technique for non-linear problems,” *International Journal of Non-Linear Mechanics*, vol. 35, no. 1, pp. 37–43, 2000.

[3] J. H. He, “Homotopy perturbation method: a new nonlinear analytical technique,” *Applied Mathematics and Computation*, vol. 135, no. 1, pp. 73–79, 2003.

[4] H. Vazquez-Leal, Y. Khan, G. Fernandez-Anaya et al., “A general solution for Troesch's problem,” *Mathematical Problems in Engineering*, vol. 2012, Article ID 208375, 14 pages, 2012.

[5] H. Vazquez-Leal, A. Sarmiento-Reyes, Y. Khan, U. Filobello-Niño, and A. Diaz-Sanchez, “Rational biparameter homotopy perturbation method and laplace-padé coupled version,” *Journal of Applied Mathematics*, vol. 2012, Article ID 923975, 21 pages, 2012.

[6] H. Vázquez-Leal, “Rational homotopy perturbation method,” *Journal of Applied Mathematics*, vol. 2012, Article ID 490342, 14 pages, 2012.

[7] Y. Khan, H. Vázquez-Leal, and L. Hernandez-Martínez, “Removal of noise oscillation term appearing in the nonlinear equation solution,” *Journal of Applied Mathematics*, vol. 2012, Article ID 387365, 9 pages, 2012.

[8] U. Filobello-Nino, H. Vazquez-Leal, Y. Khan et al., “HPM applied to solve nonlinear circuits: a study case,” *Applied Mathematical Sciences*, vol. 6, no. 85–88, pp. 4331–4344, 2012.

[9] H. Vazquez-Leal, R. Castaneda-Sheissa, A. Yıldırım et al., “Biparameter homotopy-based direct current simulation of multistable circuits,” *British Journal of Mathematics & Computer Science*, vol. 2, no. 3, pp. 137–150, 2012.

[10] U. Filobello-Nino, Hector Vazquez-Leal, R. Castaneda-Sheissa et al., “An approximate solution of Blasius equation by using HPM method,” *Asian Journal of Mathematics & Statistics*, vol. 5, no. 2, Article ID 103923, pp. 50–59, 2012.

[11] H. Vázquez-Leal, U. Filobello-Niño, R. Castañeda-Sheissa, L. Hernández-Martínez, and A. Sarmiento-Reyes, “Modified HPMs inspired by homotopy continuation methods,” *Mathematical Problems in Engineering*, vol. 2012, Article ID 309123, 19 pages, 2012.

[12] H. Vazquez-Leal, R. Castaneda-Sheissa, U. Filobello-Nino, A. Sarmiento-Reyes, and J. S. Orea, “High accurate simple approximation of normal distribution integral,” *Mathematical Problems in Engineering*, vol. 2012, Article ID 124029, 22 pages, 2012.

[13] Y. Khan, H. Vázquez-Leal, L. Hernandez-Martínez, and N. Faraz, “Variational iteration algorithm-II for solving linear and non-linear ODEs,” *International Journal of the Physical Sciences*, vol. 7, no. 25, pp. 3099–4002, 2012.

[14] J. H. He, “Variational iteration method—a kind of non-linear analytical technique: some examples,” *International Journal of Non-Linear Mechanics*, vol. 34, no. 4, pp. 699–708, 1999.

[15] J. H. He, "Variational iteration method—some recent results and new interpretations," *Journal of Computational and Applied Mathematics*, vol. 207, no. 1, pp. 3–17, 2007.

[16] J. H. He, "Variational iteration method for autonomous ordinary differential systems," *Applied Mathematics and Computation*, vol. 114, no. 2-3, pp. 115–123, 2000.

[17] M. Agida and A. S. Kumar, "A Boubaker polynomials expansion scheme solution to random Love's equation in the case of a rational Kernel," *Electronic Journal of Theoretical Physics*, vol. 7, no. 24, pp. 319–326, 2010.

[18] A. Yildirim, S. T. Mohyud-Din, and D. H. Zhang, "Analytical solutions to the pulsed Klein-Gordon equation using Modified Variational Iteration Method (MVIM) and Boubaker Polynomials Expansion Scheme (BPES)," *Computers and Mathematics with Applications*, vol. 59, no. 8, pp. 2473–2477, 2010.

