

MAESTRÍA EN MATEMÁTICAS
FACULTAD DE MATEMÁTICAS
EXAMEN DE ADMISIÓN
INGRESO EN FEBRERO DE 2014



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Nombre:

Parte A

Instrucción.- Resolver 5 problemas.

1. Sea A una matriz cuyo polinomio característico es

$$p(x) = x^{500} + x^{100} - x + 4.$$

Demuestra que A es invertible.

2. Sea S la circunferencia unitaria en \mathbb{R}^2 y $f : S \rightarrow [0, 1]$ una función continua. Demostrar que f no puede ser biyectiva.
3. Sea $V = P_n[x]$ el espacio vectorial de polinomios de grado a lo más n con coeficientes reales. Sea T la transformación lineal en V dada por

$$T = \frac{d}{dx}.$$

Calcular la representación de T en la base $\mathcal{B} = \{p_0 = 1 + x, p_1 = 1 - x, p_2 = x^2, \dots, p_n = x^n\}$, y calcular el polinomio característico.

4. Si $f > 0$ es diferenciable en $[0, \infty)$ y f' decrece a cero, demostrar que las siguientes series convergen o divergen simultáneamente:

$$\sum_{n=1}^{\infty} f'(n),$$
$$\sum_{n=1}^{\infty} \frac{f'(n)}{f(n)}.$$

Utilice un criterio de la integral.

5. Sean A y B matrices $n \times n$ sobre el campo \mathbb{F} . Probar que si $I - AB$ es invertible, entonces $I - BA$ es invertible y

$$(I - BA)^{-1} = I + B(I - AB)^{-1}A.$$

Probar que AB y BA tienen los mismos valores propios distintos de cero.

6. Considere el tetraedro T generado por 0 y los tres vectores unitarios $e_1 = (1, 0, 0)$, $e_2 = (0, 1, 0)$ y $e_3 = (0, 0, 1)$. Encuentre el volumen de T usando integración triple.

Parte B

Instrucción.- Traduzca el siguiente párrafo o bien explique brevemente lo que usted comprenda del mismo:

Logic and Intuitive Minds. A Political Aspect of the Question. After having spoken of students, let us now deal with mathematicians themselves, able not only to understand mathematical theories, but also to investigate new ones. Not only do these differ from ordinary students, but they also profoundly differ from each other. A capital distinction has been emphasized: some mathematicians are “intuitive.” and others “logical.” Poincaré has dealt with that distinction and so has the German mathematician Klein.

Other Differences in Mathematical Minds. The above question is the only one which has been examined so far, concerning different kinds of mathematical minds; but, of course, there is no doubt that mathematicians can differ from each other from various other points of view. For instance, there exists a theory, the theory of groups, the importance of which, in our science, grew increasingly for more than one century, especially since the work of Sophus Lie at the end of the nineteenth century. Some mathematicians, especially contemporary ones, have improved it by most beautiful discoveries. Some others -I confess that I belong to the latter category- though being eventually able to use it for simple applications, feel insuperable difficulty in mastering more than a rather elementary and superficial knowledge of it. Psychological reasons for that difference, which seems to me incontestable, would be interesting to find.