CHAOS MODULATION IN SOFT-WALL BILLIARDS

UNIVERSIDAD AUTÓNOMA METROPOLITANA
Casa abierta al tiempo
Unidad Iztapalapa

Xalapa, Ver. Enero 2023

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EXCITON - POLARITON BILLIARD (MOTIVATION)

- **Excitons** are collective excitations that naturally arise in semiconductors interacting with light.
- **Polaritons** are the quantum superposition of photons and excitons (quasiparticles). When the coupling between photons and excitons in a semiconductor inside a microcavity is *increased enough*.
- "Microcavity exciton-polaritons represent a unique quantum macroscopic system, which combines the main advantages of light and matter waves"[1].
- Exciton-polaritons billiards **has been studied experimentally** for Gao et al.[1] A Sinai Billiard for a condensed exciton-polaritons. It has two parameters: **thickness** of the walls and **radius** of the defect (geometry).
- They showed[1] that the inherent non-Hermitian nature of exciton-polaritons determines their basic properties, which are crucial for transport and quantum information processing.

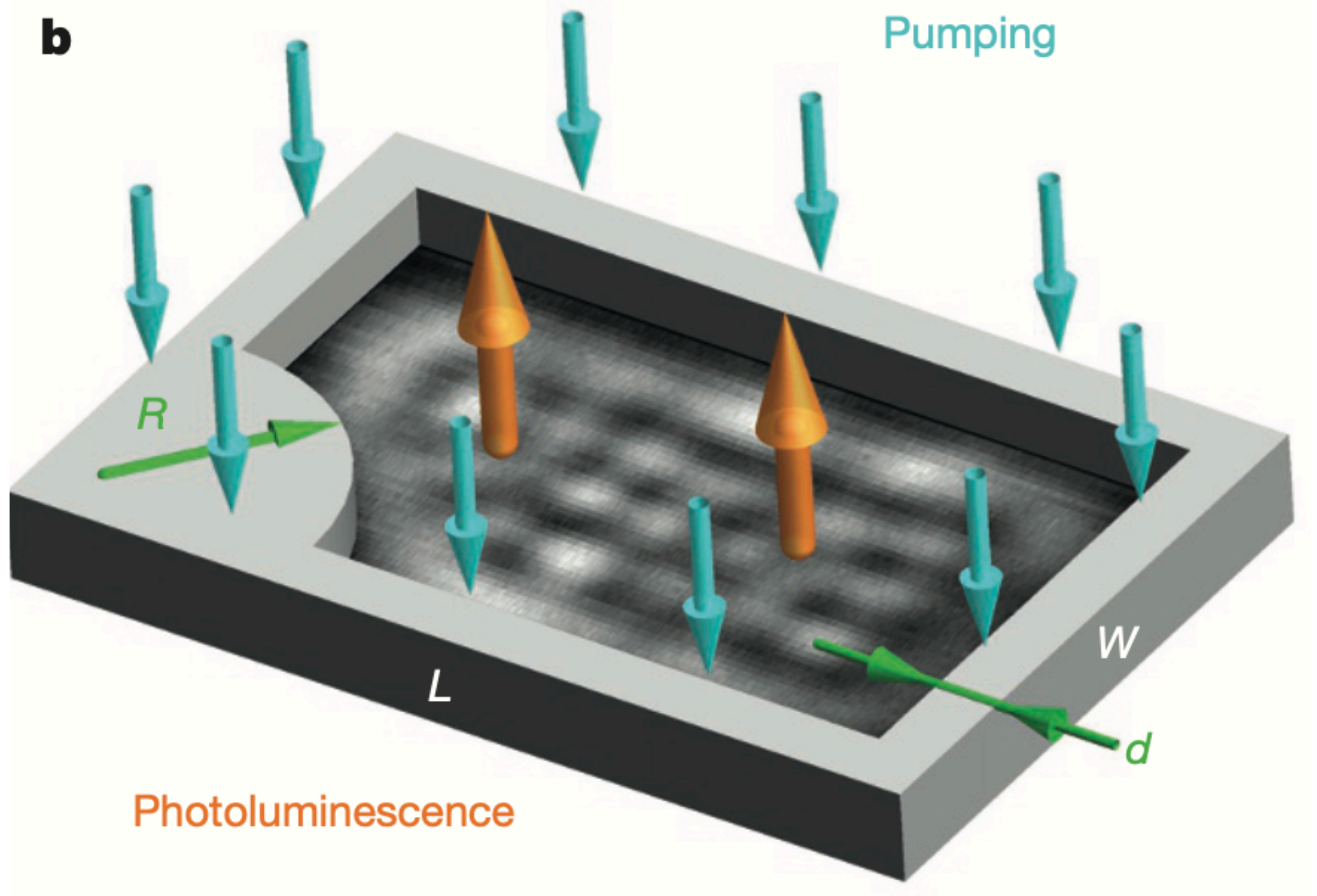
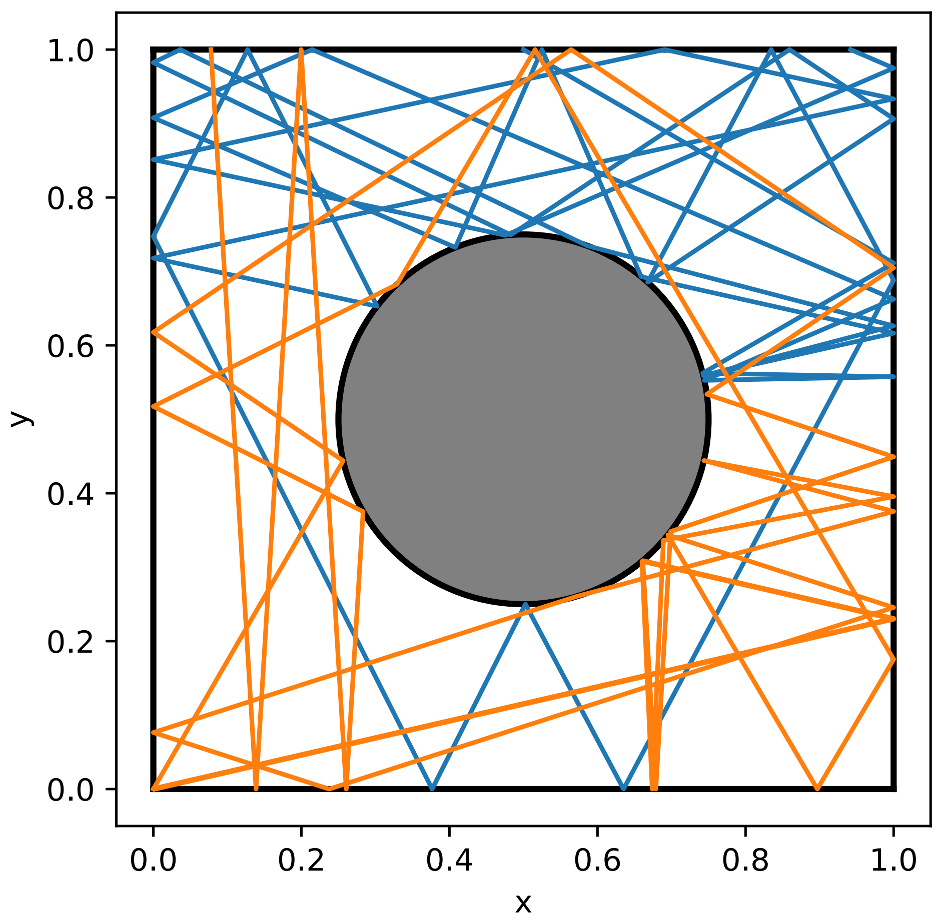


Figure of Gao et al[1].

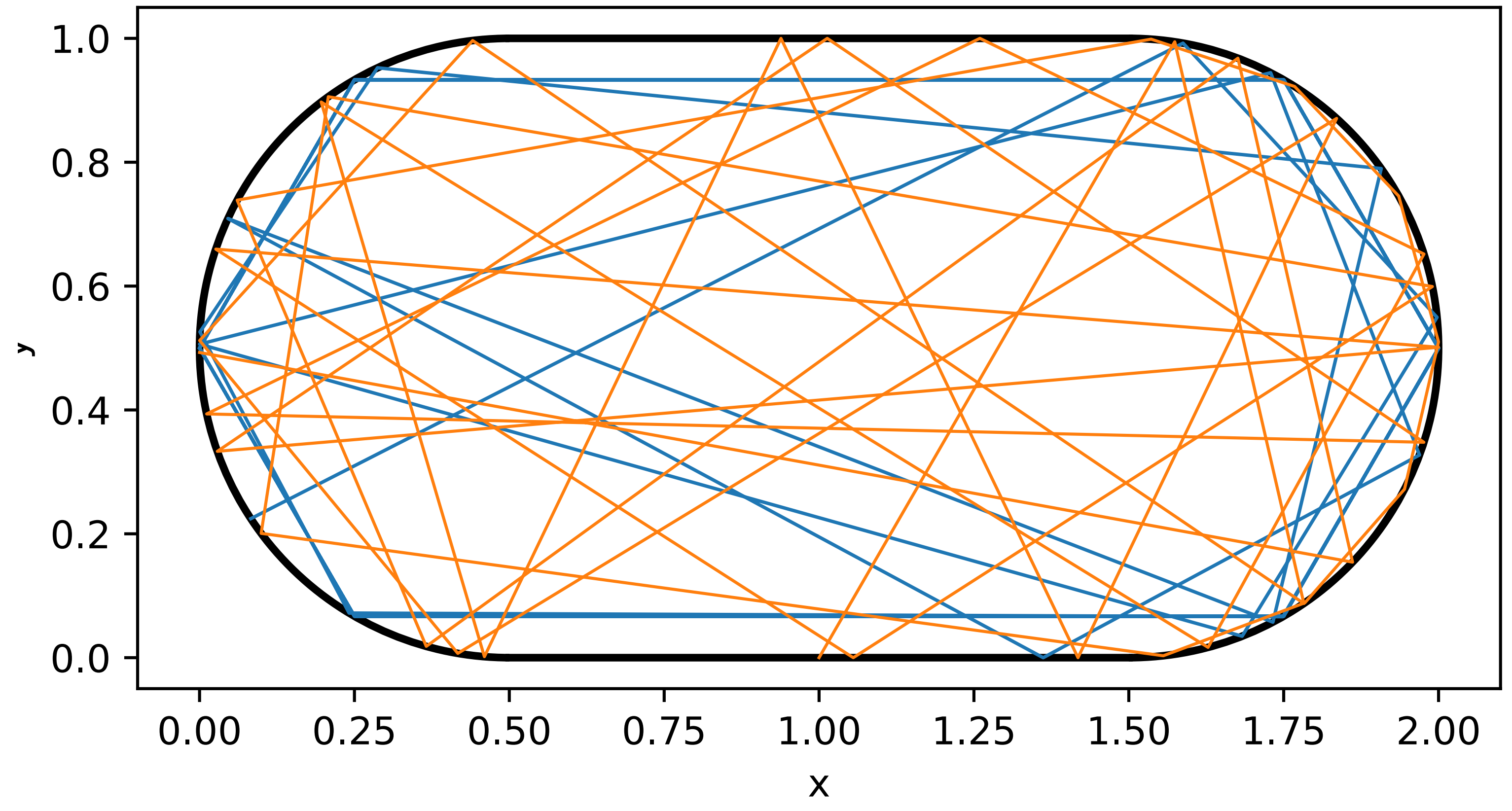
[1] T. Gao, E. Estrecho, K. Bliokh, T. Liew, M. Fraser, S. Brodbeck, M. Kamp, C. Schneider, S. Höfling, Y. Yamamoto, et al., "Observation of non-hermitian degeneracies in a chaotic exciton-polariton billiard," Nature, vol. 526, no. 7574, pp. 554–558, 2015.