[19] J. Ghanouchi, H. Labiadh, and K. Boubaker, "An attempt to solve the heat transfer equation in a model of pyrolysis spray using 4q-order m-boubaker polynomials," *International Journal of Heat and Technology*, vol. 26, no. 1, pp. 49–53, 2008.

[20] S. Slama, J. Bessrour, K. Boubaker, and M. Bouhafs, "A dynamical model for investigation of A3 point maximal spatial evolution during resistance spot welding using Boubaker polynomials," *The European Physical Journal Applied Physics*, vol. 44, no. 3, pp. 317–322, 2008.

[21] S. Slama, M. Bouhafs, and K. B. Mahmoud, "A boubaker polynomials solution to heat equation for monitoring A3 point evolution during resistance spot welding," *International Journal of Heat and Technology*, vol. 26, no. 2, pp. 141–146, 2008.

[22] S. Lazzez, K. B. Ben Mahmoud, S. Abroug, F. Saadallah, and M. Amlouk, "A Boubaker polynomials expansion scheme (BPES)-related protocol for measuring sprayed thin films thermal characteristics," *Current Applied Physics*, vol. 9, no. 5, pp. 1129–1133, 2009.

[23] T. Ghrib, K. Boubaker, and M. Bouhafs, "Investigation of thermal diffusivitymicrohardness correlation extended to surface-nitrured steel using Boubaker polynomials expansion," *Modern Physics Letters B*, vol. 22, no. 29, pp. 2893–2907, 2008.

[24] S. Fridjine, K. B. Ben Mahmoud, M. Amlouk, and M. Bouhafs, "A study of sulfur/selenium substitution effects on physical and mechanical properties of vacuum-grown ZnS1-xSex compounds using Boubaker polynomials expansion scheme (BPES)," *Journal of Alloys and Compounds*, vol. 479, no. 1-2, pp. 457–461, 2009.

[25] C. Khélia, K. Boubaker, T. B. Nasrallah, M. Amlouk, and S. Belgacem, "Morphological and thermal properties of β-SnS2 sprayed thin films using Boubaker polynomials expansion," *Journal of Alloys and Compounds*, vol. 477, no. 1-2, pp. 461–467, 2009.

[26] K. B. Mahmoud and M. Amlouk, "The 3D Amlouk-Boubaker expansivity-energy gap-Vickers hardness abacus: a new tool for optimizing semiconductor thin film materials," *Materials Letters*, vol. 63, no. 12, pp. 991–994, 2009.

[27] M. Dada, O. B. Awojoyogbe, and K. Boubaker, "Heat transfer spray model: an improved theoretical thermal time-response to uniform layers deposit using Bessel and Boubaker polynomials," *Current Applied Physics*, vol. 9, no. 3, pp. 622–624, 2009.

[28] S. A. H. A. E. Tabatabaei, T. Zhao, O. B. Awojoyogbe, and F. O. Moses, "Cut-off cooling velocity profiling inside a keyhole model using the Boubaker polynomials expansion scheme," *Heat and Mass Transfer*, vol. 45, no. 10, pp. 1247–1251, 2009.

[29] A. Belhadj, J. Bessrour, M. Bouhafs, and L. Barrallier, "Experimental and theoretical cooling velocity profile inside laser welded metals using keyhole approximation and Boubaker polynomials expansion," *Journal of Thermal Analysis and Calorimetry*, vol. 97, no. 3, pp. 911–915, 2009.

[30] A. Belhadj, O. F. Onyango, and N. Rozibaeva, "Boubaker polynomials expansion scheme-related heat transfer investigation inside keyhole model," *Journal of Thermophysics and Heat Transfer*, vol. 23, no. 3, pp. 639–640, 2009.

[31] P. Barry and A. Hennessy, "Meixner-type results for Riordan arrays and associated integer sequences," *Journal of Integer Sequences*, vol. 13, no. 9, pp. 1–34, 2010.

