Facultad de Matemáticas
16 de Mayo de 2018
Examen de admisión a la Maestría en Matemáticas

Instrucciones: Necesariamente debe resolver los problemas del 1 al 6. Y de los problemas del 7 al 12 debe resolver únicamente dos. Traduzca al español el artículo que aparece al final.

1. Sea f una función real sobre $[a, b]$. Supongamos que f es diferenciable y f' es Riemann integrable. Demostrar que
$$\int_a^b f'(x)dx = f(b) - f(a).$$

2. Sea $f : \mathbb{R}^n \to \mathbb{R}$ definida como $f(x) = \sum_{i=1}^n b_i x_i^2$ donde $b_i > 0$, $i = 1, ..., n$. Sea $a \in \mathbb{R}^n - \{0\}$ y sea M el hiperplano $M = \{x \in \mathbb{R} | ax = 1\}$. Estudiar los extremos de f en M.

3. Hallar los extremos relativos de la función $f : (0, \infty) \to \mathbb{R}$ definida como
$$f(x) = \int_0^{x^2} \sin(t)\ e^{\sin(t)} dt.$$

4. Dada una matriz cuadrada $n \times n$, A, demuestre que
$$|tr(A)| \leq n\rho(A).$$
Donde $\rho(A)$ es el máximo de los módulos de los valores propios de A.

5. Sea $T : \mathbb{R}^n \to \mathbb{R}^n$ una transformación lineal. Entonces, el núcleo de T $(KerT)$ y el rango de T $(rangT)$ son subespacios de R^n y $kerT + rangT = n$.

6. Sea
$$M = \begin{bmatrix} b & 0 & 0 & b+3 \\ 0 & 0 & 0 & b \\ b & 0 & 0 & -2 \\ 0 & 0 & 0 & b \end{bmatrix}$$
con componentes reales. Determinar su forma canónica de Jordan según los valores del parámetro b.

7. Sea $f : \mathbb{R} \to \mathbb{R}$ una función no negativa que se anula unicamente en cero. Sea $\delta(x, y) = f(x - y)$ ¿Cuándo δ es una métrica?

8. Sea $f : \mathbb{R}^n \to \mathbb{R}^n$ una trasnformación lineal que no es isomorfismo. ¿Existe una transformación lineal $g : \mathbb{R}^n \to \mathbb{R}^n$ tal que $f \circ g$ ó $g \circ f$ sea isomorfismo?

9. Determinar si $y_1 = x^{-1/2} \cos x$ y $y_2 = x^{-1/2} \sin x$ forman un conjunto fundamental de soluciones de $x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right) y = 0$ en $0 < x < \infty$. En caso afirmativo determinar la solución general de

$$x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right) y = x^{3/2}$$

10. Sea $p : \mathbb{R} \to \mathbb{R}$ una función diferenciable. Probar que la ecuación $u_t = p(u)u_x$, con $t > 0$, tiene una solución de la forma $u = f(x + p(u)t)$, donde f es una función diferenciable.

11. Sea X una variable aleatoria discreta con

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x},$$

es decir $X \sim Bin(n, p)$. Calcula la varianza de X.

12. Si G es un grupo de orden par, demostrar que el número de sus elementos de orden 2 es impar.

Who invented 'Zero'?[1] Carbon dating of an ancient Indian document, the Bakhshali manuscript, has recently placed the first written occurrence of the number zero in the third or fourth century A.D., about 500 years earlier than previously believed. While the news has no practical bearing on the infrastructure of zeros (and ones) underlying our high-tech civilization, it does remind us how indebted we are for this invention. But to whom is this debt owed? And how should it be repaid?

Chauvinistic politicians might loudly trumpet India?s role (as they have, more controversially, in the case of the Pythagorean theorem), but the history of zero remains unsettled enough to still be the subject of continuing quests. The Babylonians used it as a placeholder, an idea later developed independently by the Mayans. The Chinese, at some point in time, indicated it by an empty space in their counting-rod system. Some claim the Greeks flirted with the idea but, finding the concept of the void too frightening in their Aristotelian framework, passed it on to the Indians. The Hindus are generally acknowledged as being the first to formulate it as an independent number ? the key to using it in mathematical calculations or binary code. What?s clear is that this history is dominated by non-European civilizations. Truly an alt-right nightmare.

Obviously, there were no intellectual property rights in force back then. Had there been a patent office, it might have ruled, as courts do now, that mathematical advances uncover pre-existing knowledge rather than create anything new ? and are hence unpatentable. The conundrum of whether mathematics is discovered or invented is as old as Plato. Certainly, zero displays this duality: The void is as old as time, but it was a human innovation to harness it with a symbol.

In recognition of this innovation, and ignoring all practicalities, suppose someone, somehow, had figured out how to put a price tag on zero. The royalties generated would be staggering ? imagine the tab for just your personal use alone! This might lead to a significant redistribution of wealth, most of it going to the developing world.

[1] https://www.nytimes.com/2017/10/07/opinion/sunday/who-invented-zero.html?rref=collection%2Ftimestopic%2FMathematics