Spectral kissing and its dynamical consequences in the squeezed Kerr-nonlinear oscillator

Lea F. Santos

Department of Physics, University of Connecticut, Storrs, CT, USA

arXiv:2210.07255

Jorge Chávez

Talía Lezama

Rodrigo Cortiñas

"Curro"
Francisco Pérez-Bernal

Miguel Prado

Jorge Hirsch

Plan

1) Generalized survival probability and multifractality

2) Ground state energy distributions

3) Quantum speed limit

4) Experimental platform for ESQPT: Kerr nonlinear oscillator
(shared with Jorge: Dynamics)

Kerr + linear part (shared with Miguel)

Driven Kerr and driven Dicke model
(shared with Jorge: Driven oscillator)

Generalized Survival Probability
&
Multifractality

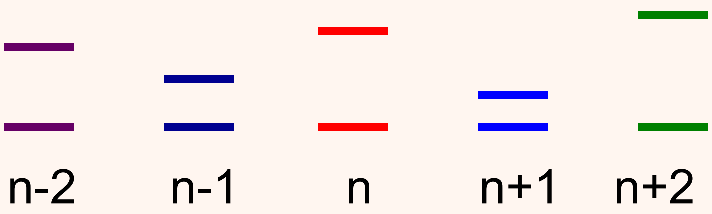
Model

The 1D XXZ model with onsite disorder: spin1/2 model.

$$H = \sum_{n=1}^{L} \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L} \frac{J}{4} \left[\sigma_n^z \sigma_{n+1}^z + \left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y \right) \right]$$

Ising interaction

Flip-flop term



Random numbers

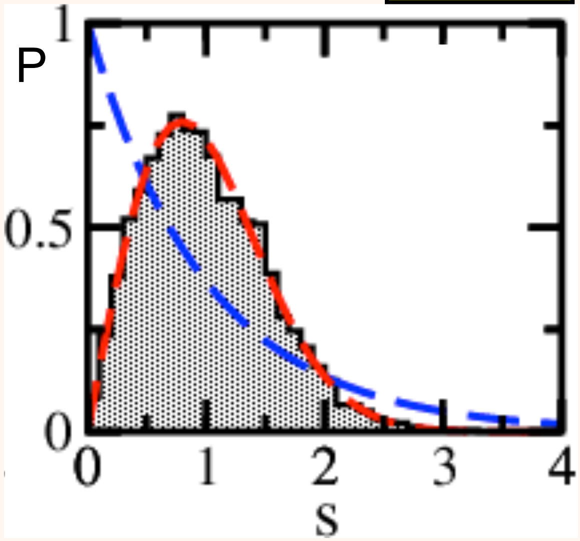
$h_n \in [-h, h]$

h: disorder strength

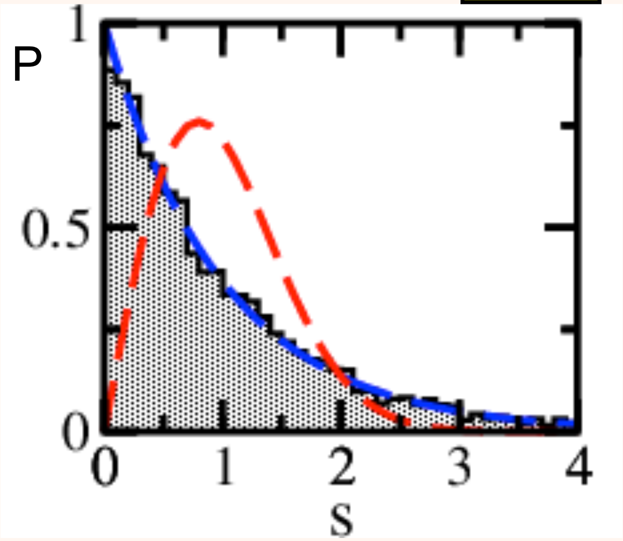
From Wigner-Dyson to Poisson

$$H = \sum_{n=1}^{L} \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L} \frac{J}{4} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \sigma_n^z \sigma_{n+1}^z) \qquad h_n \in [-h, h]$$

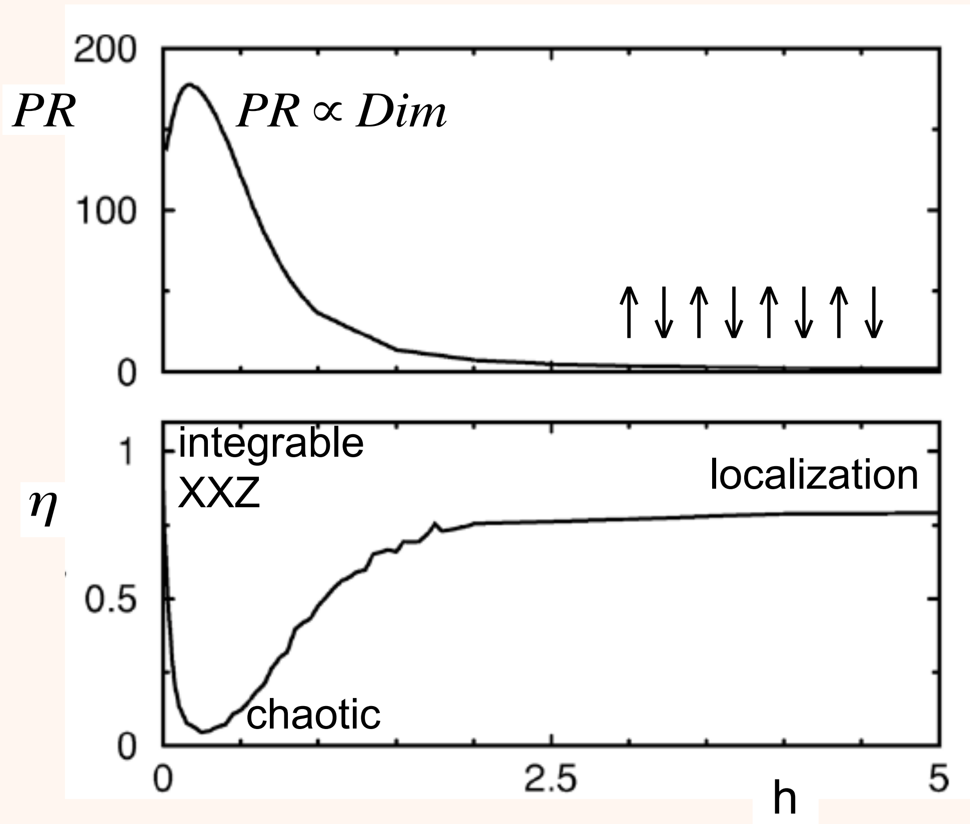
Chaotic h=0.5J



Localized $h > h_c$



Localization and entanglement



Participation Ratio

$$PR^{(\alpha)} \equiv \frac{1}{\sum_{i=1}^{D} |c_i^{(\alpha)}|^4}$$

Localization in real space

$$|\alpha\rangle = \begin{pmatrix} C^{(1)} \\ C^{(2)} \\ C^{(3)} \\ C^{(4)} \\ C^{(5)} \\ \cdots \end{pmatrix} \begin{matrix} |\uparrow\uparrow\uparrow \ldots \downarrow\downarrow\downarrow \ldots\rangle \\ |\uparrow\downarrow\uparrow \ldots \downarrow\downarrow\downarrow \ldots\rangle \\ |\uparrow\uparrow\downarrow \ldots \downarrow\downarrow\downarrow \ldots\rangle \end{matrix}$$

Inverse Participation Ratio

$$IPR_\alpha = \sum_{n=1}^{Dim} |C_\alpha^{(n)}|^4$$

LFS, Rigolin, Escobar
Entanglement versus chaos in disordered spin chains
PRA **69**, 042304 (2004)

Maximum Delocalization

Eigenstates of Full Random Matrices: GOE (real and symmetric)

$$H|\alpha\rangle = E_\alpha|\alpha\rangle$$

$$|\alpha\rangle = \begin{pmatrix} C^{(1)} \\ C^{(2)} \\ C^{(3)} \\ C^{(4)} \\ C^{(5)} \\ \dots \end{pmatrix}$$

Random vectors:

coefficients are random numbers from a Gaussian distribution (normalization)

Participation Ratio

$$PR_\alpha \equiv \frac{1}{\sum_{n=1}^{Dim} |C_\alpha^{(n)}|^4}$$

$$PR_\alpha = \frac{Dim}{3}$$

Inverse Participation Ratio

$$IPR_\alpha = \frac{3}{Dim}$$

FRACTAL

$$PR_\alpha \propto Dim^{D_2}$$

Lea F. Santos

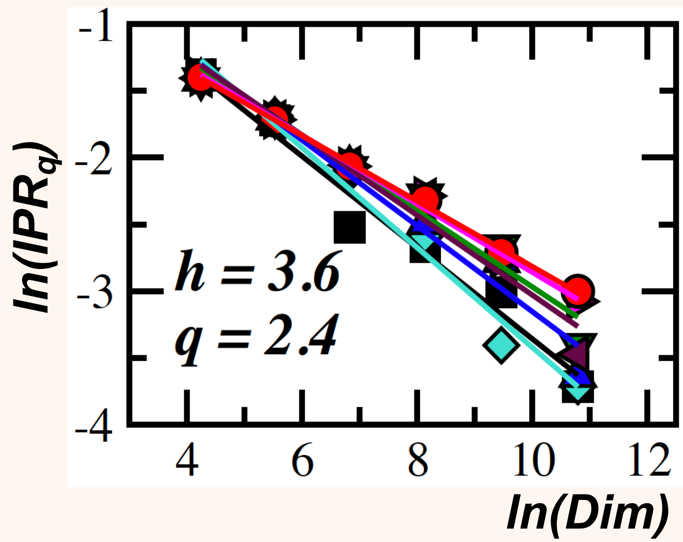
Multifractality

$$IPR_q^\alpha = \sum_k |C_k^\alpha|^{2q}$$

generalized inverse participation ratio

$$\langle IPR_q \rangle \propto Dim^{-(q-1)D_q}$$

D_q : generalized dimension



Fully delocalized states: $D_q = 1$

Multifractal states: $0 < D_q < 1$

(nonlinear dependence of D_q on q)

Localized states: $D_q = 0$

Multifractality

$$H = \sum_{n=1}^{L} \frac{h_n}{2}\sigma_n^z + \sum_{n=1}^{L} \frac{J}{4}\left[\sigma_n^z\sigma_{n+1}^z + \left(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y\right)\right]$$

$$IPR_q^\alpha = \sum_k |C_k^\alpha|^{2q}$$

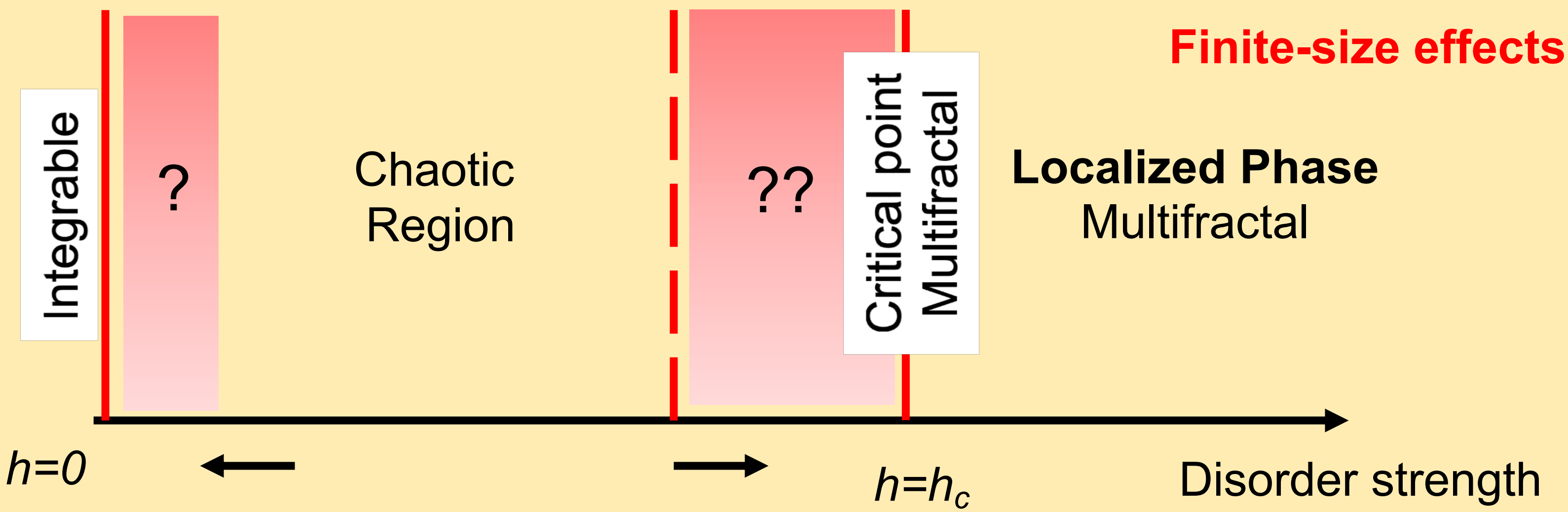
$$\langle IPR_q \rangle \propto Dim^{-(q-1)D_q}$$

