

MAESTRÍA EN MATEMÁTICAS
FACULTAD DE MATEMÁTICAS
EXAMEN DE ADMISIÓN
INGRESO EN AGOSTO DE 2014



Universidad Veracruzana

Fecha: 30 de Mayo de 2014
Nombre:

Parte A

Instrucción.- Toda respuesta debe ser justificada.

1. Sea I un intervalo y $f : I \rightarrow \mathbb{R}$ monótona sobreyectiva. Probar que f es continua.
2. Si $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, expresa A^{-1} como un polinomio en A .
3. Sea (X, d) un espacio métrico. Mostrar que si $\{x_n\}$ y $\{y_n\}$ son sucesiones de Cauchy en X , entonces $\{d(x_n, y_n)\}$ converge en \mathbb{R} .
4. Usando multiplicadores de Lagrange, encontrar el valor extremo de $z = x^2 + y$ sujeto a la condición $x + y = 1$.
5. Supóngase que W_1, W_2 son subespacios vectoriales de un espacio vectorial V . Demostrar que $W_1 + W_2$ es un subespacio vectorial. ¿ La unión de W_1 y W_2 también es un subespacio?
6. Demostrar que
$$\operatorname{div} \operatorname{rot} F = 0,$$
donde $F(x, y, z) = f_1(x, y, z)\mathbf{i} + f_2(x, y, z)\mathbf{j} + f_3(x, y, z)\mathbf{k}$ y f_1, f_2, f_3 son funciones reales diferenciables en un conjunto abierto $A \subset \mathbb{R}^3$.
7. Demuestra o proporciona un contraejemplo de la siguiente afirmación: *La suma de dos matrices nilpotentes es a su vez una matriz nilpotente.*

Parte B

Instrucción.- Traduzca el siguiente párrafo o bien explique brevemente lo que usted comprenda del mismo:

The Art of M. C. Escher.

THERE IS an obvious but superficial sense in which certain kinds of art can be called mathematical art. Op art, for instance, is "mathematical," but in a way that is certainly not new. Hardedged, rhythmic, decorative patterns are as ancient as art itself, and even the modern movement toward abstraction in painting began with the geometric forms of the cubists. When the French Dadaist painter Hans Arp tossed colored paper squares in the air and glued them where they fell, he linked the rectangles of cubism to the globs of paint slung by the later "action" painters. In a broad sense even abstract expressionist art is mathematical, since randomness is a mathematical concept.

As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.

ALBERT EINSTEIN (1879-1955)

No one knows for sure when humans started to count, that is, to measure multitude in a quantitative way. In fact, we do not even know with certainty whether numbers like "one,two,three" (the cardinal numbers) preceded numbers like "first,second,third" (the ordinal numbers), or vice versa. Cardinal numbers simply determine the plurality of a collection of items, such as the number of children in a group. Ordinal numbers, on the other hand, specify the order and succession of specific elements in a group, such as a given date in a month or a seat number in a concert hall. Originally it was assumed that counting developed specifically to address simple day-to-day needs, which clearly argued for cardinal numbers appearing first.