A MAPLE-BASED HOMOTOPIC CIRCUIT SIMULATION PACKAGE

Arturo Sarmiento-Reyes
Delf University of Technology, ITS Electronics Research Lab Mekelweg 4, 2628CD Delft The Netherlands e-mail: arturo@duteela.et.tudelft.nl

Roberto S. Murphy-Arteaga and Héctor Vázquez-Leal
National Institute for Astrophysics Optics and Electronics, Electronics Department, CAD Group P.O. Box 51, 72000 Puebla, Pue., Mexico. e-mail: hleal@inaoop.mx

ABSTRACT
This work presents HomotopyDC, a package that carries out the DC analysis of a circuit by using homotopy methods, i.e. methods that are able to find more than one DC solution. The capabilities of MAPLE have been used to their full extent in order to formulate the equilibrium equation of the circuit in a fully symbolic form, which is then used to formulate the homotopic equation. Not only are the DC solutions calculated, but also their stability is assessed. Several homotopy schemes have been implemented within the package.

1. INTRODUCTION

The DC analysis is an introduction to other types of circuit analyses, such as the AC small signal and transient analyses. Therefore, an efficient way of calculating the DC operating point is important for any circuit designer. The modern scope on the complete solution to the DC general problem of nonlinear circuits goes far beyond the simple calculation of the DC operating point. In fact, several aspects have to be studied: assessing the uniqueness, establishing an upper bound for the number of DC solutions, calculating all DC solutions and determining their stability. This paper is focussed on calculating more than one DC operating point — when the uniqueness is not guaranteed — by resorting to homotopy methods.

The problem of solving a nonlinear circuit for DC analysis leads to the problem of solving the system of nonlinear algebraic equations:

\[ f(x) = 0 \]  

where \( x \) represents the unknowns of the circuit. This equation however, can exhibit one solution, multiple solutions or no solution at all.

Homotopy methods allow us to trace a curve passing through several (at least more than one) solutions. Such a curve is called the path of solutions or the homotopy path, and it is obtained through a numeric integration procedure. Homotopy-based methods incorporate an extra parameter (\( \mu \)) into the original equation in order to form an augmented equation, which has the form:

\[ H(f(x), \mu) = 0 \]

2. PROGRAM STRUCTURE

As far as functionality is concerned, the development of HomotopyDC was guided by the following key aspects:

1. Establishing the equilibrium equation of the circuit in a completely symbolic form.
2. Formulating the homotopy relationship also in a symbolic form.
3. Implementing efficient algorithms and procedures that give as result of easy operation routines.
4. Finding all solutions.
5. Determining the stability of the solutions.

The general simulator functioning can be described as follows: (i) translate the circuit into a convenient mathematic representation, (ii) select and develop the analysis to be performed, and (iii) print/plot the obtained output data. This scheme is depicted in Figure 1.

According to the structure above, the components of HomotopyDC can be depicted as in Figure 2. These components posses the following characteristics:

1. Input stage

(a) Circuit description
The circuit description is given through an input file à la SPICE. In fact, the grammar used to describe the circuit is a super-set of the SPICE circuit description language.

(b) Input parsing
The main goal of the parsing stage is to detect input errors and formulate the basic data structure on the MAPLE V Release 5 platform. After a successful translation, the input file is parsed into a data structure representation — a MAPLE-list — which is called netlist.

(c) Data structure
It consists of a set of MAPLE-lists. Among them, we can mention:
- A list containing the circuit netlist.
- A list related to the several kinds of analyses.

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• A list containing the parameters associated to the numerical methods.
• A list containing the output items.

![Diagram](image)

Figure 1: Simulator structure

2. Analysis Stage

(a) Equation formulation
The method used for achieving the circuit analysis is the modified nodal analysis (MNA). The formulation is obtained in fully symbolic form, i.e., parameter values are not taken into account (at this point), instead, the component names are used to set-up the MNA equation. In order to use the features of MAPLE [1] [2] to their full extent, the MNA formulation has been split into a linear and non-linear parts. During the iterative process of solution, only the non-linear part is updated, thus saving CPU time.