BILLIARDS

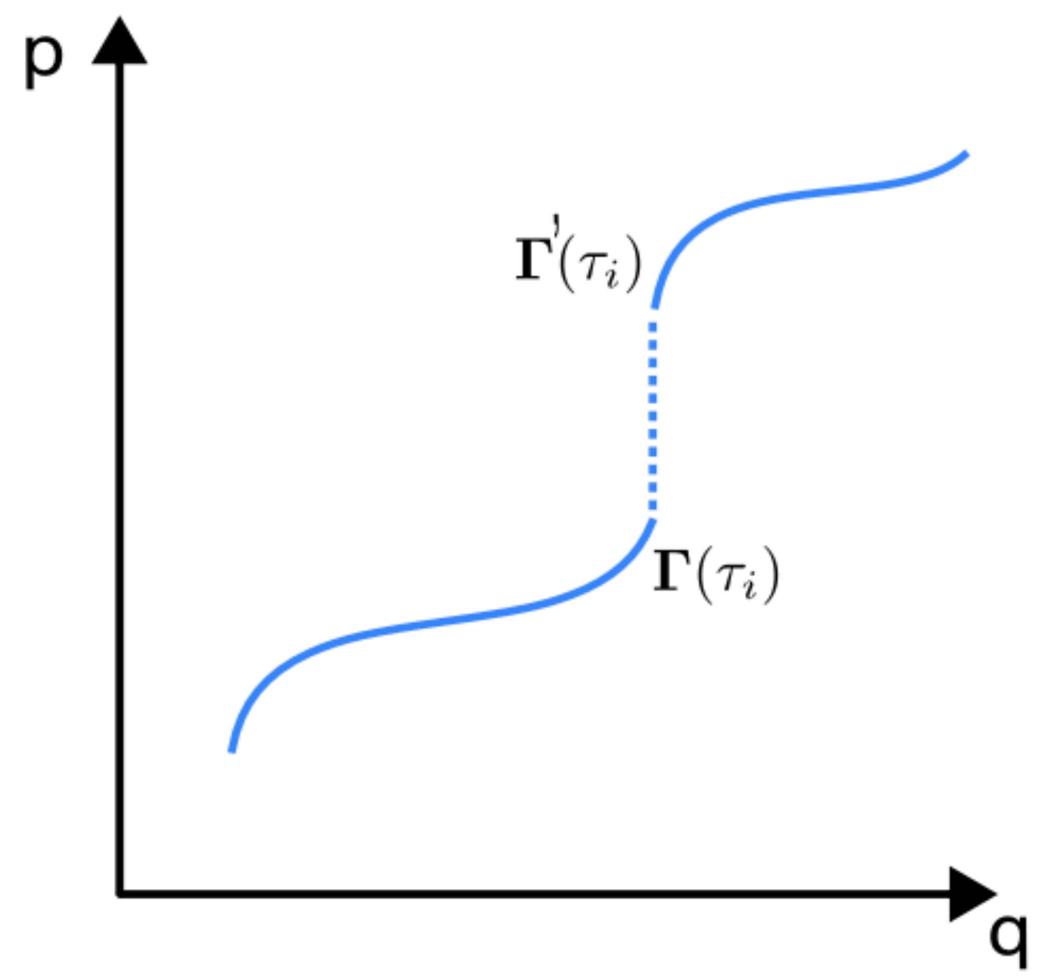
- We define a billiard table as a set of obstacles (boundaries).
- Every particle collision results in an **specular reflection**.
  Let us call this as the **hard-wall limit**.
- Classical and quantum **hard-wall billiards** can be represented by potential wells.
- Its collision space can be map via **Poincaré-Birkhoff** coordinates.