Fully delocalized states: $D_q = 1$

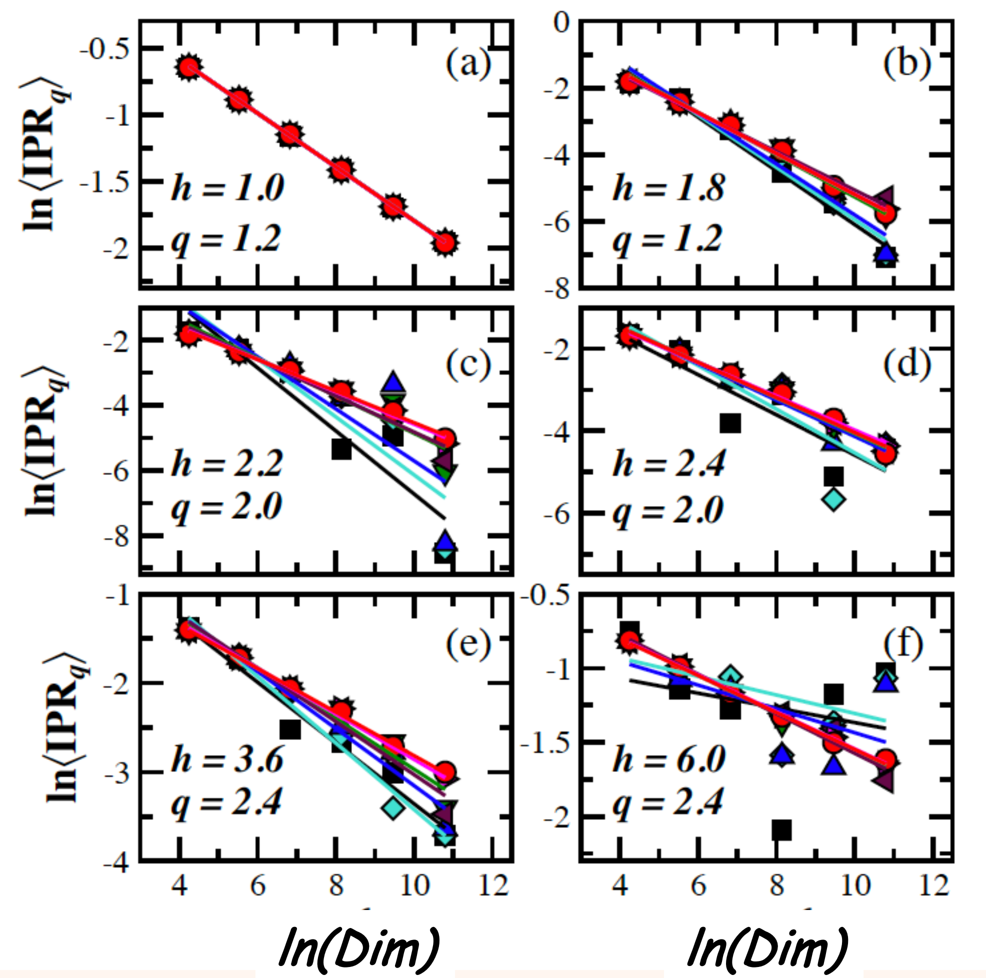
Multifractal states: $1 < D_q < 0$

Localized states: $D_q = 0$

Is there a region of multifractality before the critical point?



Lack of Self-Averaging



10^2 (black squares)

5×10^2 (turquoise diamonds)

1×10^3 (blue up triangles)

5×10^3 (green down triangles)

1×10^4 (maroon left triangles)

2×10^4 (magenta right triangles)

3×10^4 (red circles)

QUESTION:
Analysis in other bases?

Solórzano, LFS, Torres-Herrera
Phys. Rev. Research 3, 032030 (2021)

Survival Probability

Survival Probability
Return Probability
Fidelity

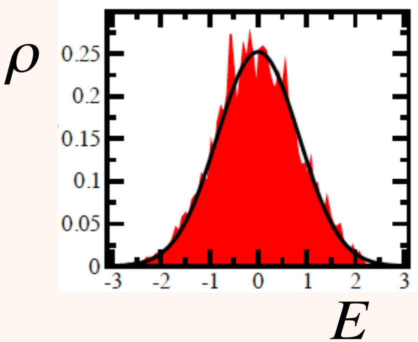
$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$$

$$\left|\Psi(0)\right\rangle = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$$

$$C_\alpha^{ini} = \langle \alpha | \Psi(0) \rangle$$

$$SP(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \left|\sum_\alpha \left|C_\alpha^{ini}\right|^2 e^{-iE_\alpha t}\right|^2 \cong \left|\int \rho_{ini}(E) e^{-iEt}\, dE\right|^2$$

Energy distribution of the initial state $\boldsymbol{\rho}_{ini}(E) = \sum_{\boldsymbol{\alpha}} \left|C_{\boldsymbol{\alpha}}^{ini}\right|^2 \boldsymbol{\delta}(E - E_{\boldsymbol{\alpha}})$



LDOS

D_2 in one-body system

$$SP(t) \propto t^{-D_2/d}$$

Huckstein & Klesse
PRB **59**, 9714 (1999)

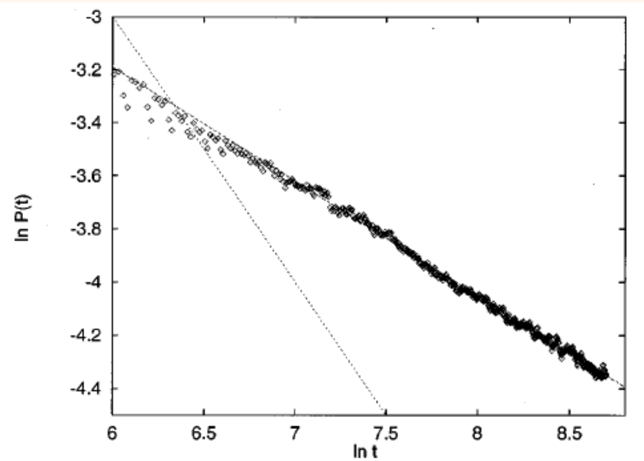


FIG. 2. Return probability for a wave packet at the Anderson transition in $d=3$. The solid line is a best fit to the data with slope $D_2/3=0.43\pm0.04$. The dashed line shows the conventional behavior $p(t)\propto t^{-3/2}$.

Ketzmerick & Geisel
PRL **69**, 695 (1992)

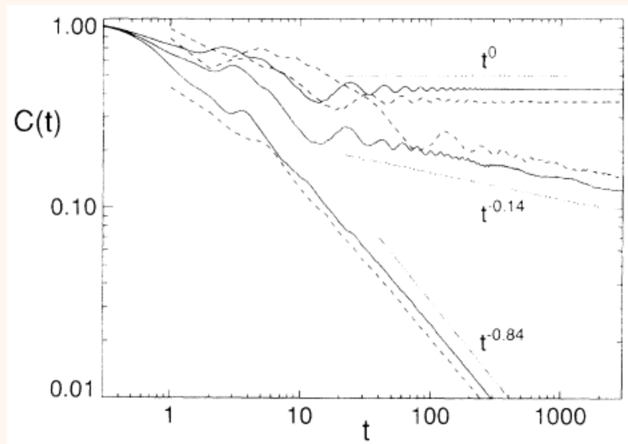


FIG. 1. Correlation function in the Harper model (solid lines) for $\sigma=1597/2584$, an approximant of the golden mean, displaying power laws $C(t)\sim t^{-\delta}$ with $\delta=0.84\pm0.01$, $\delta=0.14\pm0.01$, and $\delta=0$ for $\lambda=1$, 2, and 3, respectively. For the kicked Harper model (dashed lines) with σ the golden mean, the same asymptotic behavior is obtained for $K=6$ and $L=3$, 6, and 9 corresponding to the extended, critical, and localized regimes, respectively.

$$\psi_{n+1}+\psi_{n-1}+\lambda\cos(2\pi n\sigma-\varphi_0)\psi_n=\omega\psi_n$$

Mirlin,
Kravtsov,
Chalker, etc

Power-law Exponent and D_2

$$SP(t) = \left| \sum_{\alpha} |C_{\alpha}^{ini}|^2 e^{-iE_{\alpha} t} \right|^2 \qquad \langle SP(t) \rangle = \langle \sum_{\alpha,\beta} |C_{\alpha}^{ini}|^2 |C_{\beta}^{ini}|^2 e^{-i(E_{\alpha}-E_{\beta})t} \rangle$$

$$\langle SP(t) \rangle = \int G(E) e^{-iEt} dE$$

$$G(E) = \left\langle \sum_{\alpha\beta} |C_{\alpha}^{ini}|^2 |C_{\beta}^{ini}|^2 \delta(E_{\alpha} - E_{\beta} - E) \right\rangle$$

$$G(E \to 0) \propto E^{D_2 - 1}$$

$$\langle SP(t) \rangle = \propto t^{-D_2}$$

J. Phys. A: Math. Theor. **44** (2011) 305003 V E Kravtsov *et al*

behavior as well. The simplest correlation function involving two eigenstates corresponding to two different energies E_m and E_n can be defined as

$$C(\omega) = \sum_{\mathbf{r}} \langle |\psi_n(\mathbf{r})|^2 |\psi_m(\mathbf{r})|^2 \delta(E_m - E_n - \omega) \rangle. \quad (2)$$

As any other correlator at criticality $C(\omega)$ is expected to decay in a power-law fashion

$$C(\omega) \propto (E_0/\omega)^{\mu}, \quad \Delta \ll \omega \ll E_0, \quad (3)$$

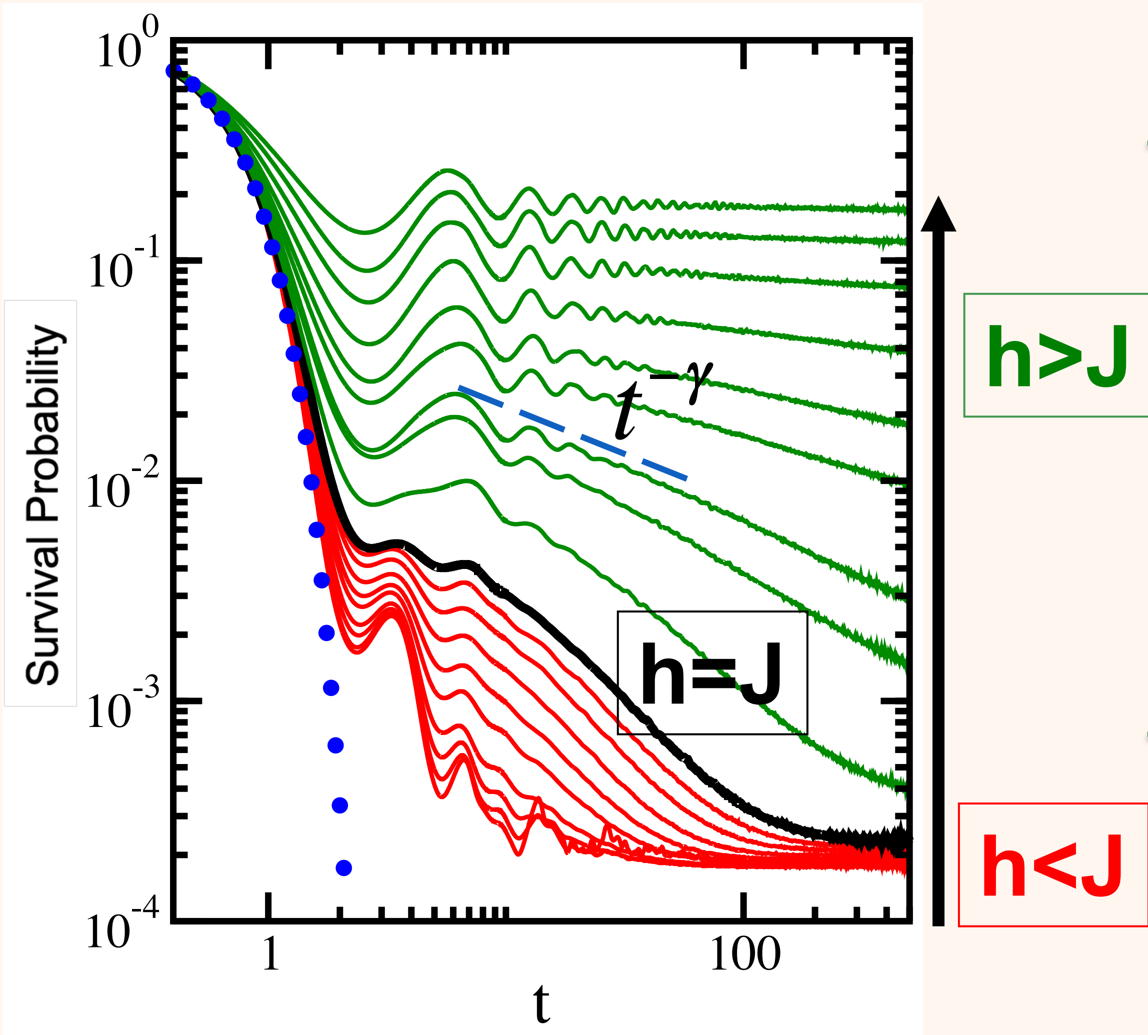
where Δ is the mean level spacing and E_0 is a high-energy cutoff. What is more surprising is the fact that the dynamical exponent μ is related to the fractal dimension d_2 in a simple way