(b) Homotopic solution
Currently, several homotopy schemes have been incorporated to HomotopyDC.

i. Braining's method.
This method [3] is not exactly a continuation method in the sense that an embedded parameter appears in the homotopic process. However, the continuation procedure is carried out by changing the sign of the determinant after a solution is found. This method is based on the integration of a related system of differential equations which is obtained by adding an artificial parameter derivative to the equilibrium equation:

$$\frac{df}{dp} + f(x) = 0$$  

(3)

ii. Chua's method.
This method [4] selects an initial point which lies on a curve passing through all the solutions, provided that this curve is continuously differentiable. The homotopy is given as:

$$\frac{df_i}{dp}(x(p)) + f_i(x(p)) = 0$$  

(4)

with $f_i(x(0)) = 0$ for $i = 1, 2, \ldots, n - 1$. The $n$-th equation is given as:

$$\frac{df_n}{dp}(x(p)) \pm f_n(x(p)) = 0$$  

(5)

On one side, the $n$-th equation changes sign when the Jacobian changes sign, in order to avoid moving away from the path. On the other side, it also changes sign at the solutions points, to allow the search of further solutions.

![Diagram](image)

Figure 2: Program structure

iii. Chua's method.
This method [5] uses a parameter $\mu$ and an arbitrary positive constant which serves as a damping factor, yielding the following homotopy relation:
$$\frac{df(x, \mu)}{dp} + cf(x, \mu) = 0 \quad (6)$$

where $f$ is the augmented system:

$$\dot{f}(x, \mu) = f(x) + (\mu - 1)f(x_0) = 0$$

and $c$ is the damping factor. Equation (6) however, represents a system of $n$ equations with $n+1$ unknowns. Therefore, an additional equation is needed:

$$\text{sgn} \left( \frac{dx_k}{dp} \right) \frac{dx_k}{dp} = 1 \quad (7)$$

which determines the continuation procedure.

3. **Output Stage**

HomotopyDC has several features regarding the way the results are shown to the user.

(a) The main output data from the homotopy schemes is given in the form of MAPLE-lists:

i. A list of the roots ($x^*$).

ii. A list containing the numerical values of the the solution path, i.e $x^*$ vs $\mu$.

(b) Several printing options were implemented in order to plot the values of the nonlinear elements during the iterative process.

3. **CASE STUDY**

In order to illustrate the execution of HomotopyDC, the circuit depicted in Figure 3 is analyzed. It is a very simple nonlinear circuit composed of a series connection of an independent voltage source, a linear resistor and a nonlinear conductance having a third-order polynomial $i-u$ branch relationship. The components of the circuit are listed in Table 1.

![Figure 3: Circuit](image)

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>1 kΩ</td>
</tr>
<tr>
<td>$V_1$</td>
<td>5 V</td>
</tr>
<tr>
<td>$K_1$</td>
<td>$i = (0.8u^3 - 5.25u^2 + 9u) \times 10^{-3}$ A</td>
</tr>
</tbody>
</table>

Table 1: Description of the circuit components

The input file with the circuit description is:

```
A SIMPLE NONLINEAR CIRCUIT
V1 1 0 5
K1 1 2 i=(0.8*u^3-5.25*u^2+9*u)*1e-3
R1 2 0 1000
# Analysis statements
DC OP
end
```

In this case more than one solution can be assessed. Here, we report three solutions that were found by using Chao’s method, the results arising from the other homotopy methods also agree however.

\[
\begin{bmatrix}
V_1 \\
V_2 \\
I_{V_1}
\end{bmatrix}
= \begin{bmatrix}
4.99999670 \\
1.35183938 \\
-0.0013610744
\end{bmatrix}
= \begin{bmatrix}
4.999465709 \\
2.726056267 \\
-0.00272605662
\end{bmatrix}
\]

Solution ①

\[
\begin{bmatrix}
V_1 \\
V_2 \\
I_{V_1}
\end{bmatrix}
= \begin{bmatrix}
4.219477563 \\
5.004537763 \\
-0.0042947554
\end{bmatrix}
\]

Solution ②

\[
\begin{bmatrix}
V_1 \\
V_2 \\
I_{V_1}
\end{bmatrix}
= \begin{bmatrix}
4.219477563 \\
5.004537763 \\
-0.0042947554
\end{bmatrix}
\]

Solution ③

The homotopic path is shown in Figure 4. Herein, it can be noticed that Solution ② lies on the negative-resistance region of the tunnel characteristic which means that this solution is unstable, while Solution ① and Solution ③ are stable.

![Figure 4: Homotopic path](image)
4. CONCLUSIONS

HomotopyDC, a versatile DC simulation program, has been fully developed under MAPLE V. It uses symbolic techniques in order to setup the resulting equilibrium equation of the circuit. An important feature resides in the fact that the input language of the simulator constitutes a super-set of the SPICE input language. The main result is the calculation of multiple DC operating points and the homotopy path passing along multiple DC solutions. Besides, the program determines the stability of all the operating points.

5. REFERENCES