Sinai billiard

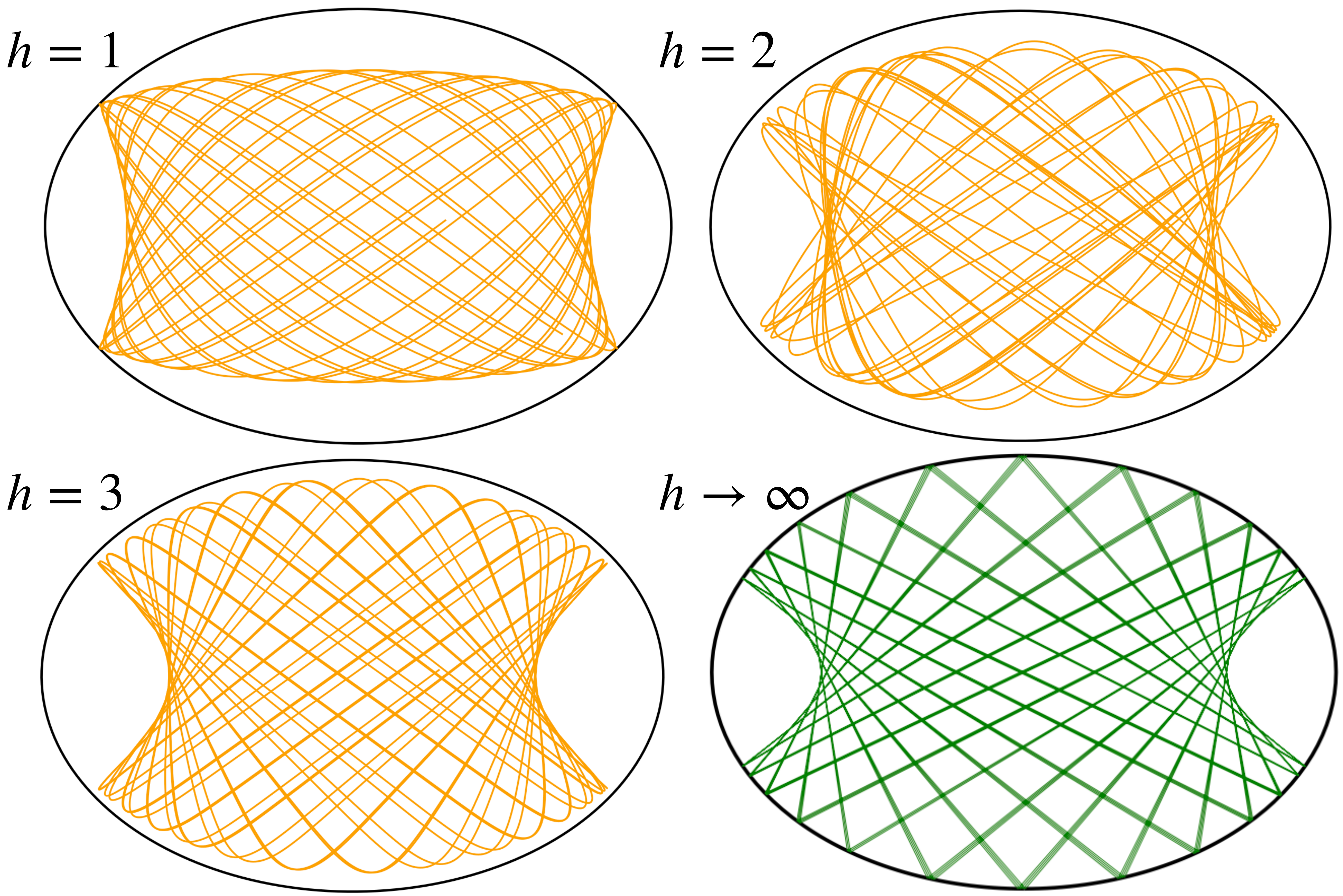


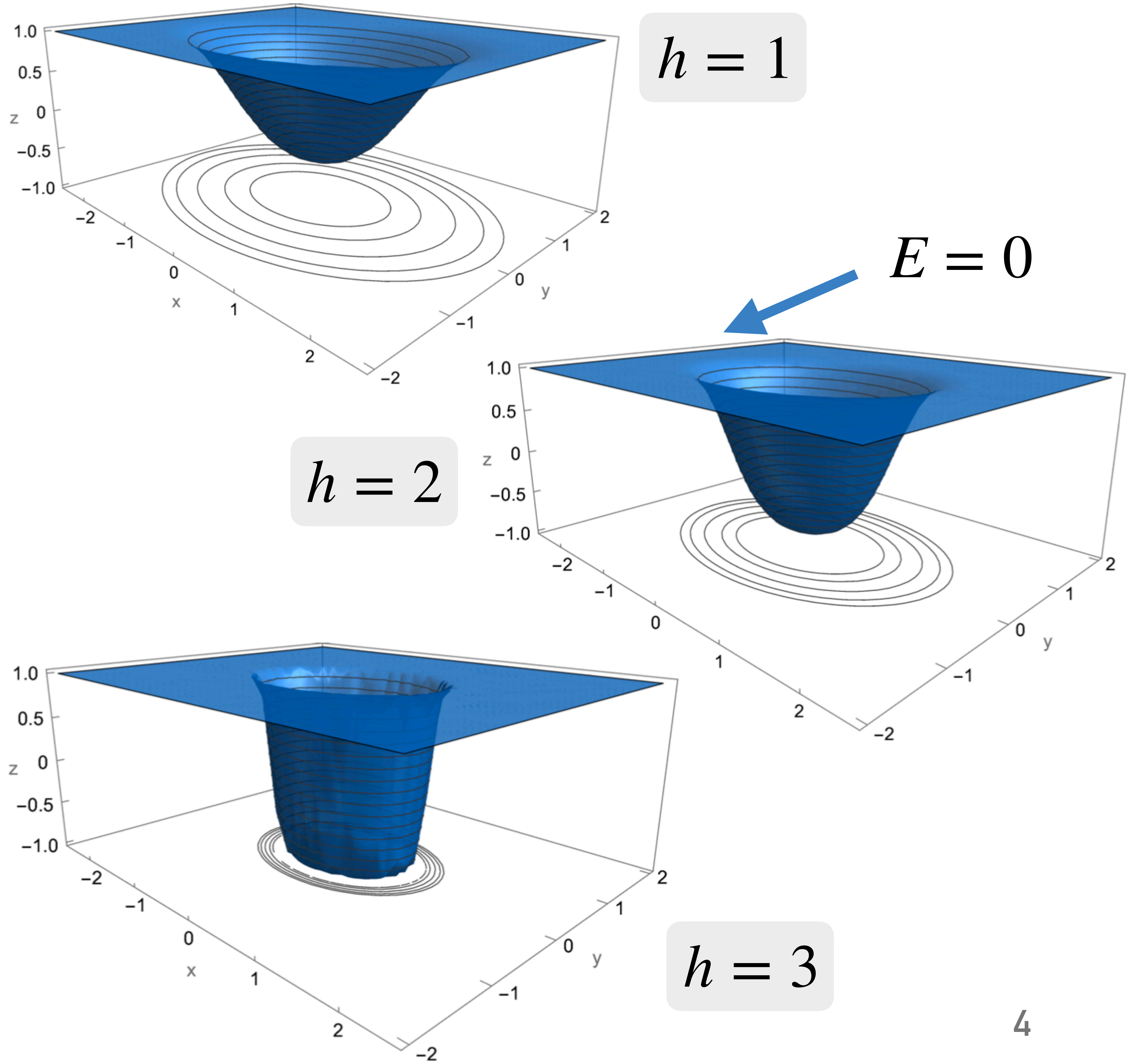
Bunimovich stadium



HARD WALL VS. SOFT WALL

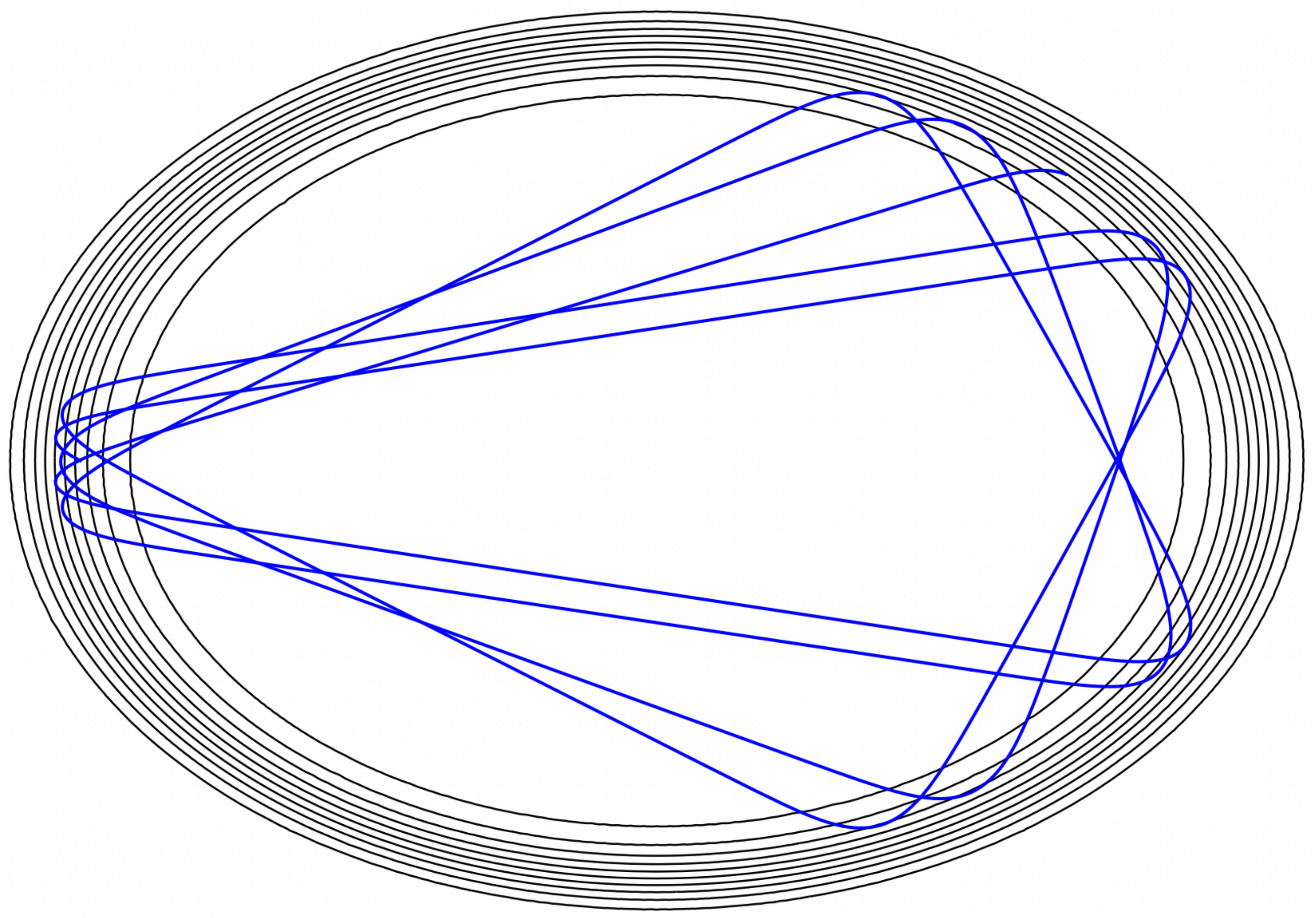
We introduce a *hardness* parameter h which makes a smooth potential well. We achieve this using error function.

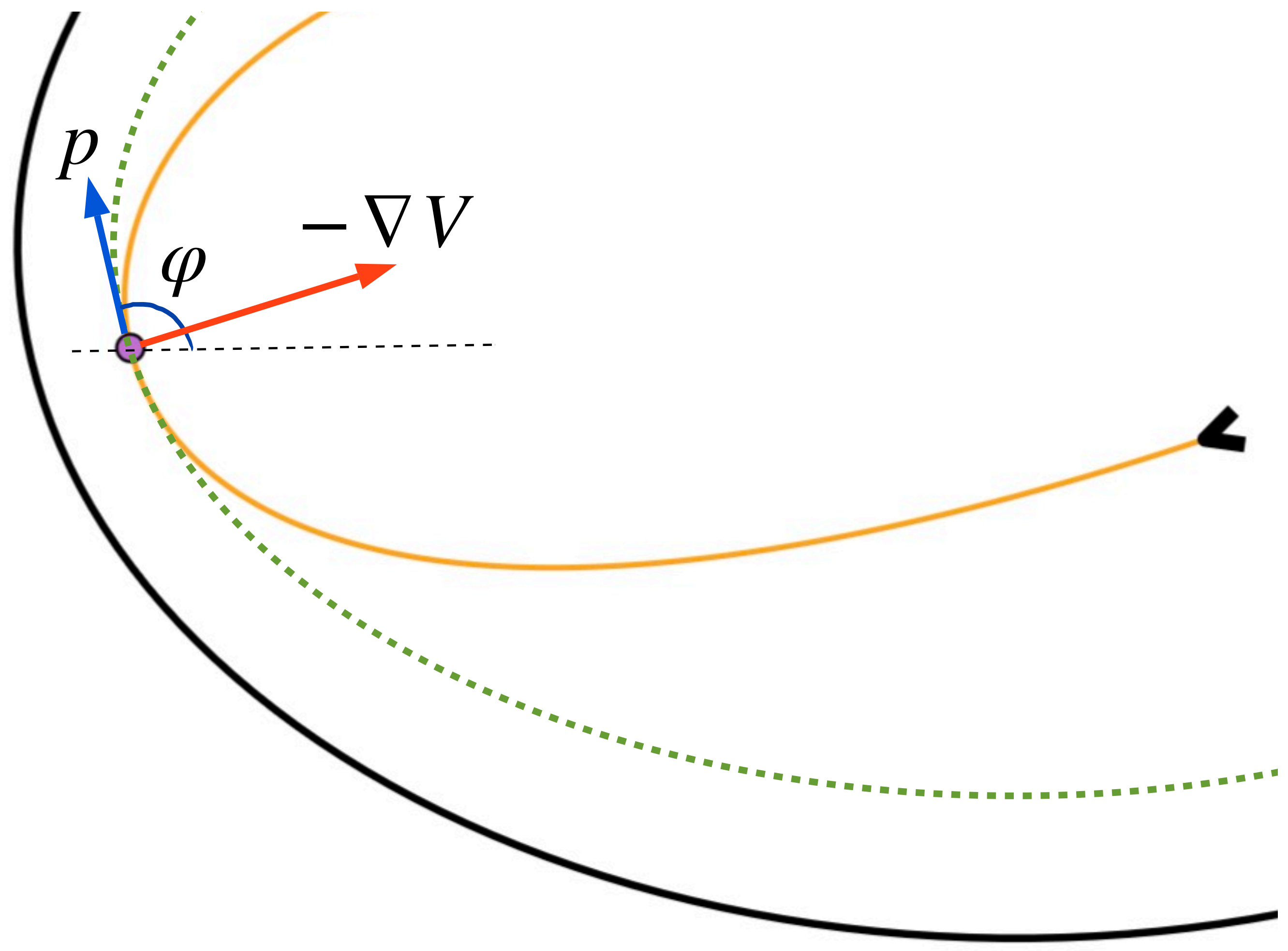




HARD WALL VS. SOFT WALL

- For **hard-wall** billiards there are discontinuities at the boundaries. A map is needed to handle the evolution at the boundaries.
- Evolution for **soft-wall** billiards is totally given by its Hamiltonian.
- Defining the **collision space** is a subtle task, turning the Poincaré-Birkhoff coordinates around.
- Has been found that softness is a parameter for **chaos modulation**[2], as the geometry.





[2] *T. Kroetz, H. A. Oliveira, J. S. Portela, and R. L. Viana, "Dynamical properties of the soft- wall elliptical billiard," Physical Review E, vol. 94, no. 2, p. 022218, 2016.*

CHAOS INDICATORS AND OUR SYSTEM

- Lyapunov spectrum.
- Poincaré surface of section
- Geometry, energy and hardness.

$$\frac{dp_y}{dt} = -\frac{\partial H}{\partial y} = -\frac{4hy}{b^2\sqrt{\pi}} exp\left[-h^2\left(x^2 + \frac{y^2}{b^2} - 1\right)\right]$$

$$\frac{dp_x}{dt} = -\frac{\partial H}{\partial x} = -\frac{4hx}{\sqrt{\pi}} exp\left[-h^2\left(x^2 + \frac{y^2}{b^2} - 1\right)\right]$$

$$\dot{x} = \frac{\partial H}{\partial p_x}, \qquad \dot{y} = \frac{\partial H}{\partial p_y},$$

Our main **goal** is to find a tool that **compute** Lyapunov exponents, PSOS as well as evolves the dynamic.