$$\mu = 1 - d_2/d. \quad (4)$$

This relation was suggested by Chalker and Daniel [2, 3] and confirmed by a great number of computer simulations [2, 4, 5] thereafter. As $E_0/\omega \gg 1$ and $\mu > 0$, equation (3) implies an

Lea F. Santos

Power-law Exponent and D_2



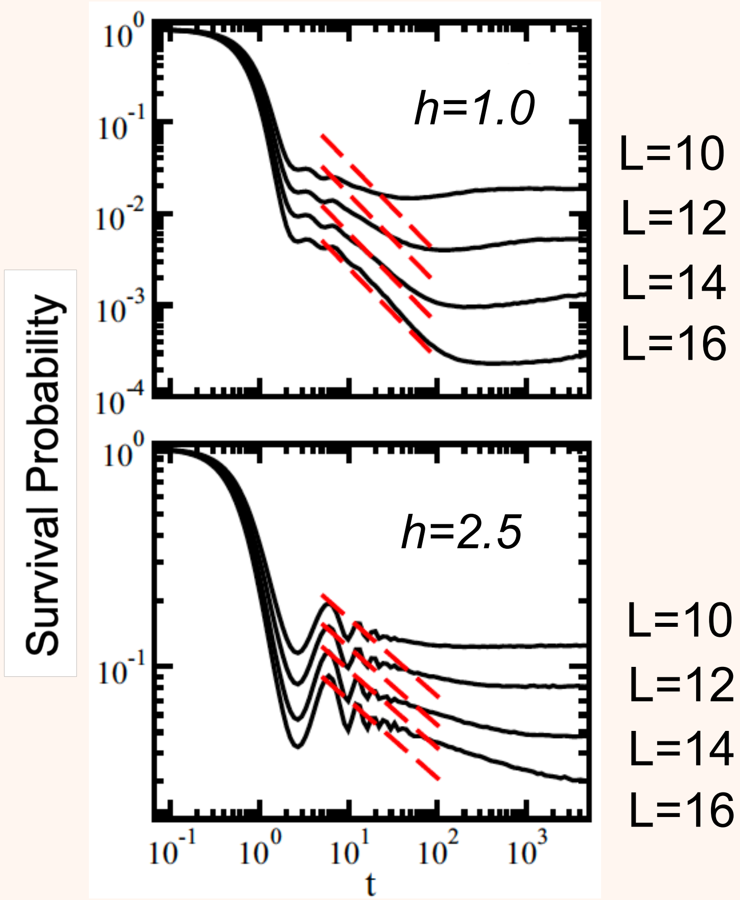
Power-law exponent coincides with the generalized dimension D_2

$$t^{-\gamma}$$ $$\gamma < 1$$

$$PR \propto Dim^{D_2}$$

Torres & LFS
PRB **92**, 01420 (2015)

Power-law Exponent and D_2



Torres & LFS
PRB **92**, 01420 (2015)

Power-law exponent coincides with the generalized dimension D_2

$$t^{-\gamma}$$

$$PR^{(\alpha)} \propto Dim^{D_2}$$

ADVANTAGE:

When studying dynamics, we can deal with larger systems, than when we analyze the structure of the eigenstates.

Generalized Survival Probability

$$SP_q(t) = \frac{1}{\mathcal{N}_q^2} \left| \sum_{\alpha=1}^{\mathcal{D}} |C_\alpha^{(0)}|^q e^{-iE_\alpha t} \right|^2 = \left| \int \rho_q(E) e^{-iE_\alpha t} dE \right|^2$$

$$\mathcal{N}_q = \sum_{\alpha=1}^{\mathcal{D}} |C_\alpha^{(0)}|^q$$

Generalized LDOS

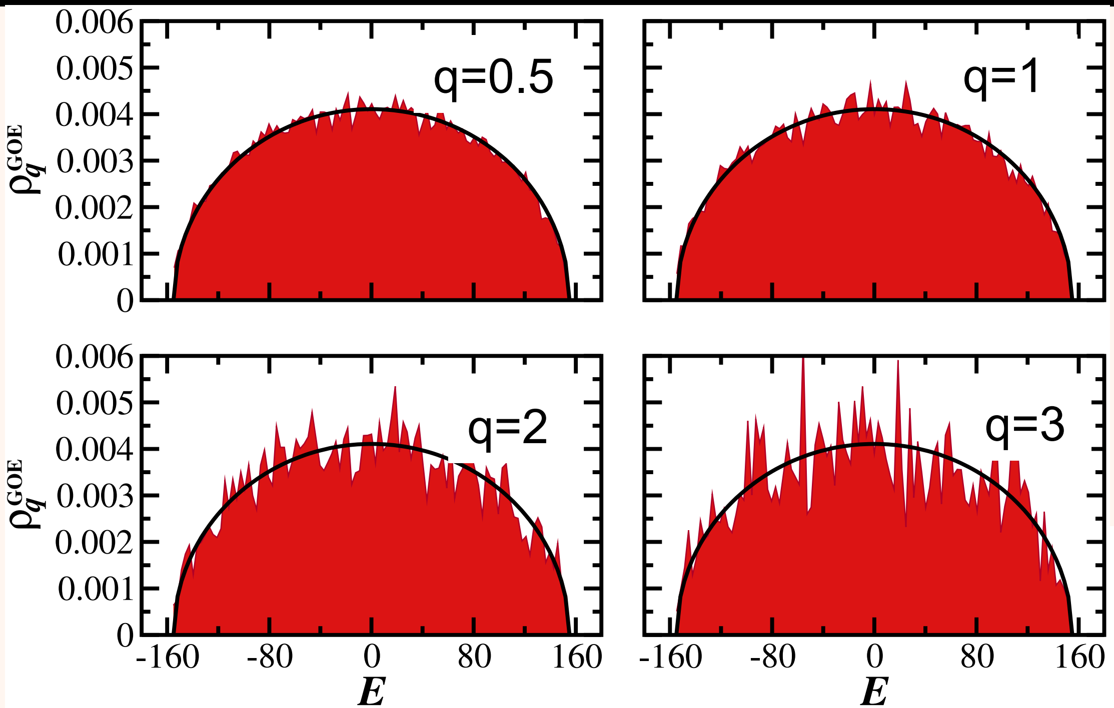
$$\rho_q(E) = \frac{1}{\mathcal{N}_q} \sum_{\alpha=1}^{\mathcal{D}} |C_\alpha^{(0)}|^q \delta(E_\alpha - E)$$

$$E_q^{(0)} = \frac{1}{\mathcal{N}_q} \sum_{\alpha=1}^{\mathcal{D}} |C_\alpha^{(0)}|^q E_\alpha \quad \text{and} \quad \sigma_q^2 = \frac{1}{\mathcal{N}_q} \sum_{\alpha=1}^{\mathcal{D}} |C_\alpha^{(0)}|^q (E_\alpha - E_q^{(0)})^2.$$

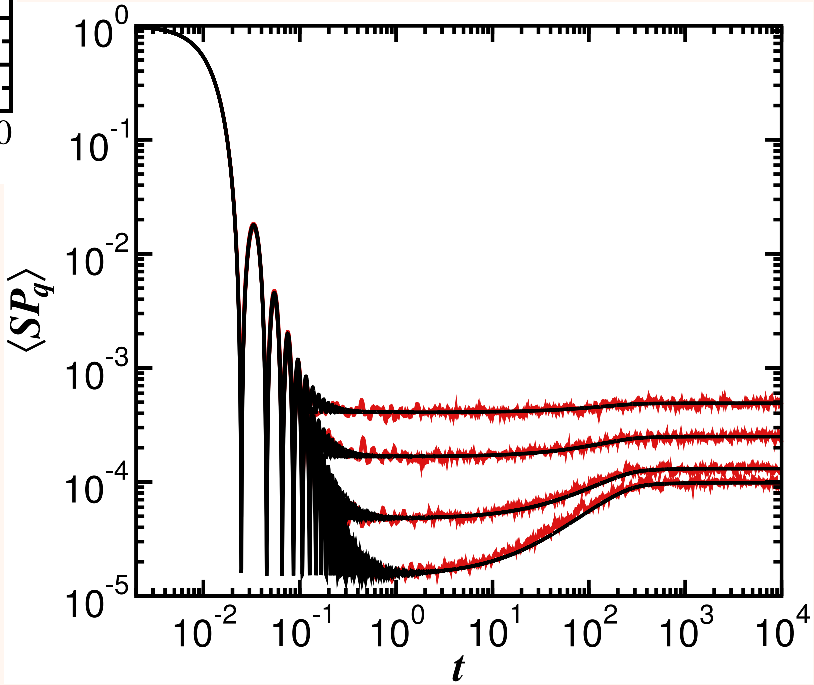
q=0: $SP_q(t) = SFF(t)$

Getting analytical results for the $SP_q(t)$ using GOE…

Generalized Survival Probability: GOE



Generalized LDOS
does not depend on q

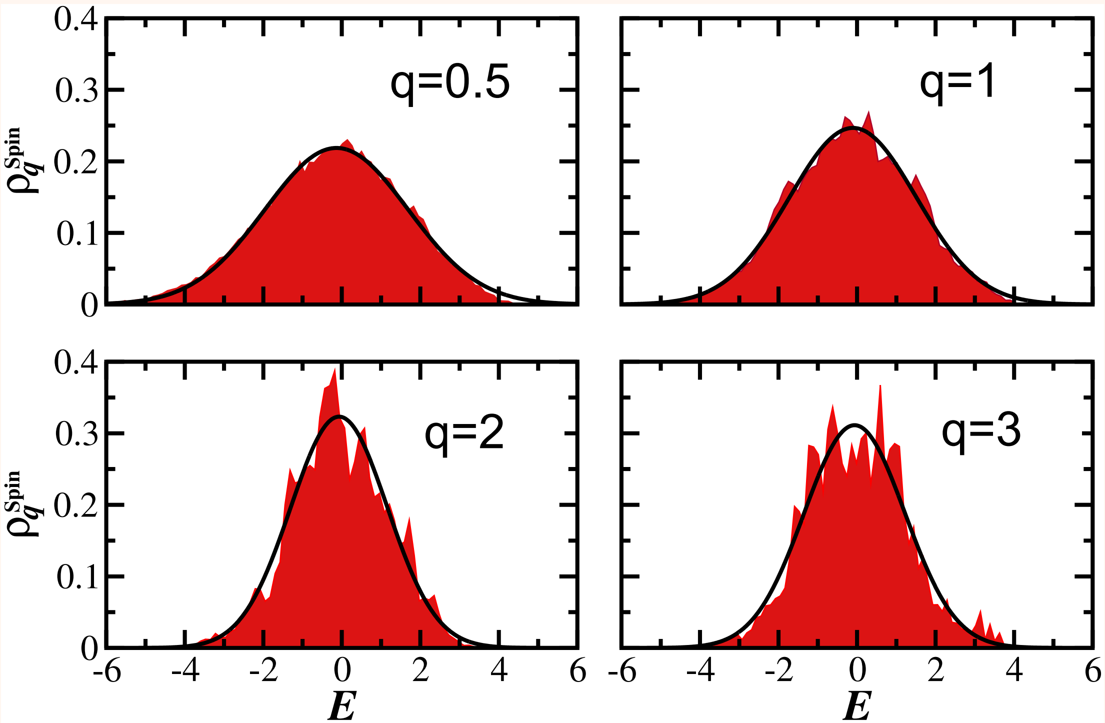


One can then state that the robustness of the generalized LDoS for different values of q is a sign of the ergodicity of the eigenstates of the system.