[32] A. S. Kumar, "An analytical solution to applied mathematics-related Love's equation using the Boubaker polynomials expansion scheme," *Journal of the Franklin Institute*, vol. 347, no. 9, pp. 1755–1761, 2010.

[33] S. Fridjine and M. Amlouk, "A new parameter: an ABACUS for optimizig functional materials using the Boubaker polynomials expansion scheme," *Modern Physics Letters B*, vol. 23, no. 17, pp. 2179–2191, 2009.

[34] M. Benhaliliba, C. E. Benouis, K. Boubaker, M. Amlouk, and A. Amlouk, "A new guide to thermally optimized doped oxides monolayer spray-grown solar cells: the amlouk-boubaker optothermal expansivity Ψab," in *Solar Cells—New Aspects and Solutions*, L. A. Kosyachenko, Ed., pp. 27–41, InTech, 2011.

[35] A. Milgram, "The stability of the Boubaker polynomials expansion scheme (BPES)-based solution to Lotka-Volterra problem," *Journal of Theoretical Biology*, vol. 271, no. 1, pp. 157–158, 2011.

[36] H. Rahmanov, "A solution to the non lLinear korteweg-de-vries equation in the particular case dispersion-adsorption problem in porous media using the spectral boubaker polynomials expansion scheme (BPES)," *Studies in Nonlinear Sciences*, vol. 2, no. 1, pp. 46–49, 2011.

[37] M. Koksal and S. Herdem, "Analysis of nonlinear circuits by using differential Taylor transform," *Computers and Electrical Engineering*, vol. 28, no. 6, pp. 513–525, 2002.

[38] U. Filobello-Nino, H. Vazquez-Leal, Y. Khan et al., "Perturbation methodand Laplace-Padé approximation to solve nonlinear problems," *Miskolc Mathematical Notes*. In press.

[39] H. Vazquez-Leal, U. Filobello-Nino, A. Yildirim et al., "Transient and DC approximate expressions for diode circuits," *IEICE Electronics Express*, vol. 9, no. 6, pp. 522–530, 2012.

[40] H. Vazquez-Leal, L. Hernandez-Martinez, A. Sarmiento-Reyes, R. Castañeda-Sheissa, and A. Gallardo-Del-Angel, "Homotopy method with a formal stop criterion applied to circuit simulation," *IEICE Electronics Express*, vol. 8, no. 21, pp. 1808–1815, 2011.

[41] H. Vazquez-Leal, L. Hernandez-Martinez, and A. Sarmiento-Reyes, "Double-bounded homotopy for analysing nonlinear resistive circuits," in *Proceeding of the IEEE International Symposium on Circuits and Systems (ISCAS '05)*, pp. 3203–3206, Kobe, Japan, May 2005.

[42] H. Vazquez-Leal, L. Hernandez-Martinez, A. Sarmiento-Reyes, and R. Castañeda-Sheissa, "Numerical continuation scheme for tracing the double bounded homotopy for analysing nonlinear circuits," in *Proceedings of the International Conference on Communications, Circuits and Systems*, pp. 1122–1126, Hong Kong, China, May 2005.

[43] W. H. Enright, K. R. Jackson, S. P. Norsett, and P. G. Thomsen, "Interpolants for runge-kutta formulas," *ACM Transactions on Mathematical Software*, vol. 12, no. 3, pp. 193–218, 1986.

[44] E. Fehlberg, "Klassischerunge-kutta-formelnvierter und niedrigererordnung mitschrittweiten-kontrolle und ihreanwendung

auf waermeleitungsprobleme," *Computing*, vol. 6, no. 1-2, pp. 61–71, 1970.

[45] E. Tlelo-Cuautle, J. M. Muñoz-Pacheco, and J. Martínez-Carballido, "Frequency scaling simulation of Chua's circuit by automatic determination and control of step-size," *Applied Mathematics and Computation*, vol. 194, no. 2, pp. 486–491, 2007.

[46] L. Portero, A. Arrarás, and J. C. Jorge, "Variable step-size fractional step Runge-Kutta methods for time-dependent partial differential equations," *Applied Numerical Mathematics*, vol. 62, no. 10, pp. 1463–1476, 2012.