Starting point is studying **elliptical billiard**. Because its *geometry* depends mainly in one parameter: eccentricity. As well, we already have a parametrization for the billiard.

$$V(x,y) = \mathrm{erf}\left[h\left(x^2 + \frac{y^2}{b^2} - 1\right)\right]$$

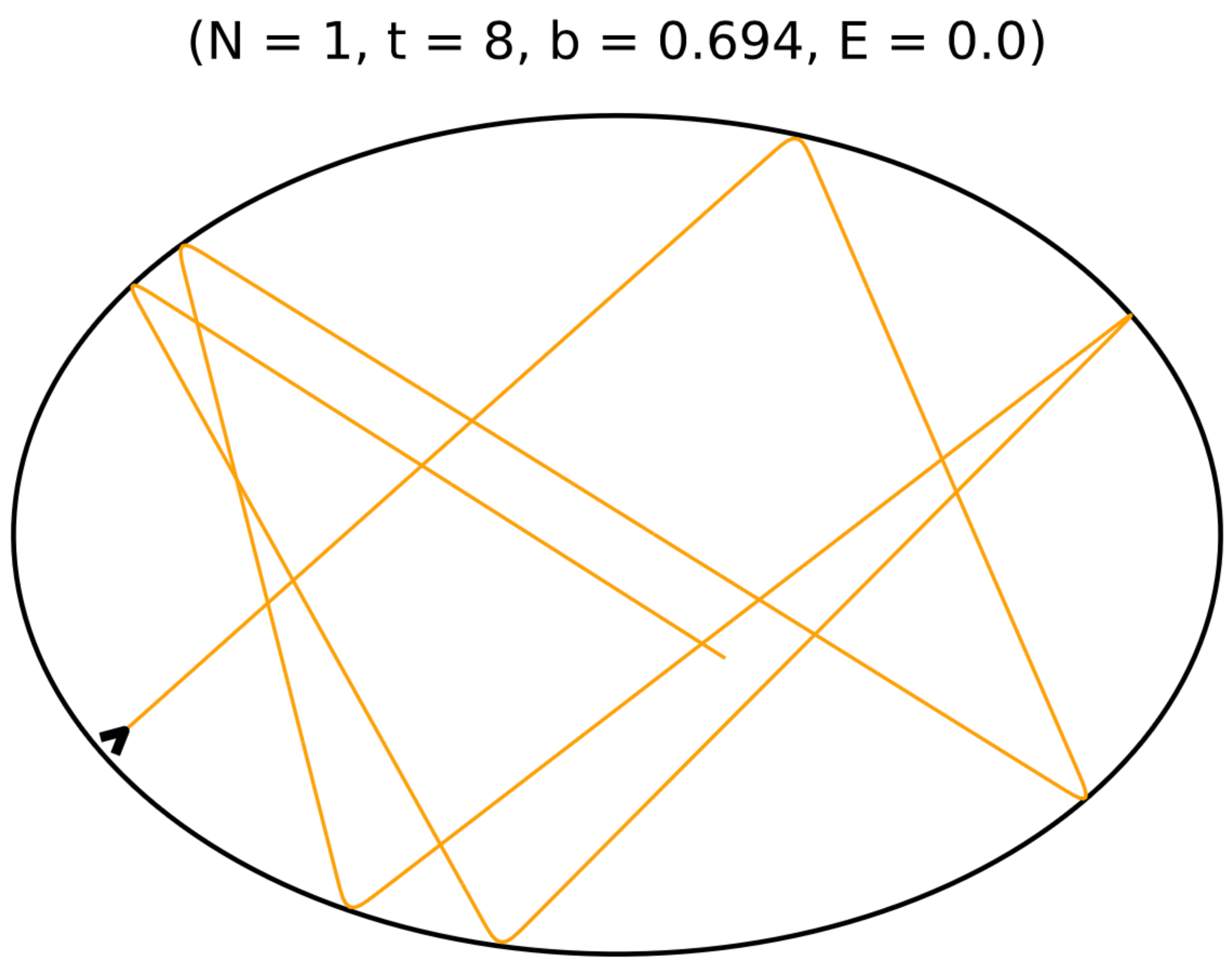
CHAOS INDICATORS AND OUR SYSTEM

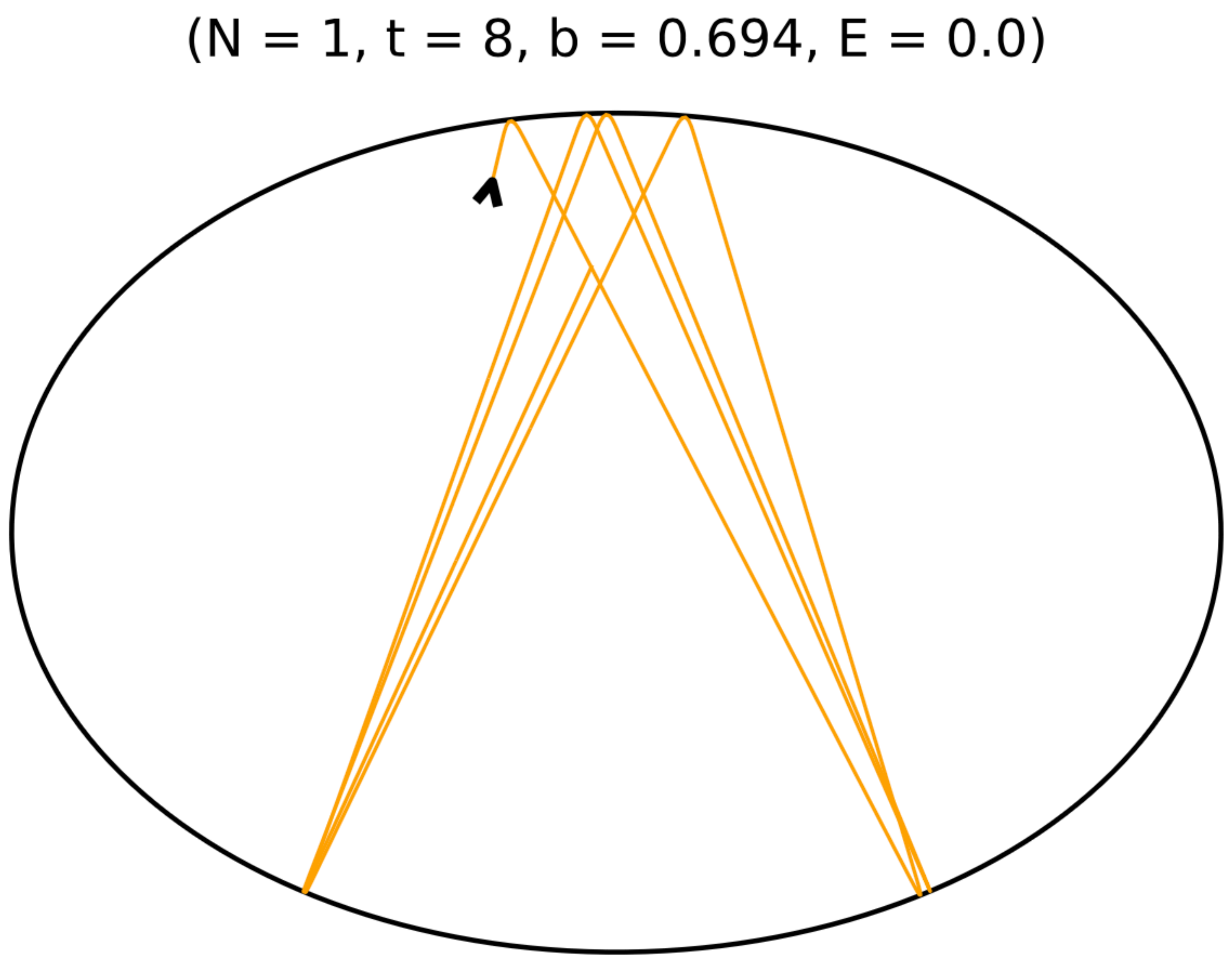
We will share some of the main **issues** we confront when analyzing and computing chaos indicators and even solving the dynamic.

- Stiffness
- Energy dependence initial conditions
- Long time computing Lyapunov exponents
- Defining collision
- Construct collision space
- Not expected regions in PSOS for higher hardness values

SOLVING MOVEMENT EQUATIONS

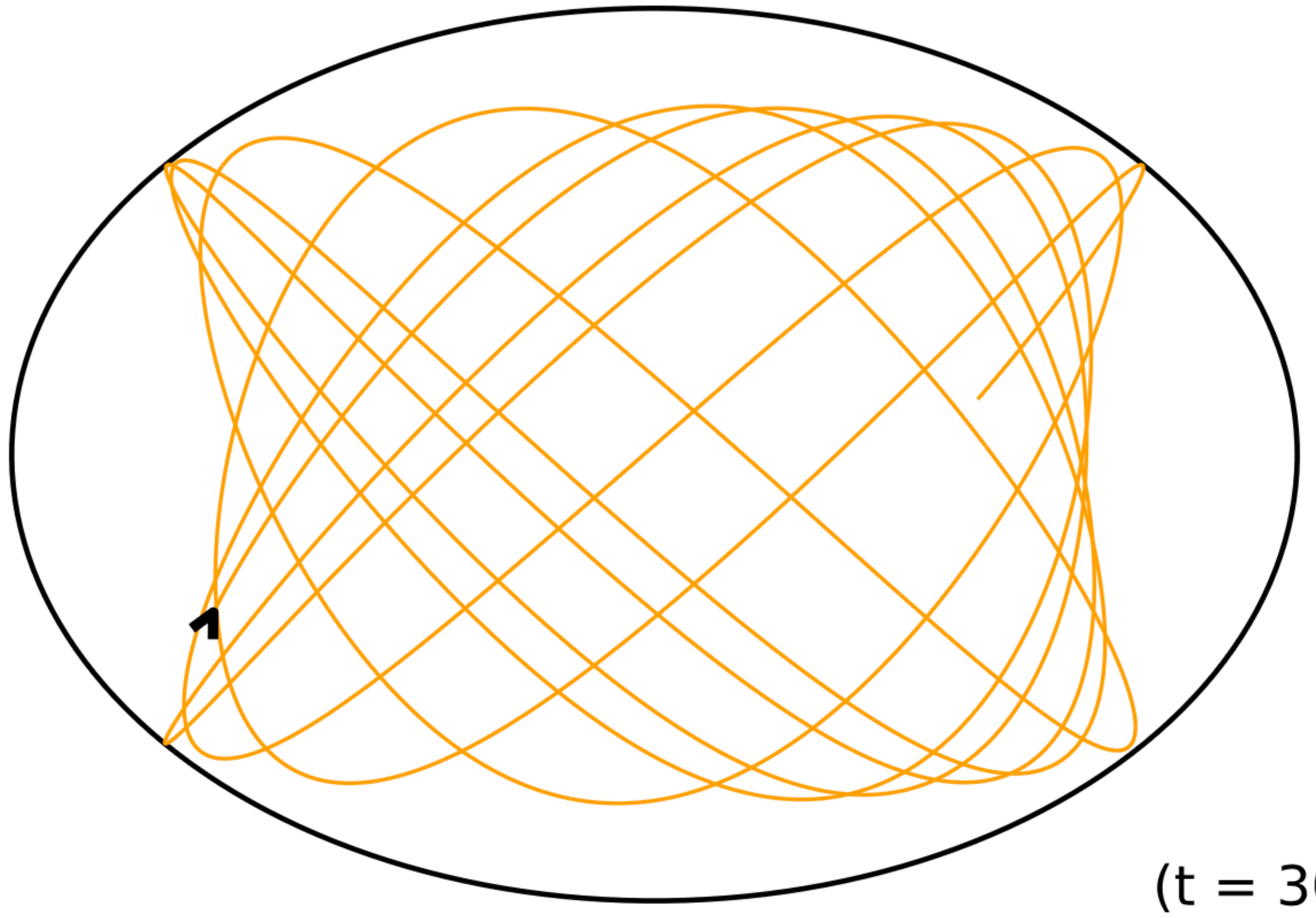
- The **stiffness** of the movement equations for increasing **hardness** values makes getting solutions a rough task.
- We attack this using an **adaptive step solver**. Taking smaller steps where potential changes abruptly.
- Useful for long trajectories for computing Lyapunov exponents.