Lea F. Santos

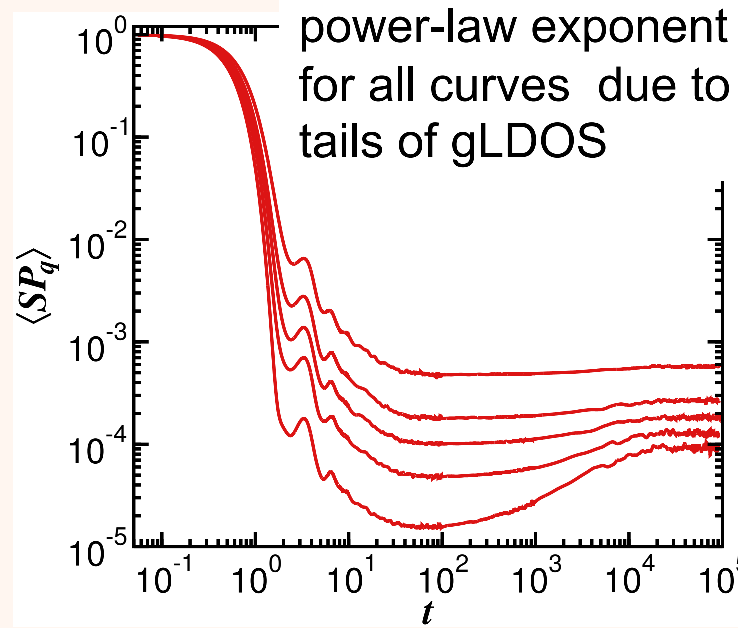
Generalized Survival Probability: Chaotic Spin Model

The dependence of the width of the gLDoS on q reveals the limited degree of ergodicity of physical systems, even deep in the chaotic regime



h=0.5

We expect the same power-law exponent for all curves due to tails of gLDOS

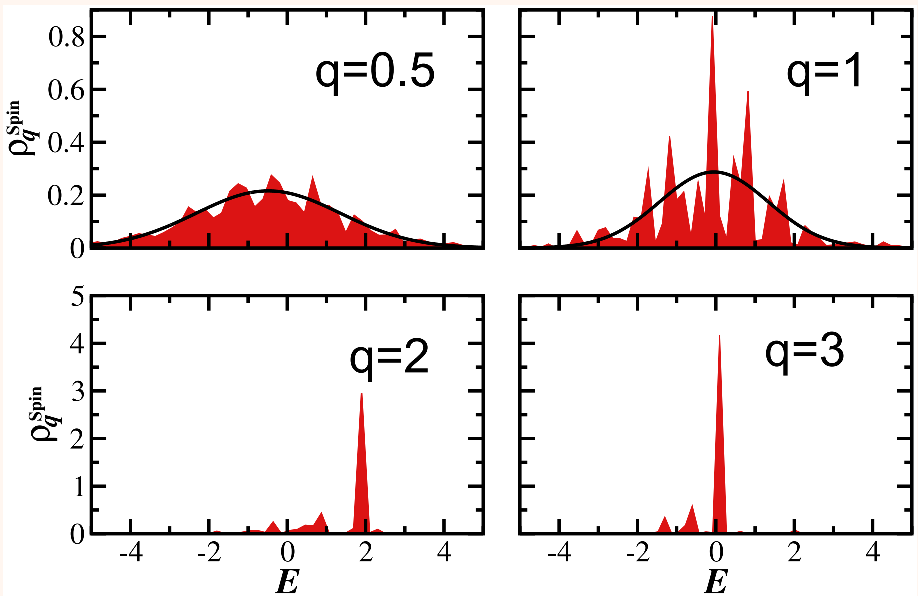


The components at the tails of the initial-state energy distribution, where chaotic states are nonexistent, get erased.

Lea F. Santos

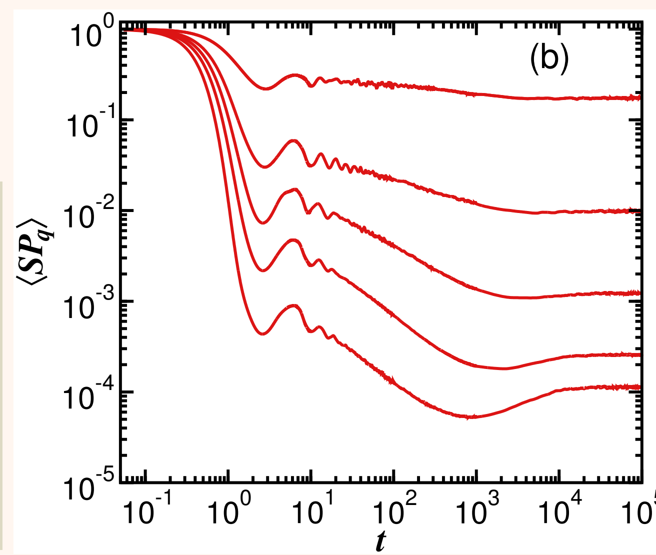
Generalized Survival Probability: Spin Model for h=2

For finite-size systems, several numerical studies supported that the eigenstates of the disordered spin model should become multifractal in its transition to the many-body localized phase, although this has not been confirmed in the thermodynamic limit.



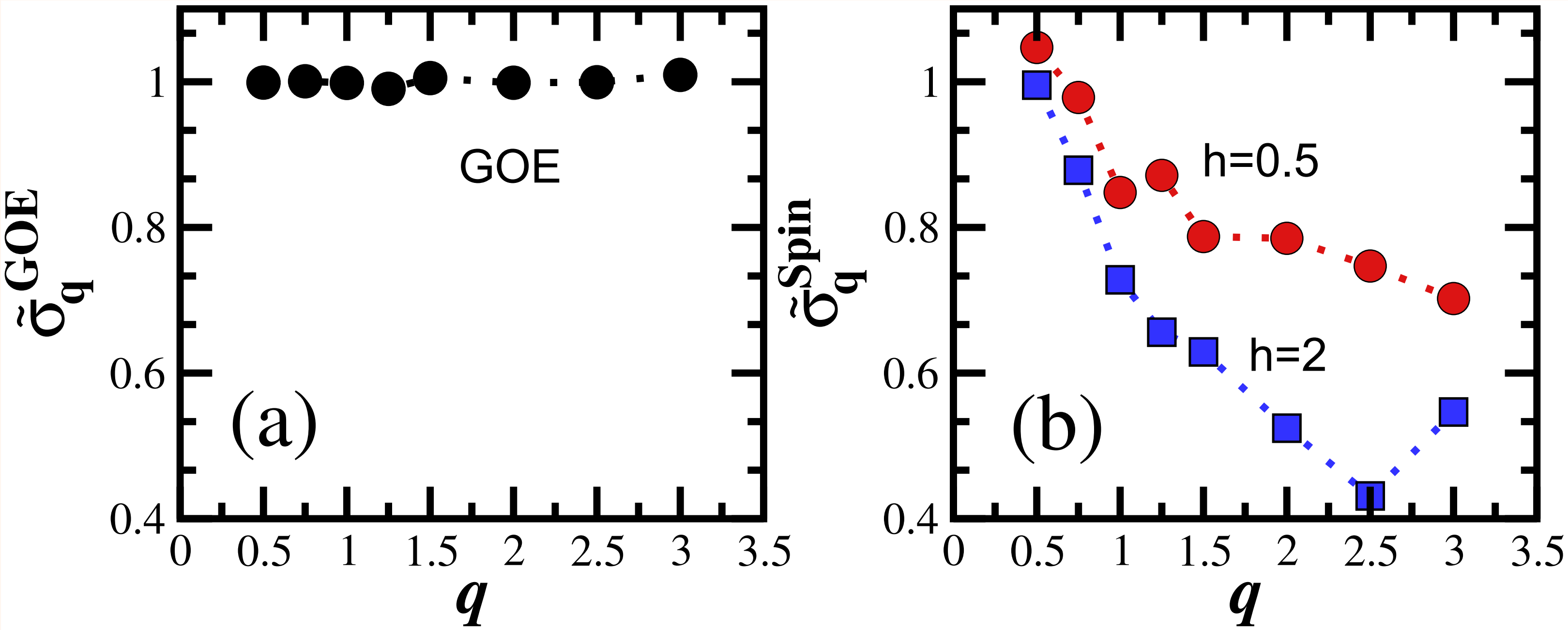
The patterns observed in (a)-(b) suggest fractality.

One may be able to get D_q from SP_q (right).
It is clear that γ_q decreases as q increases and the minimum of the correlation hole takes longer to be reached. In this case, the power-law behavior reflects the correlations among the components of the initial state, which get enhanced for larger values of q.



Width of the gLDOS

The results suggest that even the WIDTH of the gLDOS can say something about multifractality!



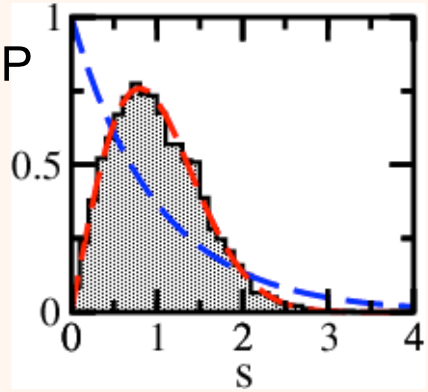
$$D_q \ \ vs \ \ \sigma_q$$

*) Analysis for various values of h
*) Analysis also of the power-law decay
*) Analysis of the time to reach the hole

Discussion: SP for Dicke averaged over initial states; disordered Bose-Hubbard, models with long-range couplings …

Distribution of ground-state energy

Ground state energy: Nuclear Physics



- bulk of the spectrum
- How about the **distribution of the lowest energy level**?

Extreme-value statistics concerns the study of **rare events**, such as tsunamis, floods, earthquakes, and large variations in the stock market. It has been employed in the context of the Griffiths phase, in the study of the fluctuations of the smallest (largest) eigenvalue of random matrices (applications in analyses of the stability of dynamical systems with interactions and of the equilibrium properties of disordered systems at low temperatures)

To understand the predominance of 0^+ ground states.

VOLUME 87, NUMBER 2 PHYSICAL REVIEW LETTERS 9 JULY 2001

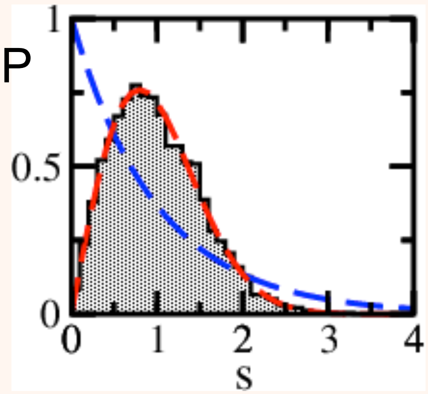
Comment on "Two-Body Random Ensembles: From Nuclear Spectra to Random Polynomials"

R. Bijker[1] and A. Frank[1,2]

Kusnezov
Zelevinsky

An investigation of the spectroscopy of even-even nuclei with random one- and two-body interactions has led to the surprising result that the ground state has $J^P = 0^+$ in typically 60%–70% of the cases. This holds for both the nuclear shell model [1,2] and the interacting boson model [3]. To understand this phenomenon it was suggested that one should study the distribution of the lowest eigenvalues

Tracy-Widom distribution



Our results:
Agreement with random matrix theory for the energy levels in the bulk of the spectrum does **NOT** imply the same for the ground-state energy distribution!

Tracy-Widom

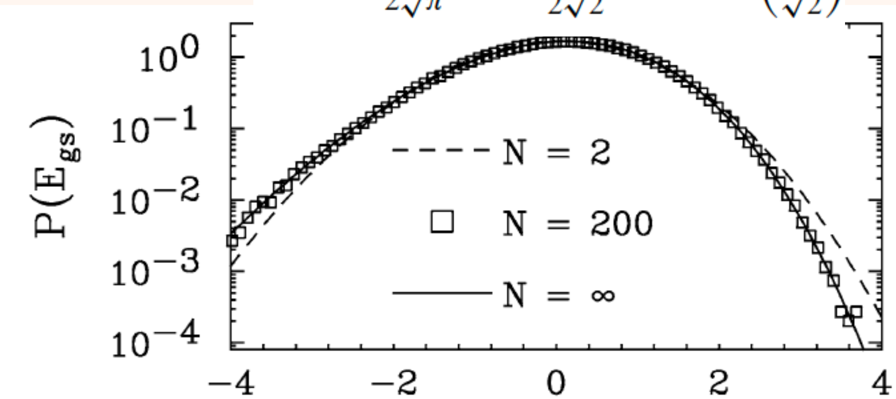
$$P(E_0) = \sqrt{F_2(-E_0)} \exp\left[\frac{1}{2}\int_{-E_0}^{\infty} q(x)dx\right], \qquad (3)$$

with

$$F_2(x) = \exp\left[-\int_{x}^{\infty}(z-x)q^2(x)dz\right], \qquad (4)$$

where $q(x)$ is the solution of the Painlevé II differential equation $q'' = xq + 2q^3$ subjected to the boundary condition $q(x) \sim \mathrm{Ai}(x)$ for $x \to \infty$, with $\mathrm{Ai}(x)$ denoting the Airy function.

2x2 GOE: $P(E_0) = \frac{1}{2\sqrt{\pi}}e^{-E_0^2} - \frac{1}{2\sqrt{2}}e^{-\frac{1}{2}E_0^2}E_0\mathrm{erfc}\left(\frac{E_0}{\sqrt{2}}\right)$

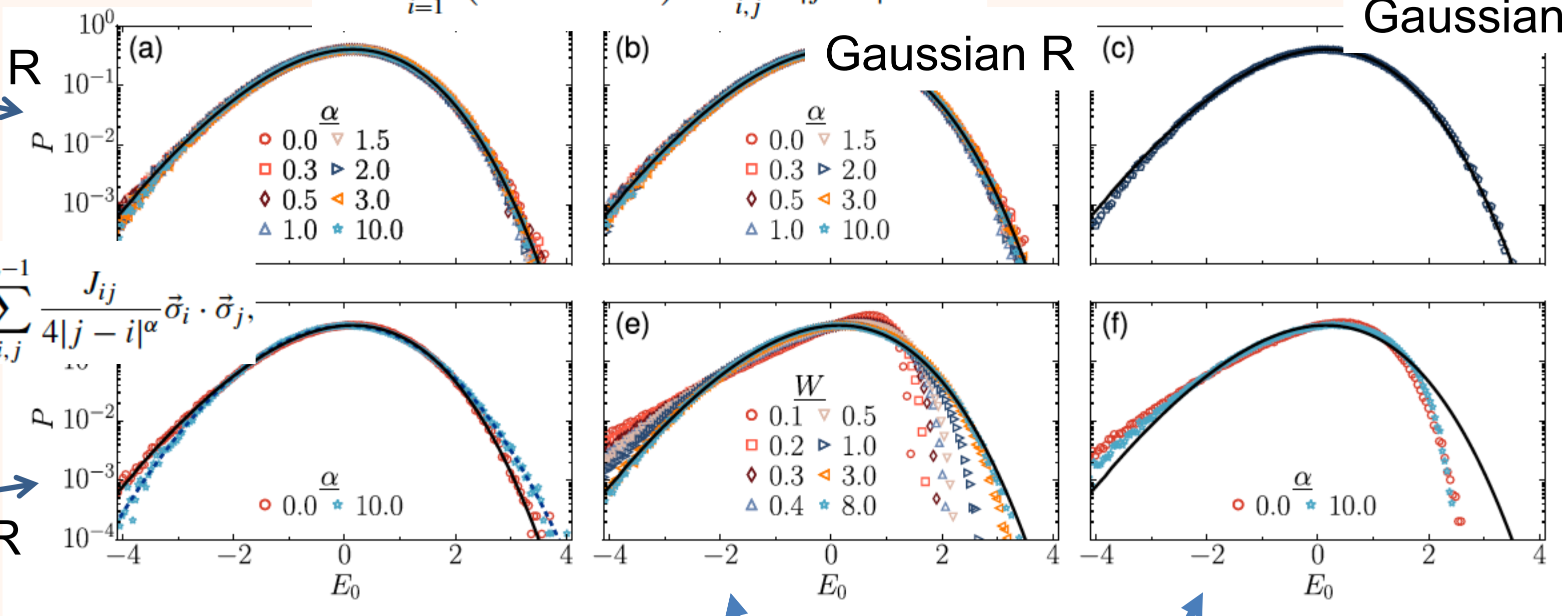


Tracy-Widom distribution in physical models

$$H = \sum_{i=1}^{L}\left(\frac{h_i^z}{2}\sigma_i^z + \frac{h_i^x}{2}\sigma_i^x\right) + \sum_{i,j}^{L-1}\frac{J_{ij}}{4|j-i|^\alpha}\sigma_i^z\sigma_j^z,$$

$$H = \sum_{i=1}^{L-2} J_{i+1}P_i\sigma_{i+1}^x P_{i+2} + J_1\sigma_1^x P_2 + J_L P_{L-1}\sigma_L^x,$$

$$H = \sum_{i=1}^{L}\frac{\epsilon_i}{2}\sigma_i^z + \sum_{i,j}^{L-1}\frac{J_{ij}}{4|j-i|^\alpha}\vec{\sigma}_i\cdot\vec{\sigma}_j,$$



$$H = W\sum_{i=1}^{L}\frac{h_i}{2}\sigma_i^z + \frac{J}{4}\sum_{i=1}^{L-1}\vec{\sigma}_i\cdot\vec{\sigma}_{i+1}.$$

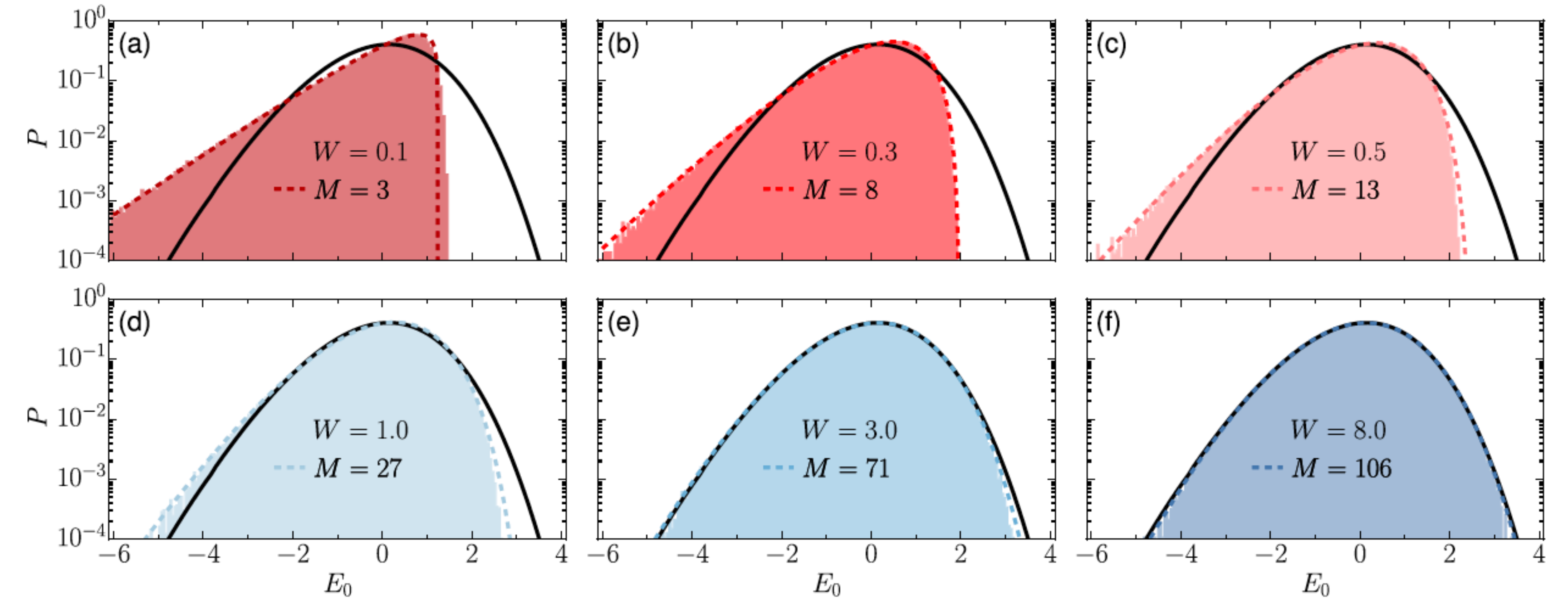
$$H = \sum_{i=1}^{L}\frac{U_i}{2}n_i(n_i-1) - \sum_{i,j}^{L-1}\frac{J_{i,j}}{|j-i|^\alpha}(a_i^\dagger a_j + a_j^\dagger a_i),$$

FIXED (not random) coupling.
DEVIATIONS from Tracy-Widom and from 2x2GOE

Why? Which other models deviate? Random Dicke? Few vs Many degrees…

Chi-square distribution

$$P(E_0) = \frac{1}{2^{M/2}\Gamma(M/2)} e^{-E_0/2} E_0^{M/2-1} \qquad (E_0 \geqslant 0)$$



Just as the Brody and Izrailev distributions reproduce any of the level spacing distributions between Poisson and Wigner-Dyson, the χ2 distribution captures any of the lowest-energy distributions, including TW, 2x2GOE and Gaussian.

$$H = W \sum_{i=1}^{L} \frac{h_i}{2} \sigma_i^z + \frac{J}{4} \sum_{i=1}^{L-1} \vec{\sigma}_i \cdot \vec{\sigma}_{i+1}.$$

Open Questions

Why are there deviations?

Why does the chi-square distribution capture most of E_0-distributions for disordered many-body quantum systems? Does it work for systems with few degrees of freedom (random Dicke)?

What happens to the structure of the ground states? Are they multifractal? Is the structure of the eigenstates related with the kind of E_0-distribution that we get?

How about the prevalence of 0^+? Is there an analog for spin models?

Analog of 0^+ in spin models?

L=10 sites

Total dimension : 2^(10) = 1024

$$\sum_i h_i \sigma_i^z + \sum_{i<j} J_{ij} \sigma_i^z \sigma_j^z$$

SzT = -10 Occurrences = 0.1 %

SzT = -8 Occurrences = 0.89 %

SzT = -6 Occurrences = 4.44 %

SzT = -4 Occurrences = 11.81 %

SzT = -2 Occurrences = 20.96 %

SzT = 0 Occurrences = 24.79 %

SzT = 2 Occurrences = 20.46 %

SzT = 4 Occurrences = 11.38 %

SzT = 6 Occurrences = 4.24 %

SzT = 8 Occurrences = 0.85 %

SzT = 10 Occurrences = 0.08 %

Sector SzT=-10 means all spins pointing down

Number of states: 1

(same for the Sector SzT=+10)

Percentage:

```
(1. / 1024) × 100
0.0976563
```

Sector SzT=-8 means 9 spins pointing down and 1 up

Number of states: 10

(same for the Sector SzT=+8)

Percentage:

```
(10 / 1024.) × 100
0.976563
```

Sector SzT=0 means 5 spins pointing down and 5 up

Number of states: 252

Percentage:

```
10! / 5!^2
(252 / 1024.) × 100
252
24.6094
```

Lea F. Santos

Analog of 0^+ in spin models?