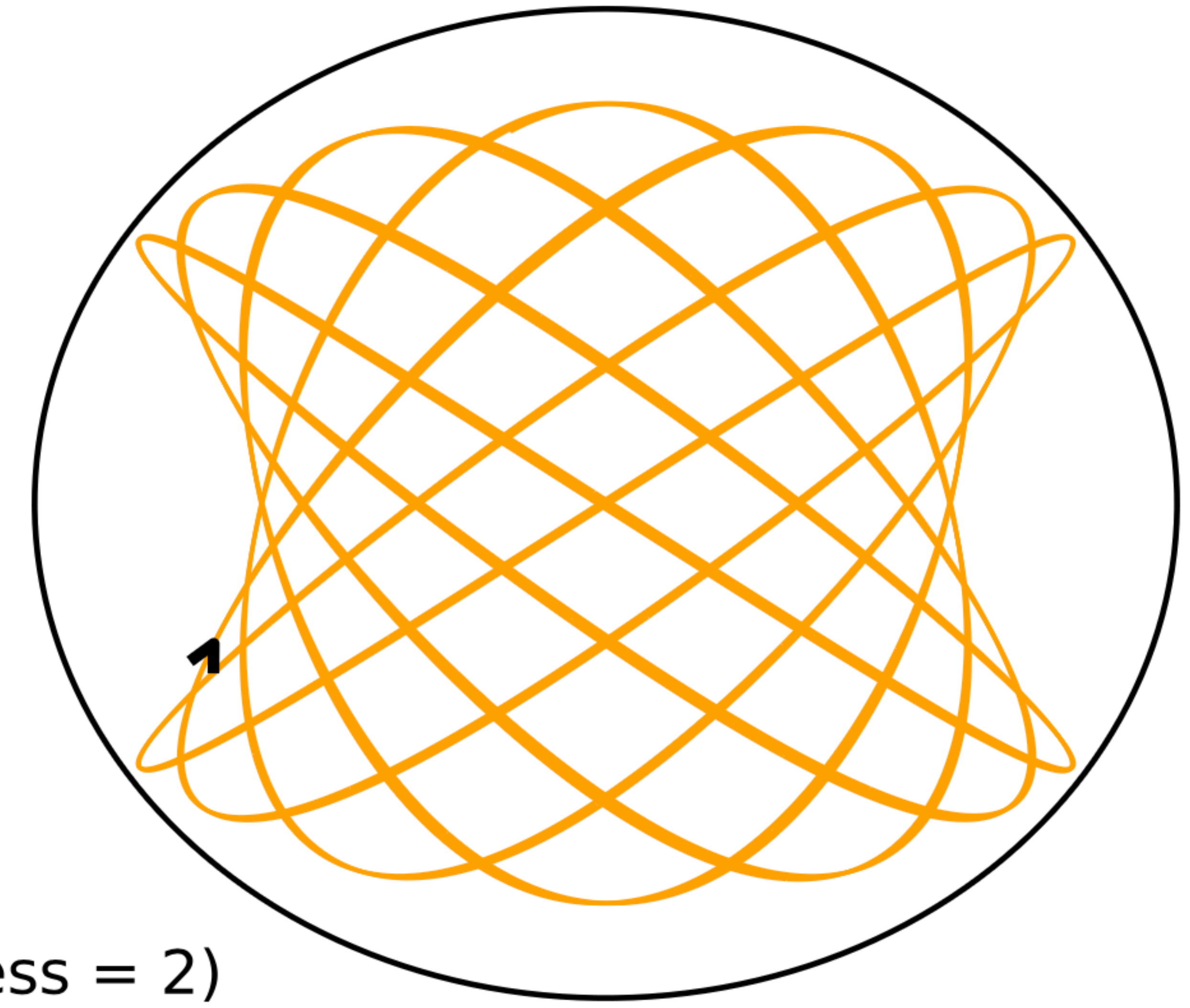


PARAMETER DEPENDENCE

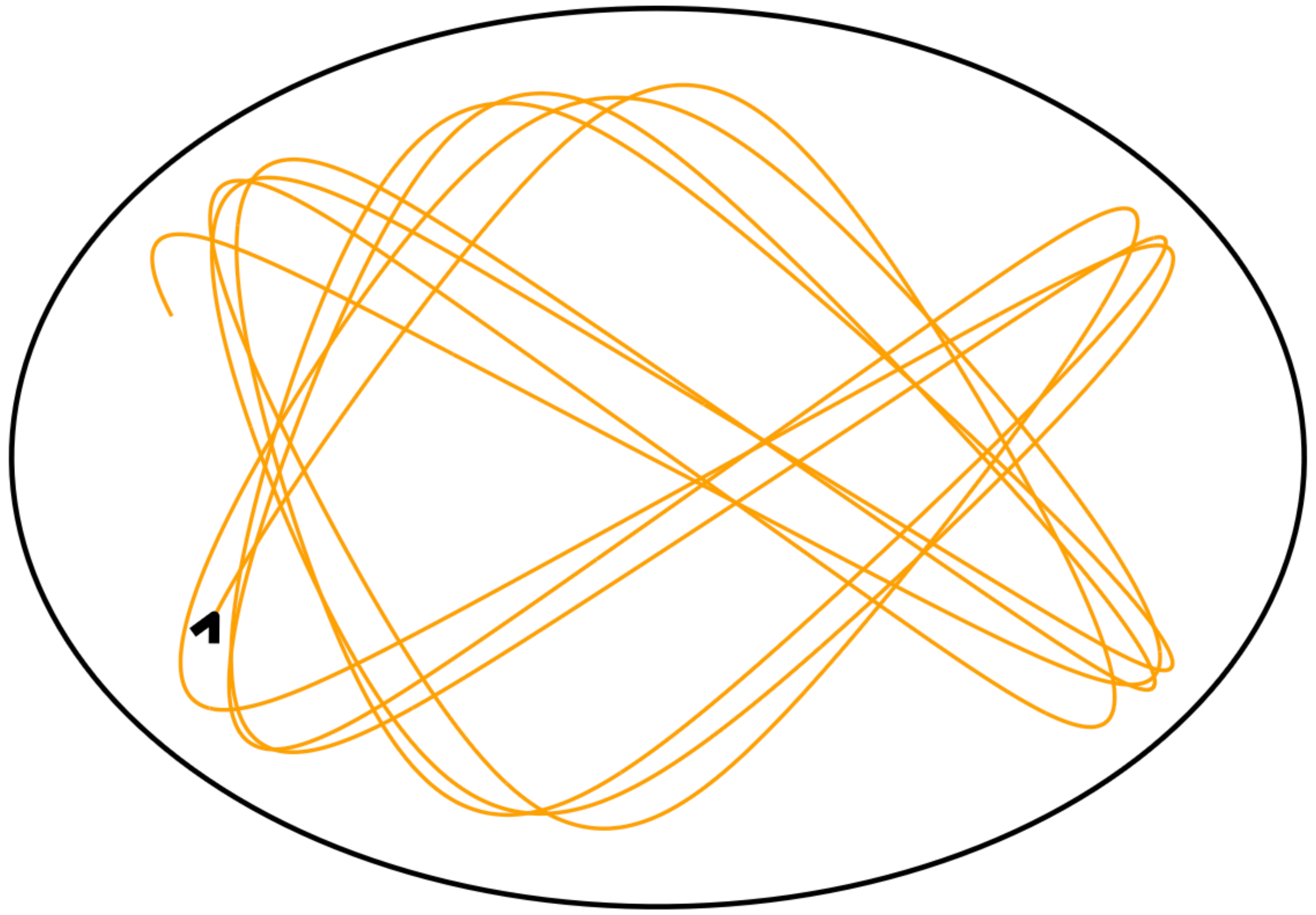
(t = 30, b = 0.694, E = 0.0, hardness = 1)



(t = 100, b = 0.866, E = -0.046, hardness = 1)