L=10 sites
Total dimension : 2^(10) = 1024

$$\sum_i h_i \sigma_i^z + \sum_{i<j} J_{ij} (\sigma_i^z \sigma_j^z + \sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y)$$

SzT = -10 Occurrences = 0.1 %
SzT = -8 Occurrences = 0.89 %
SzT = -6 Occurrences = 4.44 %
SzT = -4 Occurrences = 11.81 %
SzT = -2 Occurrences = 20.96 %
SzT = 0 Occurrences = 24.79 %
SzT = 2 Occurrences = 20.46 %
SzT = 4 Occurrences = 11.38 %
SzT = 6 Occurrences = 4.24 %
SzT = 8 Occurrences = 0.85 %
SzT = 10 Occurrences = 0.08 %

Sz=0 sector gets enhanced.

Why?

SzT = -10 Occurrences= 0.0 %
SzT = -8 Occurrences= 0.08 %
SzT = -6 Occurrences= 1.2 %
SzT = -4 Occurrences= 7.86 %
SzT = -2 Occurrences= 23.79 %
SzT = 0 Occurrences= 35.02 %
SzT = 2 Occurrences= 23.55 %
SzT = 4 Occurrences= 7.3 %
SzT = 6 Occurrences= 1.12 %
SzT = 8 Occurrences= 0.07 %
SzT = 10 Occurrences= 0.01 %

Quantum Speed Limit

https://www.quantphys.com/2022/01/mandelstam-tamm-uncertainty-relation.html

Mandelstam-Tamm energy-time uncertainty relation

Derivation of the Mandelstam-Tamm energy-time relation

$$\Delta E \Delta t_A \geq \frac{\hbar}{2}$$

Δt_A time required for a significant change of the expectation value of an observable A

$$\Delta E = \sqrt{\langle H^2 \rangle - \langle H \rangle^2}\,,$$
$$\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}\,,$$

Assume that H and A do not commute

$$\Delta E \Delta A \geq \frac{|\langle HA - AH \rangle|}{2}$$

Since the rate of change of the expectation value of A is

$$\frac{d\langle A \rangle}{dt} = \frac{i}{\hbar}\langle HA - AH \rangle$$

We have that

$$\Delta E \Delta A \geq \frac{\hbar}{2}\left|\frac{d\langle A \rangle}{dt}\right|$$

Defining

$$\Delta t_A = \left|\frac{d\langle A \rangle}{dt}\right|^{-1} \Delta A$$

Quantum Speed Limit

$$\Delta E \Delta A \geq \frac{\hbar}{2}\left|\frac{d\langle A\rangle}{dt}\right|$$

$$\Delta E = \sqrt{\langle H^2\rangle - \langle H\rangle^2}\,,$$
$$\Delta A = \sqrt{\langle A^2\rangle - \langle A\rangle^2}\,,$$

change of the expectation value of an observable A

$$\langle A\rangle = \langle\Psi(t)|A|\Psi(t)\rangle$$

Since $\langle H\rangle = \langle\Psi(t)|H|\Psi(t)\rangle = \sum_{\alpha,\beta} C_\alpha C_\beta^* e^{iE_\beta t} e^{-iE_\alpha t}\langle\beta|H|\alpha\rangle = \sum_\alpha |C_\alpha|^2 E_\alpha$

ΔE is the uncertainty in the energy of the initial state = width of the LDOS = Γ

Let's choose A = projection into the initial state $A = |\Psi(0)\rangle\langle\Psi(0)|$

Therefore $\langle A\rangle = \langle\Psi(t)|\Psi(0)\rangle\langle\Psi(0)|\Psi(t)\rangle = SP(t)$

$$\langle A^2\rangle = \langle\Psi(t)|\Psi(0)\rangle\langle\Psi(0)|\Psi(0)\rangle\langle\Psi(0)|\Psi(t)\rangle = SP(t)$$

Quantum Speed Limit

$$\Delta E \Delta A \geq \frac{\hbar}{2} \left| \frac{d\langle A \rangle}{dt} \right|$$

$$\Delta E = \sqrt{\langle H^2 \rangle - \langle H \rangle^2} \,,$$

$$\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2} \,,$$

ΔE is the uncertainty in the energy of the initial state = width of the LDOS = Γ

$$\langle A \rangle = \langle \Psi(t) | \Psi(0) \rangle \langle \Psi(0) | \Psi(t) \rangle = SP(t)$$

$$\langle A^2 \rangle = \langle \Psi(t) | \Psi(0) \rangle \langle \Psi(0) | \Psi(0) \rangle \langle \Psi(0) | \Psi(t) \rangle = SP(t)$$

$$\Gamma \sqrt{SP - SP^2} \geq \frac{1}{2} \left| \frac{dSP}{dt} \right|$$

Doing the integral

$$\arccos(\sqrt{SP(t)}) \geq \Gamma t \Rightarrow SP(t) \geq \cos^2(\Gamma t)$$

d>1, Effectively Break the Chain

integrable

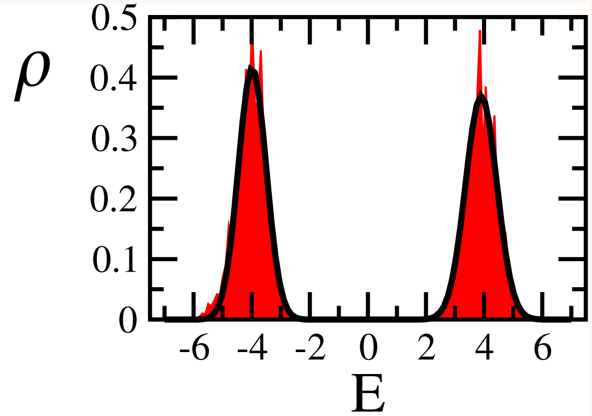
chaotic (impurity model)

$$H_{initial} = H_{XXZ} = \sum_{n=1}^{L-1} J(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + \Delta S_n^z S_{n+1}^z) \longrightarrow H_{final} = H_{XXZ} + dJS_{L/2}^z$$

$d > 1$ breaks the chain

$$|\psi_\alpha\rangle = c_1|1001\rangle + c_2|0101\rangle$$

DENSITY OF STATES



$$|\psi_\alpha\rangle = c_1|0011\rangle + c_2|0110\rangle$$

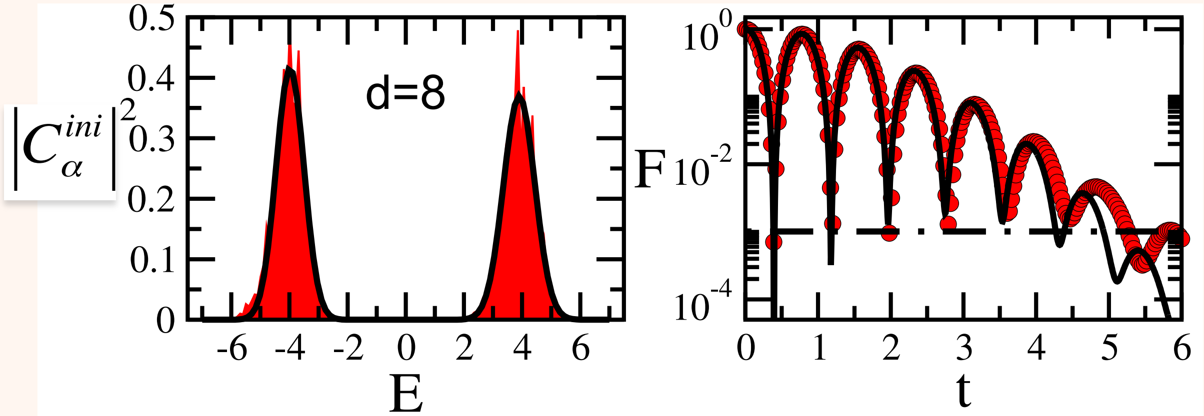
Quantum Speed Limit

integrable — chaotic

$$H_{initial} = H_{XXZ} = \sum_{n=1}^{L-1} J(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + \Delta S_n^z S_{n+1}^z) \longrightarrow H_{final} = H_{XXZ} + dJS_{L/2}^z$$

$$F(t) = \cos^2(dt/2)\exp(-\sigma^2 t^2)$$

LDOS



L=16, 8 up spins $\Delta = 0.48$

Torres & LFS
PRA **90** (2014)

Can we get a more realistic and experimentally feasible bimodal distribution for the LDOS in the Dicke model or other experimental model?

Superconducting Circuits
(Squeezed Kerr Nonlinear Oscillator)

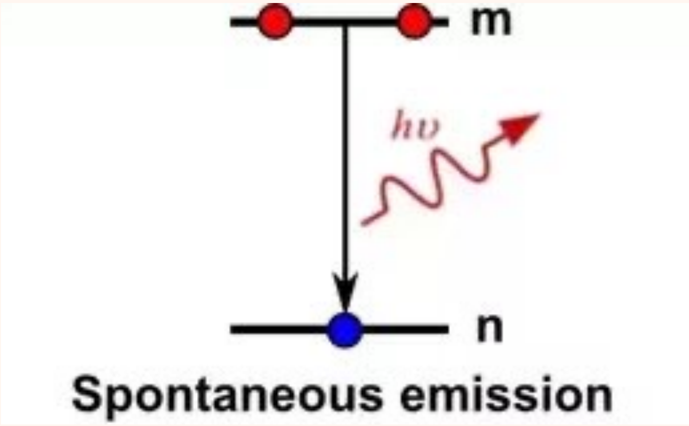
https://www.youtube.com/watch?v=-7b-ngIgcyw

arXiv:2210.07255

Spontaneous Emission

SPONTANEOUS EMISSION:

Excited atom discharges its excess energy in the form of photons that escape to infinity at the speed of light.
(as uncontrollable and as irreversible as the explosion of fireworks)



Atom is embedded in vacuum fluctuations (atom-vacuum system).

Electron is coupled with the quantized electromagnetic field of the vacuum.

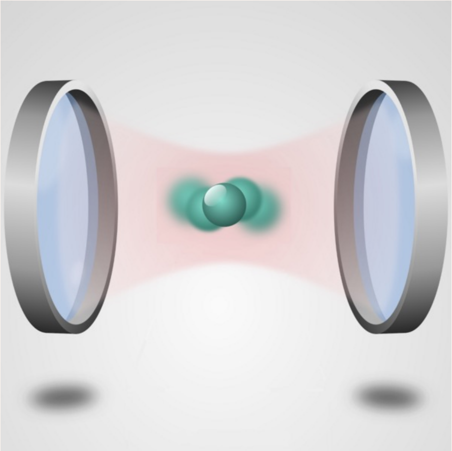
Photon has many vacuum modes to propagate into.

Cavity QED
(cavity quantum electrodynamics)

SPONTANEOUS EMISSION:

We can control and manipulate spontaneous emission by placing the atom in a small box with reflecting walls (optical/microwave CAVITY)

CAVITY QUANTUM ELECTRODYNAMICS (QED):
is the study/control of the interaction between light and matter (atom/particle) confined in a cavity.



Circuit QED
(circuit quantum electrodynamics)

CIRCUIT QUANTUM ELECTRODYNAMICS (QED):

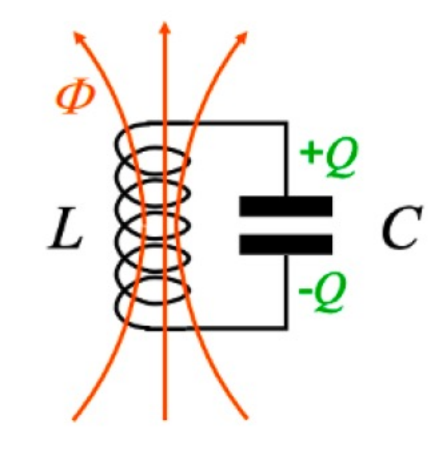
was inspired by atomic cavity QED;

is the study of the interaction between light and matter;

light = microwave photons (quantized electromagnetic fields)
(photons stored in high-quality coplanar waveguide resonators);

artificial atom = nonlinear superconducting circuit.

LC Circuit



Small circuits: "coordinate" Φ and the "momentum" Q become noncommuting quantum observables

$$[\Phi, Q] = i\hbar$$

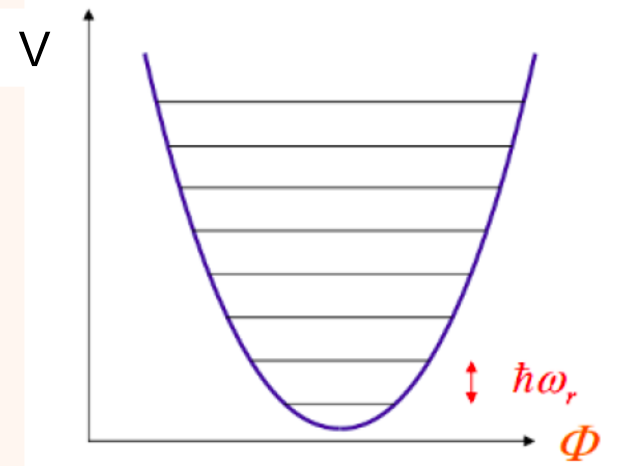
$$\hat{\Phi} = \Phi_{\text{zpf}}(\hat{a}^\dagger + \hat{a})$$

$$\hat{Q} = iQ_{\text{zpf}}(\hat{a}^\dagger - \hat{a})$$

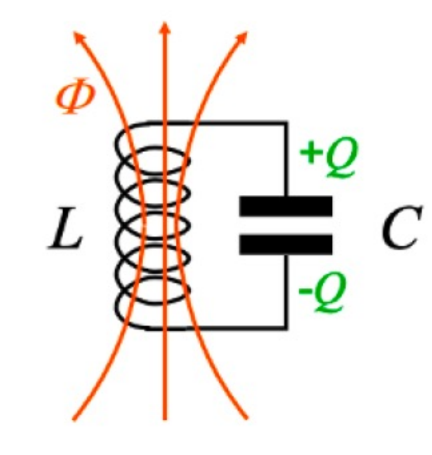
$$\hat{H}_{LC} = \hbar\omega_r(\hat{a}^\dagger\hat{a} + 1/2)$$

$$H_{LC} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

Quantum harmonic oscillator



LC Circuit



Small circuits: “coordinate” Φ and the “momentum” Q become noncommuting quantum observables

$$[\Phi, Q] = i\hbar$$

Low temperature: circuit is a superconducting, resistance vanishes, electrons -> Cooper pairs.

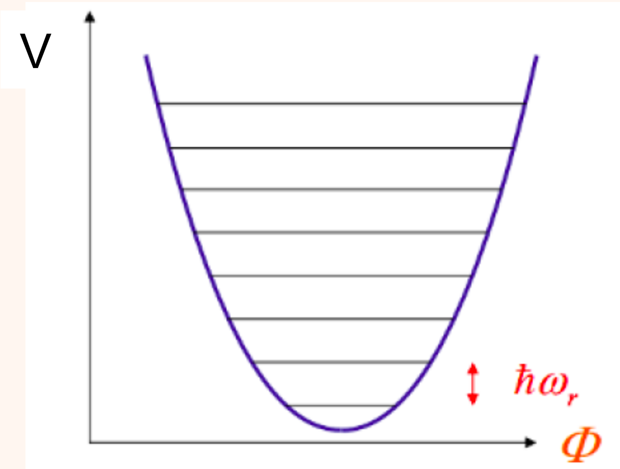
$$H_{LC} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

$$\hat{\Phi} = \Phi_{\mathrm{zpf}}(\hat{a}^\dagger + \hat{a})$$

$$\hat{Q} = iQ_{\mathrm{zpf}}(\hat{a}^\dagger - \hat{a})$$

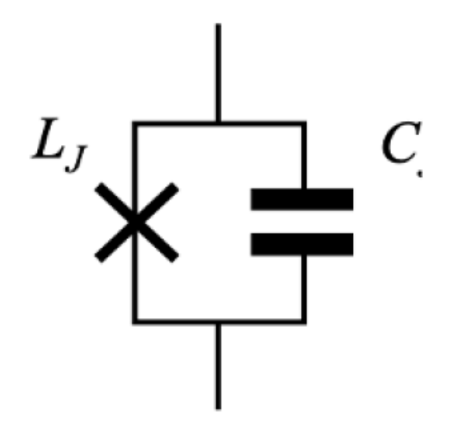
$$\hat{H}_{LC} = \hbar\omega_r(\hat{a}^\dagger\hat{a} + 1/2)$$

Quantum harmonic oscillator

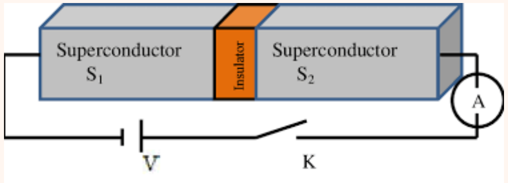


Non-addressable energy levels

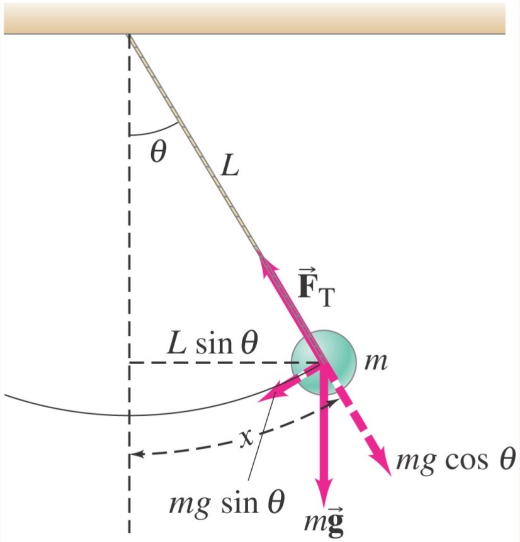
Anharmonic LC Circuit



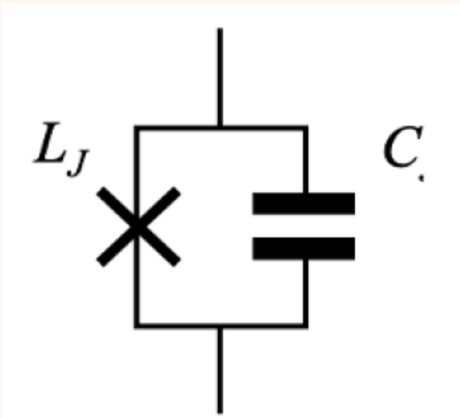
Josephson junction is a nonlinear circuit element



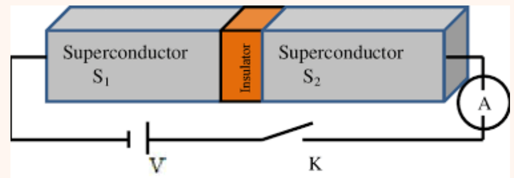
$$\hat{H} = \frac{1}{2C}\hat{Q}^2 - E_J \cos\left(\frac{2\pi}{\Phi_0}\hat{\Phi}\right)$$



Anharmonic LC Circuit

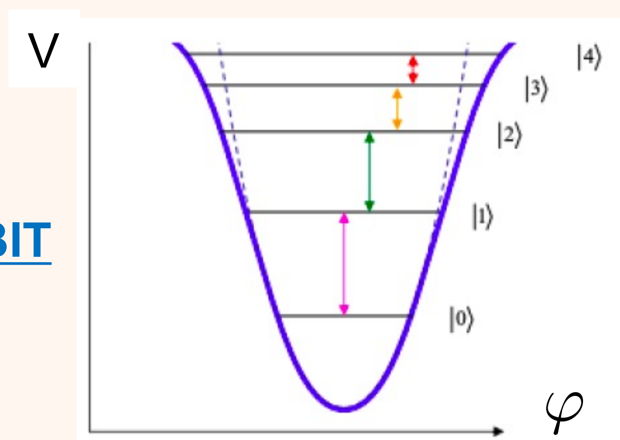


Josephson junction is a nonlinear circuit element



$$\hat{H} = \frac{1}{2C}\hat{Q}^2 - E_J \cos\left(\frac{2\pi}{\Phi_0}\hat{\Phi}\right)$$

TRANSMON QUBIT

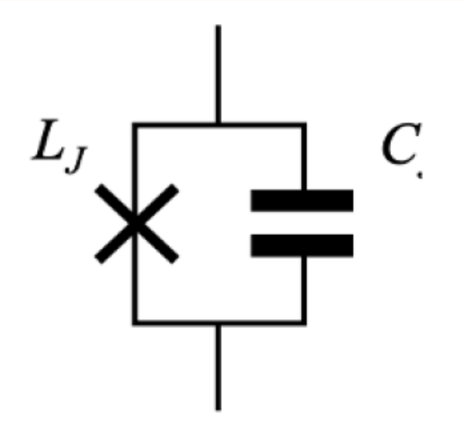


Quantum **AN**harmonic oscillator (pendulum)

Addressable energy levels

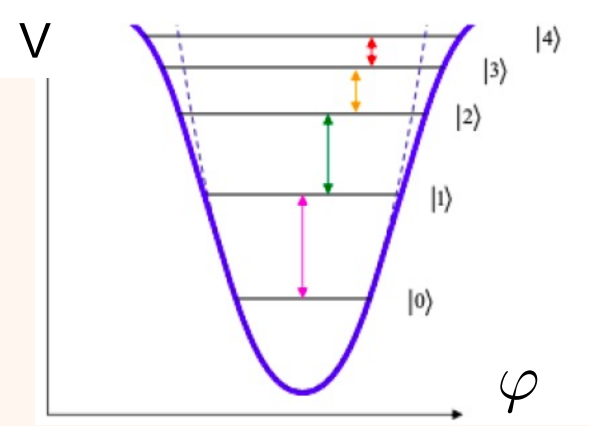
It allows for **selective** transitions between energy levels

Kerr Nonlinear Oscillator



$$\hat{H} = \frac{1}{2C}\hat{Q}^2 - E_J \cos\left(\frac{2\pi}{\Phi_0}\hat{\Phi}\right)$$

$$\hat{Q} = iQ_{\text{zpf}}(\hat{a}^\dagger - \hat{a}) \qquad \hat{\Phi} = \Phi_{\text{zpf}}(\hat{a}^\dagger + \hat{a})$$



$$-K\hat{a}^{\dagger 2}\hat{a}^2$$

Squeezed Kerr-Nonlinear Oscillator

In the case of the SNAIL transmon, the Hamiltonian of the driven circuit, which is built by an arrangement of a few Josephson junctions, reads

$$\hat{H}(t)/\hbar = \omega_o \hat{a}^\dagger \hat{a} + \sum_{m=3}^{\infty} \frac{g_m}{m} (\hat{a} + \hat{a}^\dagger)^m - i\Omega_d(\hat{a} - \hat{a}^\dagger)\cos\omega_d t$$

microwave drive

$$\hat{\mathcal{H}}(t) = \omega_o \hat{a}^\dagger \hat{a} + \frac{g_3}{3}(\hat{a} + \hat{a}^\dagger)^3 + \frac{g_4}{4}(\hat{a} + \hat{a}^\dagger)^4 - i\Omega_d(\hat{a} - \hat{a}^\dagger)\cos\omega_d t,$$

rotating frame

Static effective Hamiltonian

$$\hat{H} = \Delta \hat{a}^\dagger \hat{a} - K \hat{a}^{\dagger 2} \hat{a}^2 + \epsilon_2 (\hat{a}^{\dagger 2} + \hat{a}^2)$$

Kerr nonlinearity

Squeezing amplitude

Squeezed Kerr-Nonlinear Oscillator

$\Delta = 0$

Jorge's talk

$$\frac{\hat{H}}{\hbar} = -K\hat{a}^{\dagger 2}\hat{a}^2 + \epsilon_2(\hat{a}^{\dagger 2} + \hat{a}^2)$$

Jorge will also talk about
1) Our ongoing studies of the original driven system (with Diego Wisniacki)
2) Our idea of adding time dependence to the effective H (with Curro)

$\Delta \neq 0$

Miguel's talk

$$\hat{H} = \Delta\hat{a}^{\dagger}\hat{a} - K\hat{a}^{\dagger 2}\hat{a}^2 + \epsilon_2(\hat{a}^{\dagger 2} + \hat{a}^2)$$

Miguel will also talk about
the parallel with the double-well Bose-Hubbard model (with Jorge Hirsch)