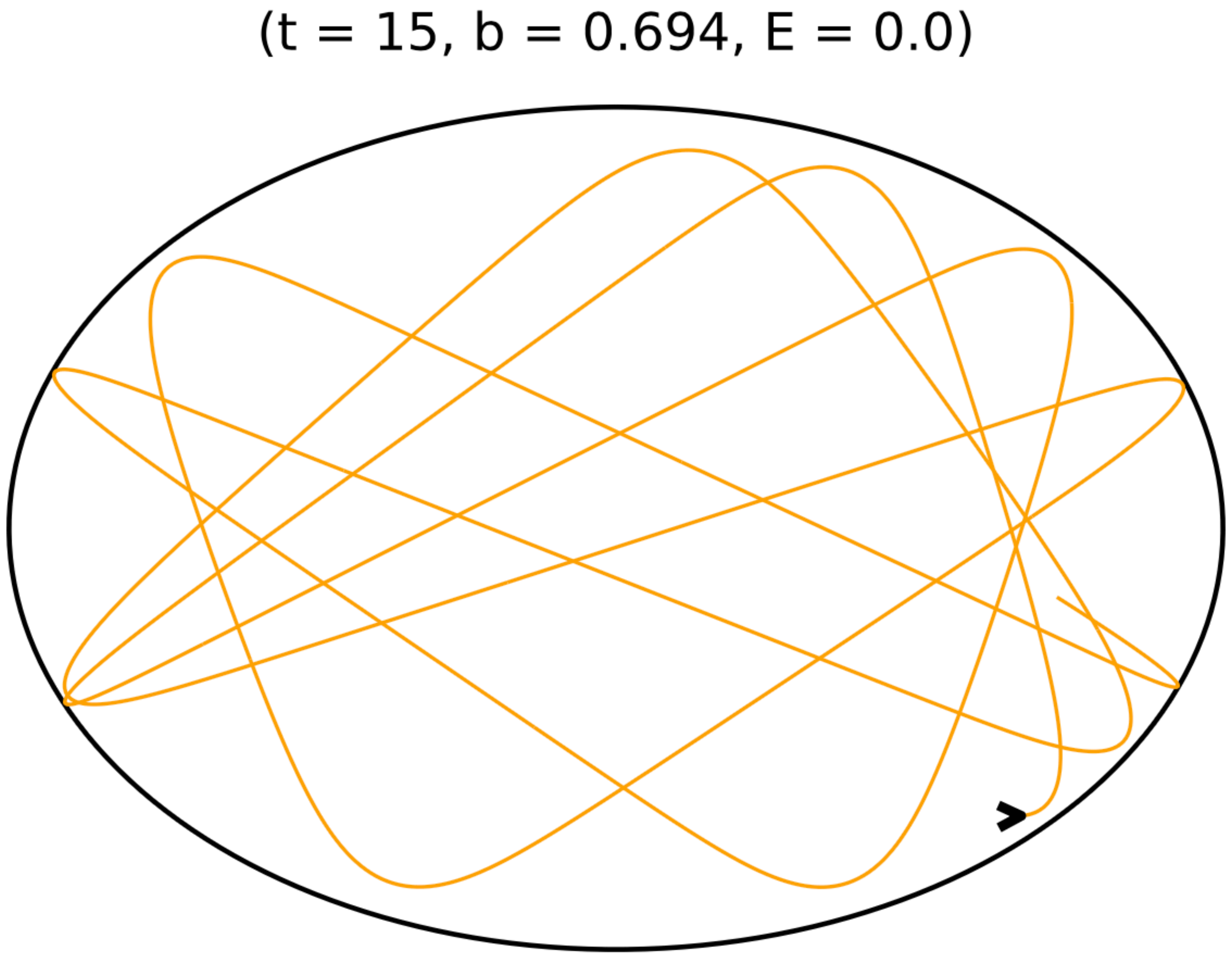


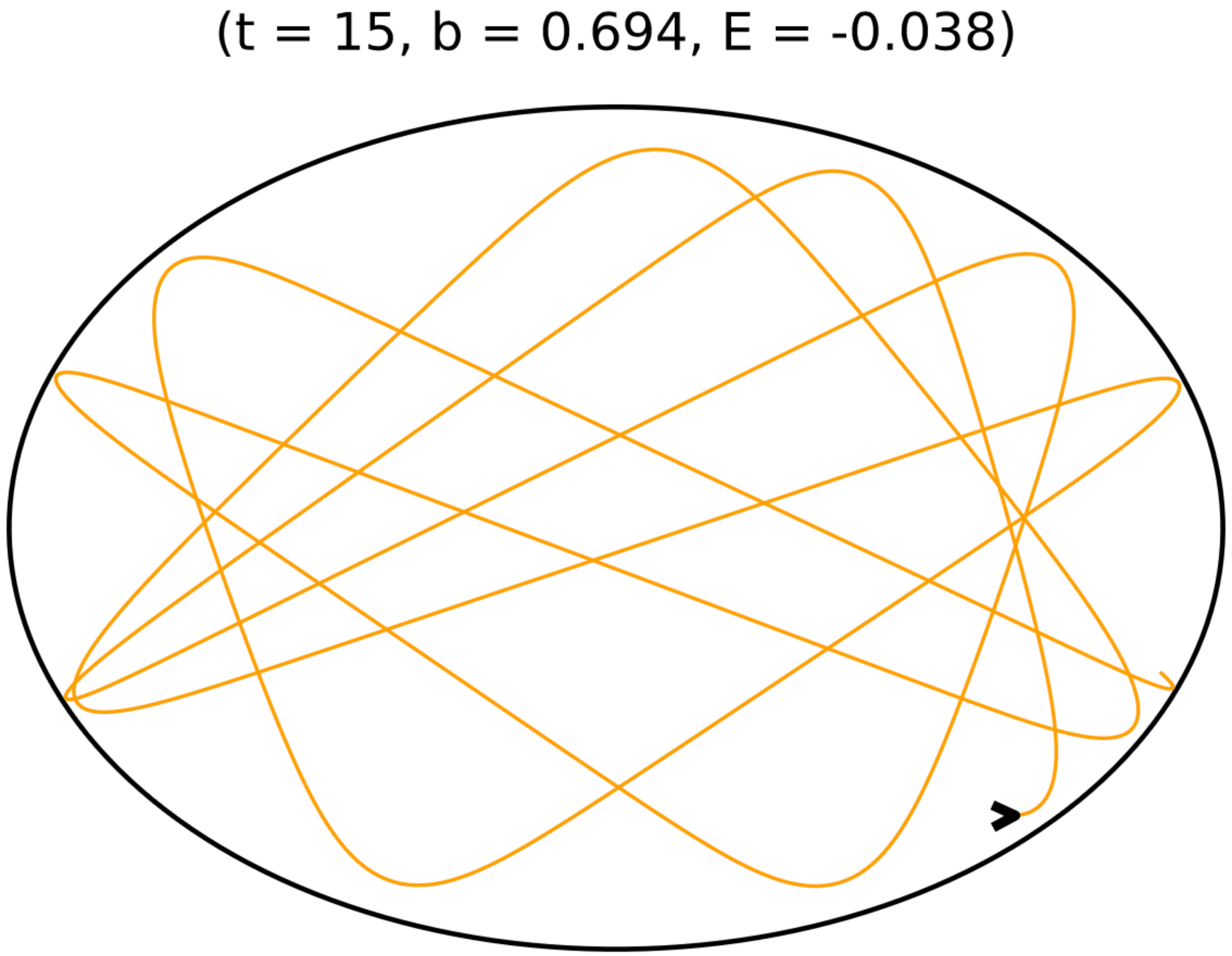
(t = 30, b = 0.694, E = -0.311, hardness = 2)



PARAMETER DEPENDENCE

For hard-wall billiards is usual to test trajectories within a vicinity of initial conditions.



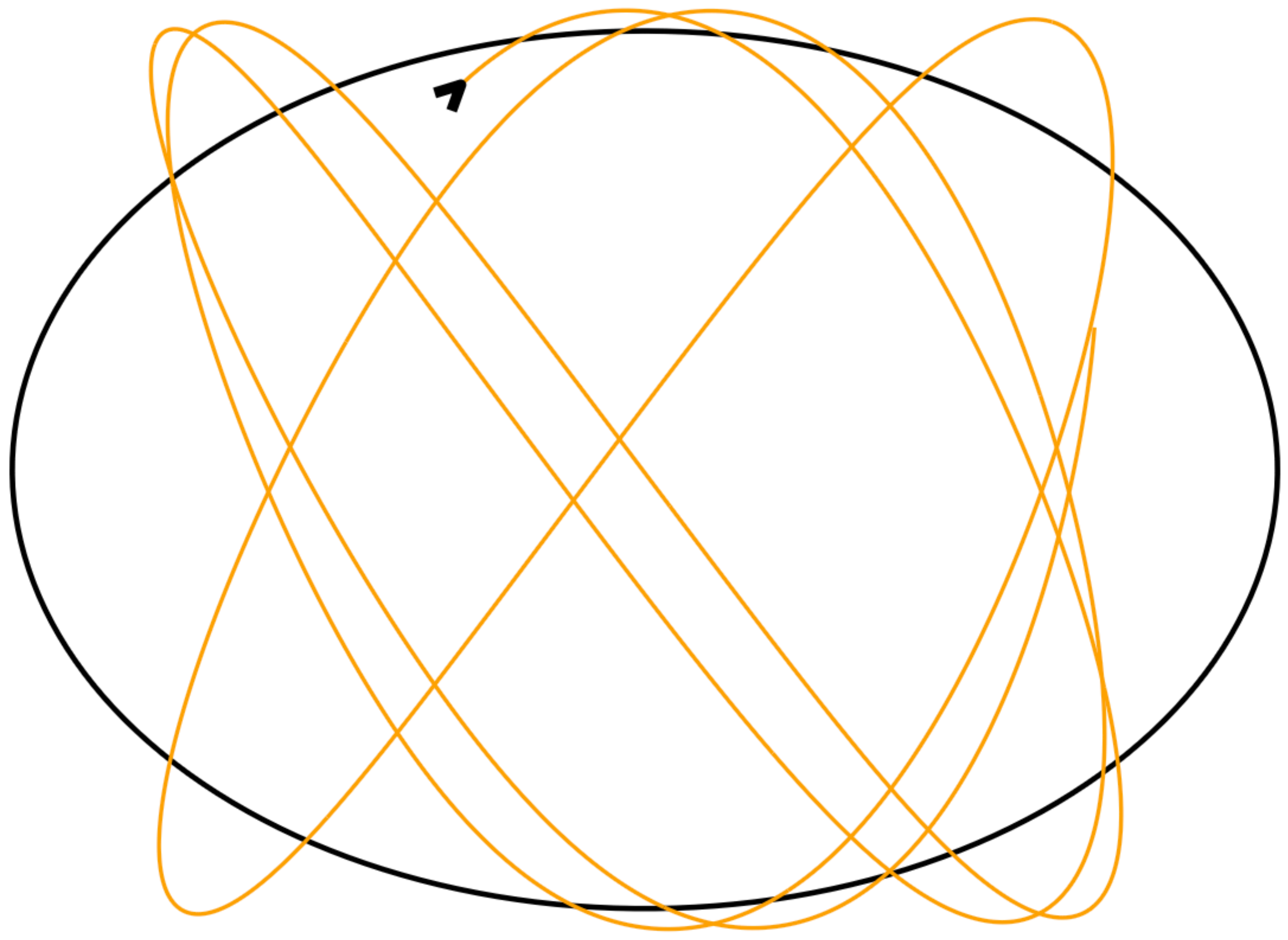


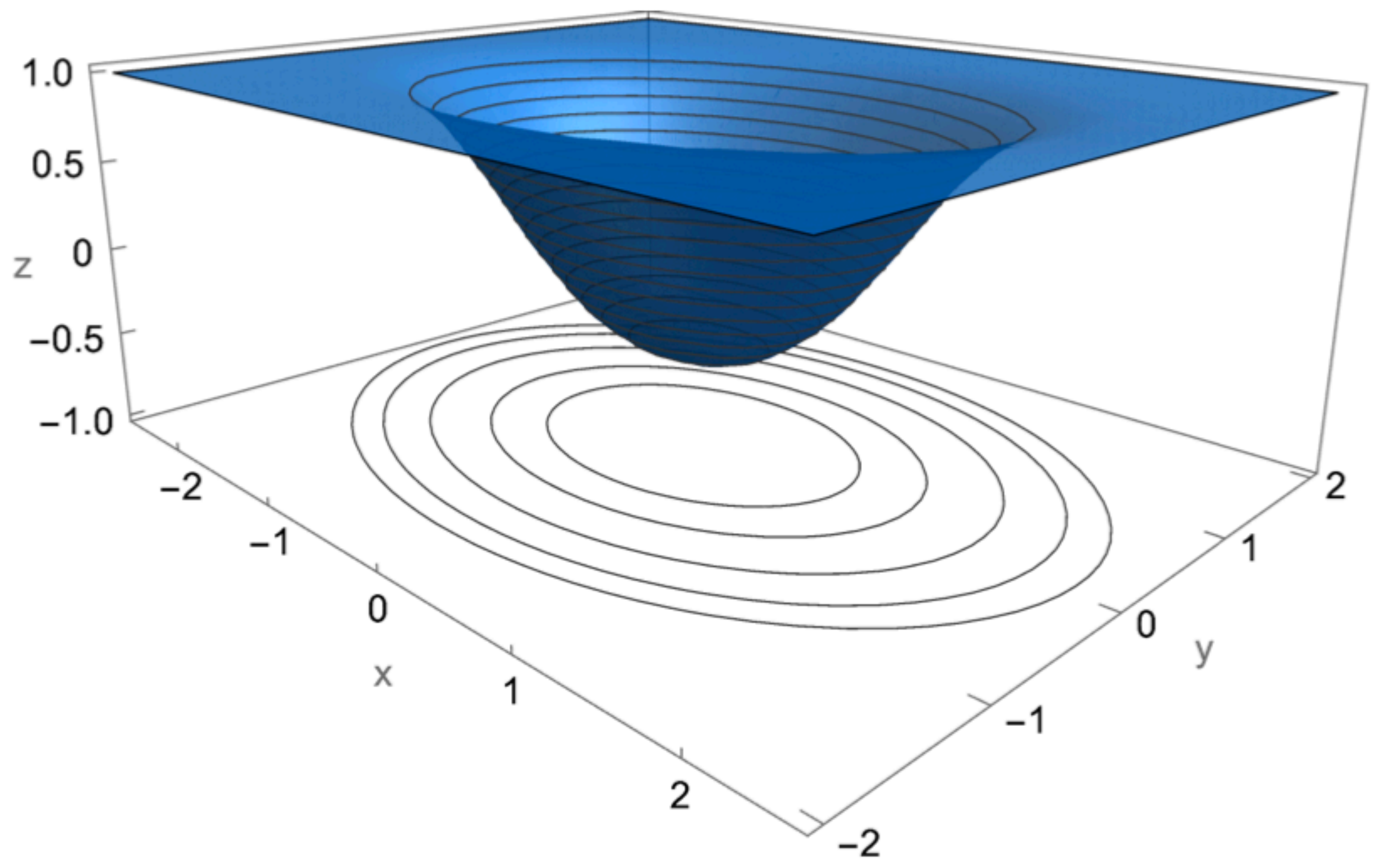
$$q_x : 0.66 \rightarrow 0.65$$

PARAMETER DEPENDENCE

There is a range of valid energy values.

(t = 15, b = 0.694, E = 0.62)





LONG TIME COMPUTING LYAPUNOV EXPONENTS

h = 4.5 is complex to solve
the equations

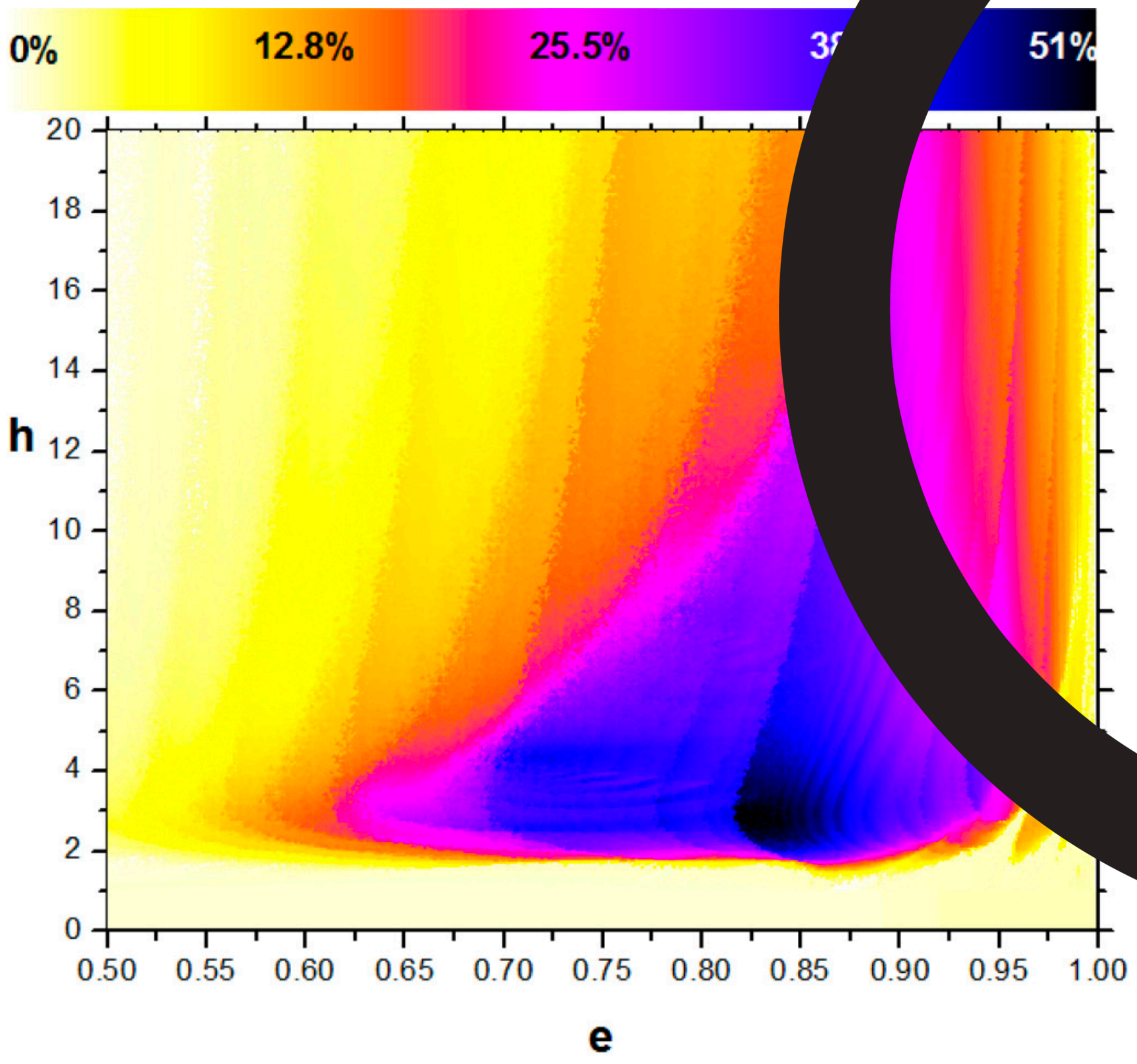
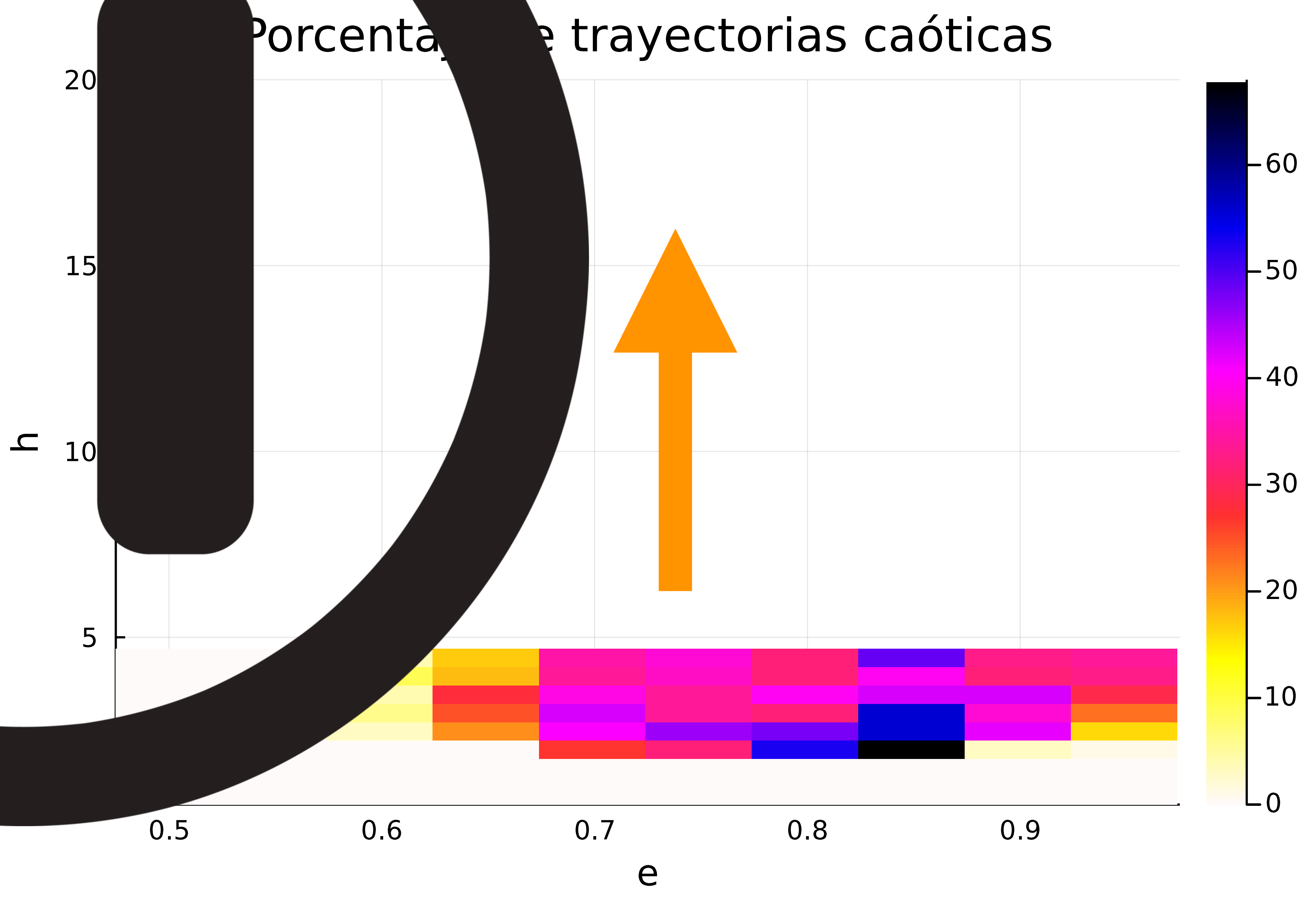
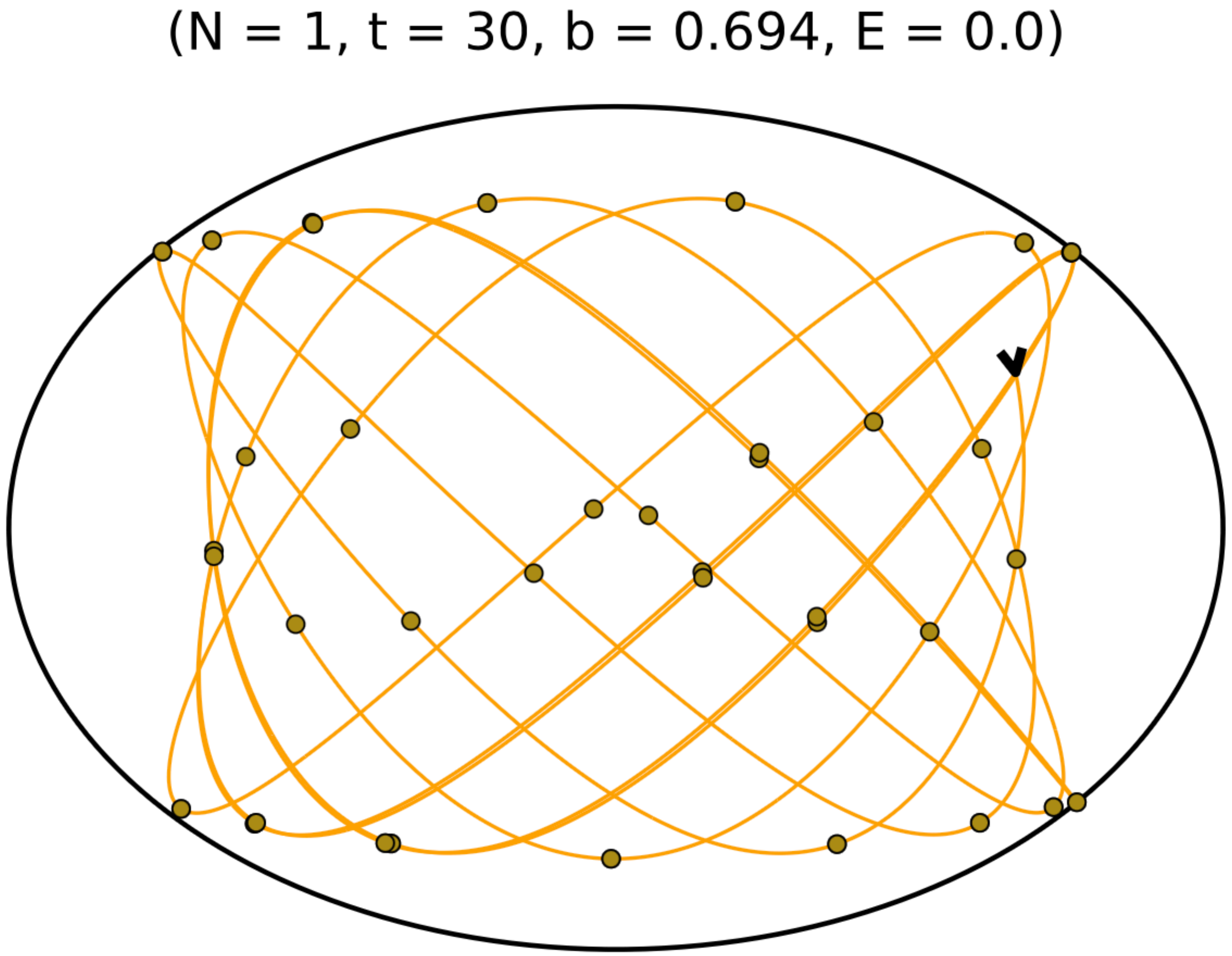