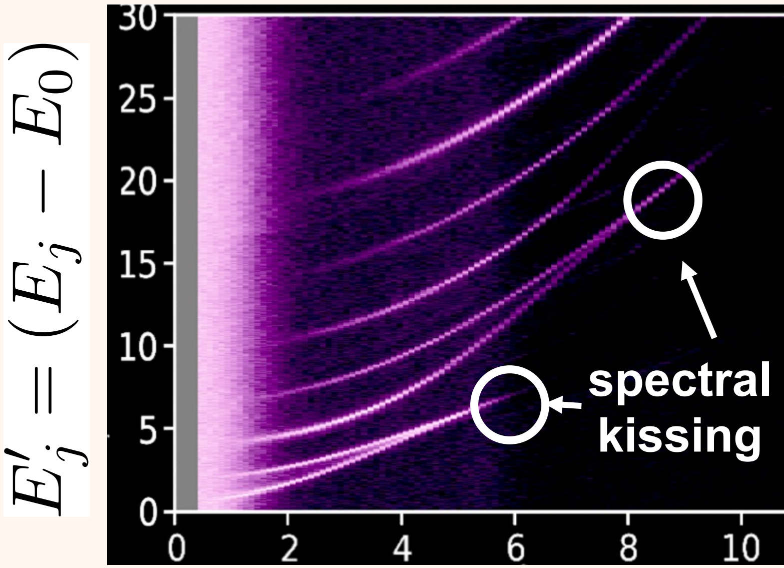
Squeezed Kerr-Nonlinear Oscillator

arXiv: 2209.03934

$$\frac{\hat{H}}{\hbar} = -K\hat{a}^{\dagger 2}\hat{a}^2 + \epsilon_2(\hat{a}^{\dagger 2} + \hat{a}^2)$$

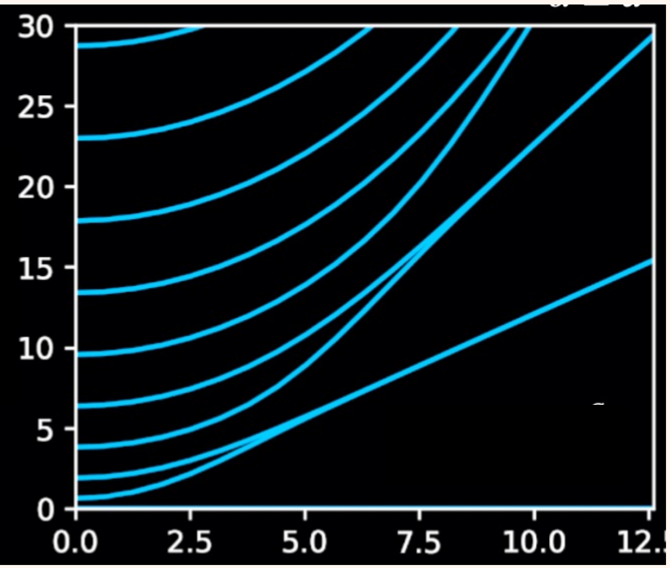
K $= 0.319\,\mathrm{MHz}$

EXPERIMENT



ϵ_2/K

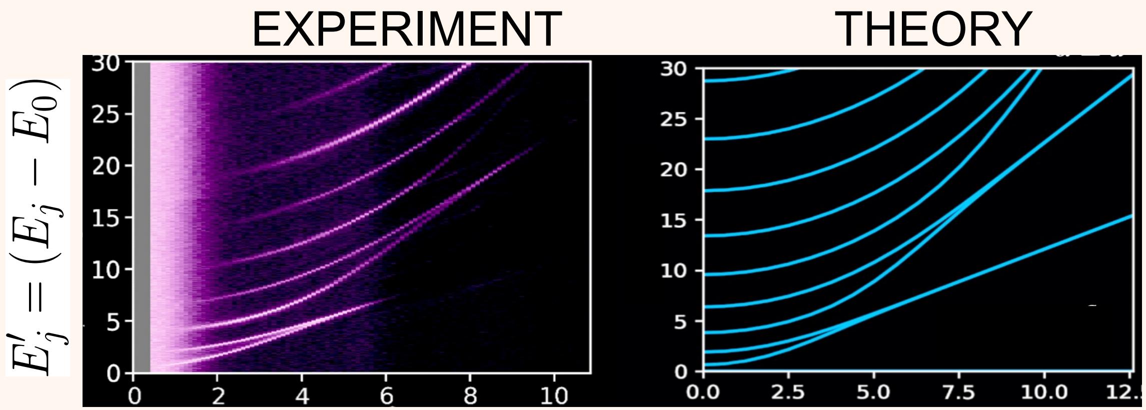
THEORY

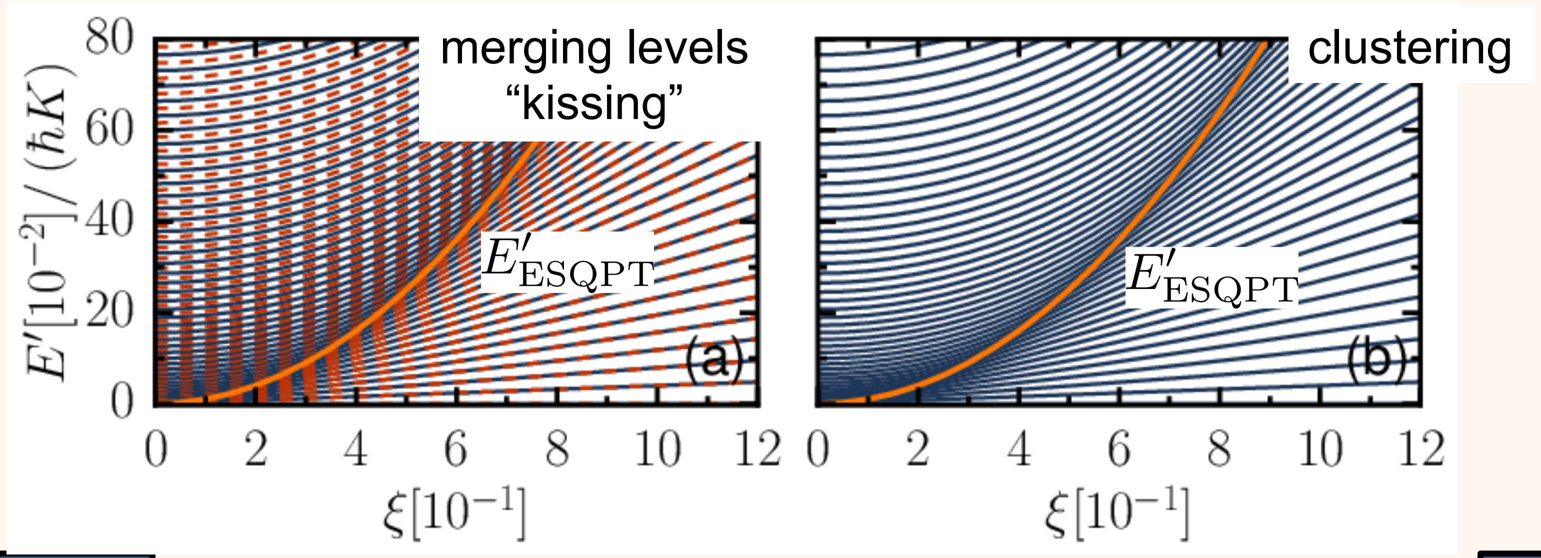


ϵ_2/K

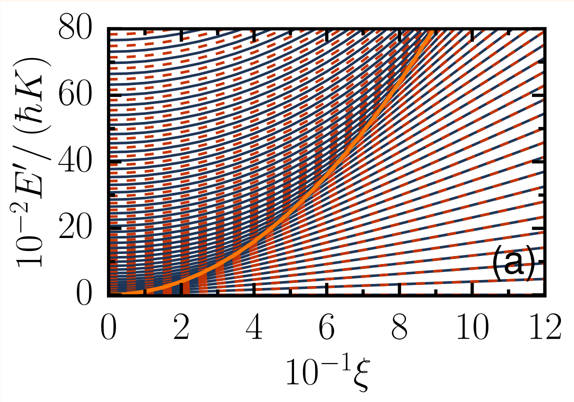
Vanishing of Energy Separations

arXiv: 2209.03934





Advantages of the Experimental Platform



There is no other platform for the analysis of ESQPTs where the spectrum can be measured as a **function** of the control parameter.

We bring to the theoreticians, a highly **controllable** platform in which new experiments can be proposed and performed.

We bring to the superconducting circuit community insights on what can be explored with their developing technology.

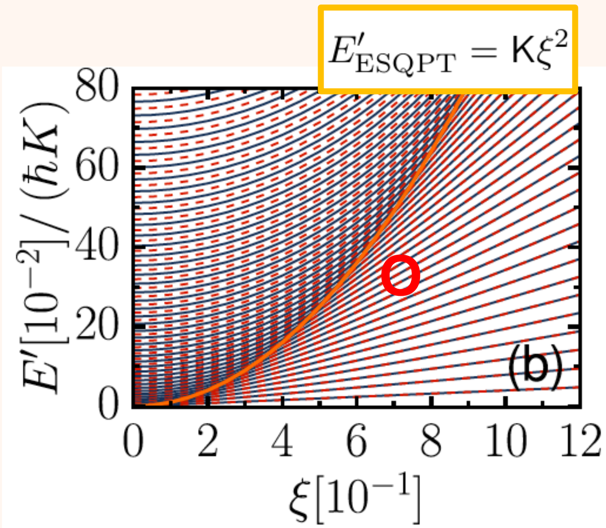
The superconducting circuit platform that we consider is **unique** for studies of ESQPTs because **both spectrum and dynamics** can be measured simultaneously.

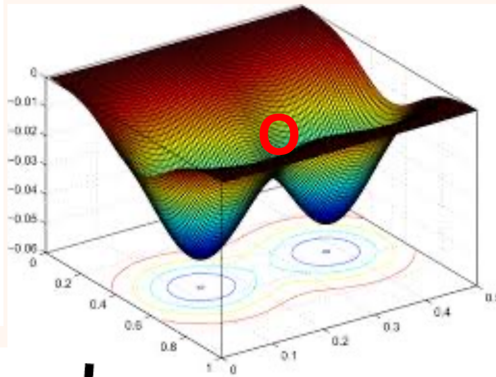
Dynamics in phase space

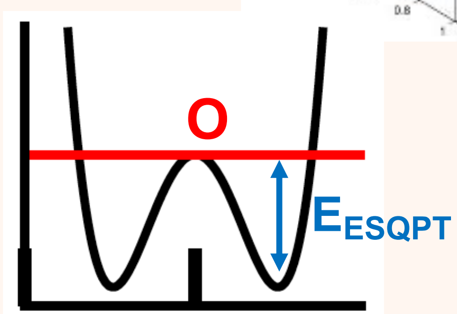
- Cavity QED provides unmatched fidelities for direct reconstruction and real-time observations.
- Wigner/Husimi function can be experimentally accessed in real-time.

Quantum simulator for nuclear, molecular, and condensed matter systems that present ESQPTs and related phenomena.

Critical Point

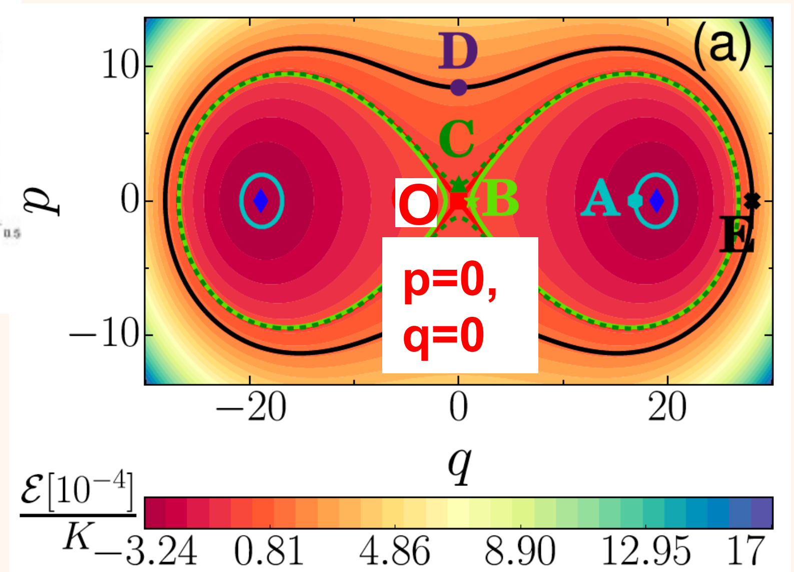






Classical Hamiltonian

$$\frac{H_{cl}}{K} = \frac{1}{4}(q^2 + p^2)^2 - \xi(q^2 - p^2)$$



The model is similar to the Lipkin model (1 degree of freedom), but the spectrum is unbounded.

Jorge's talk