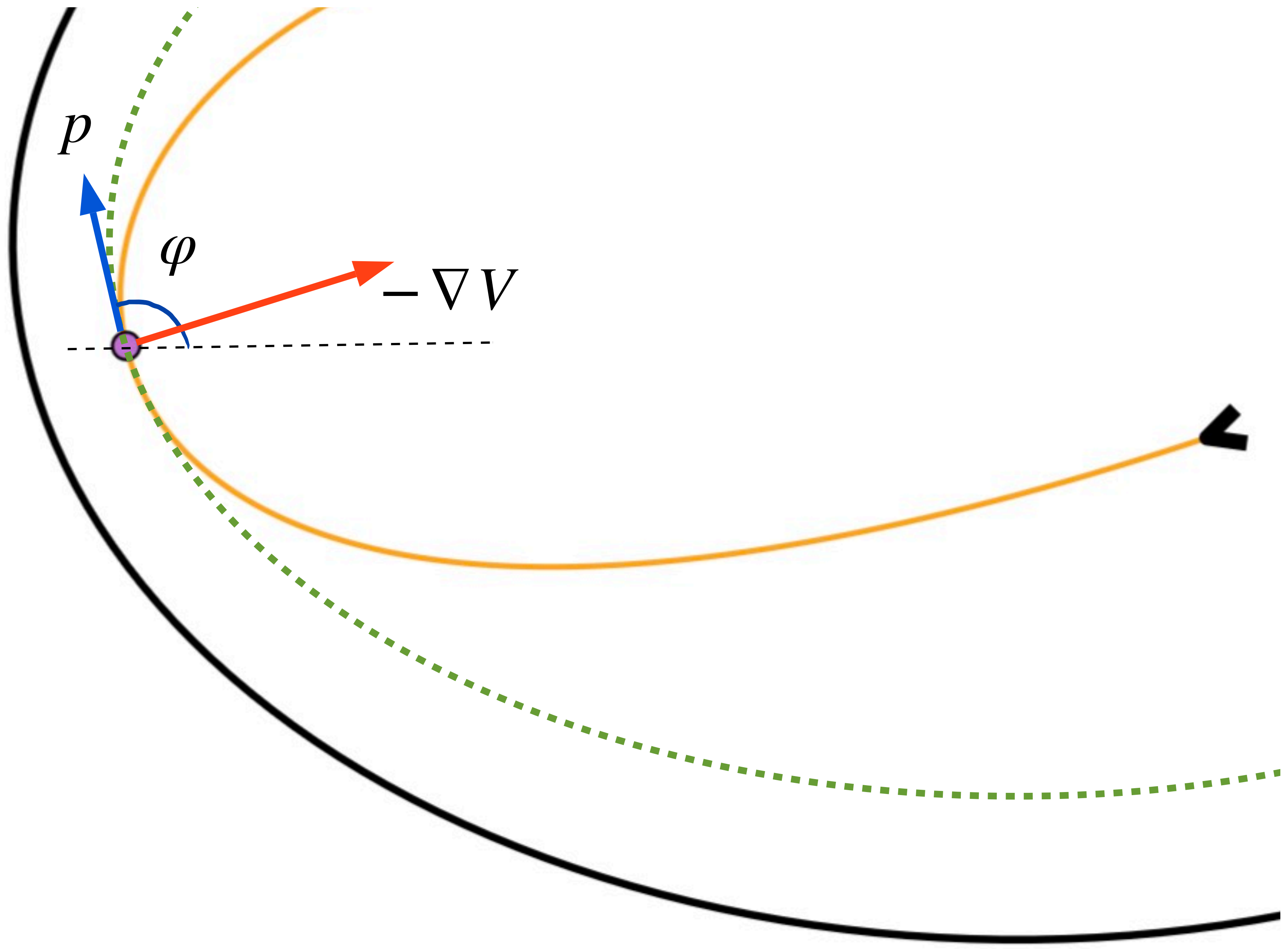
Figure of Kroetz et al.



DEFINING COLLISIONS

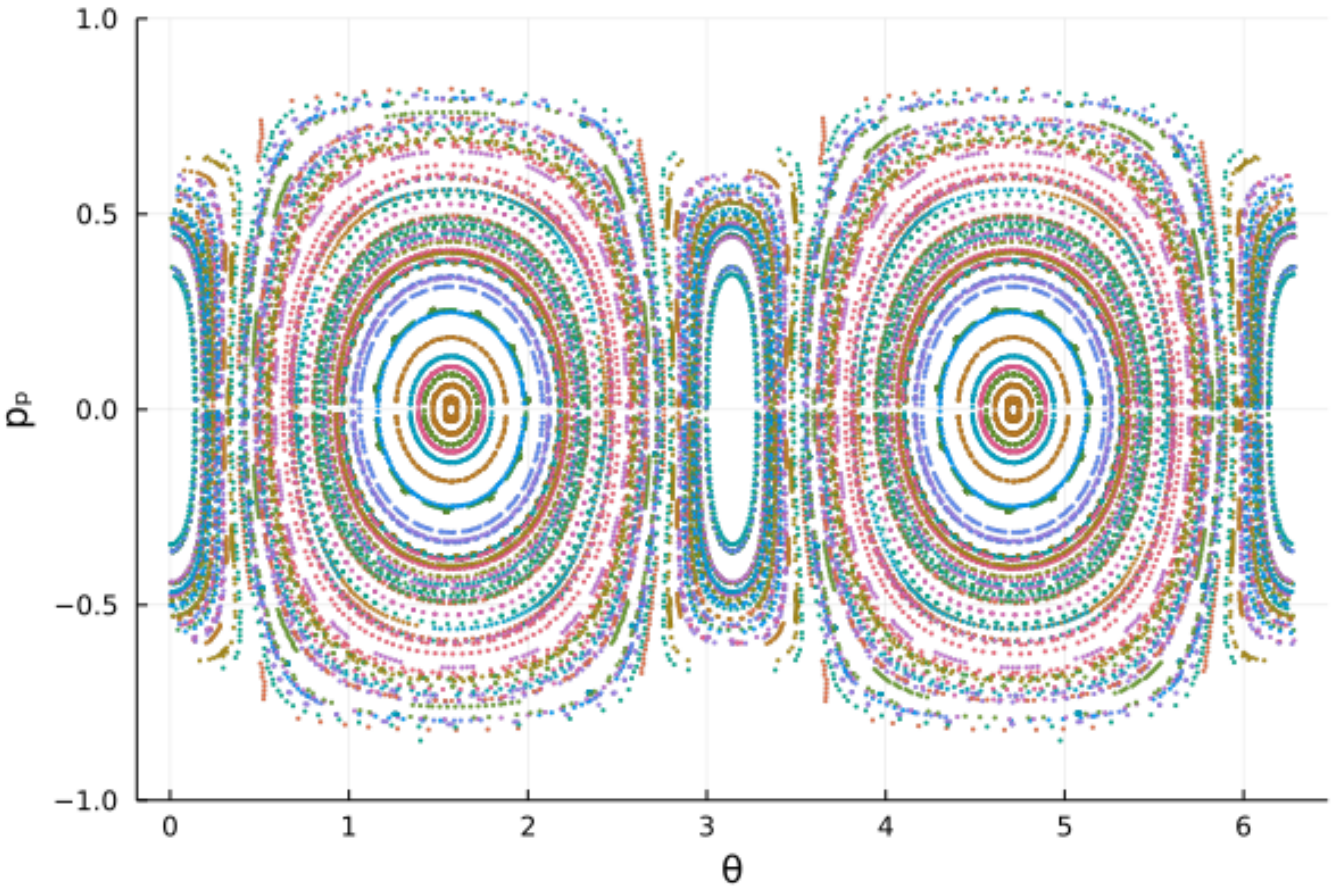
We look for the instant when particle moment is totally parallel to the equipotential curve.



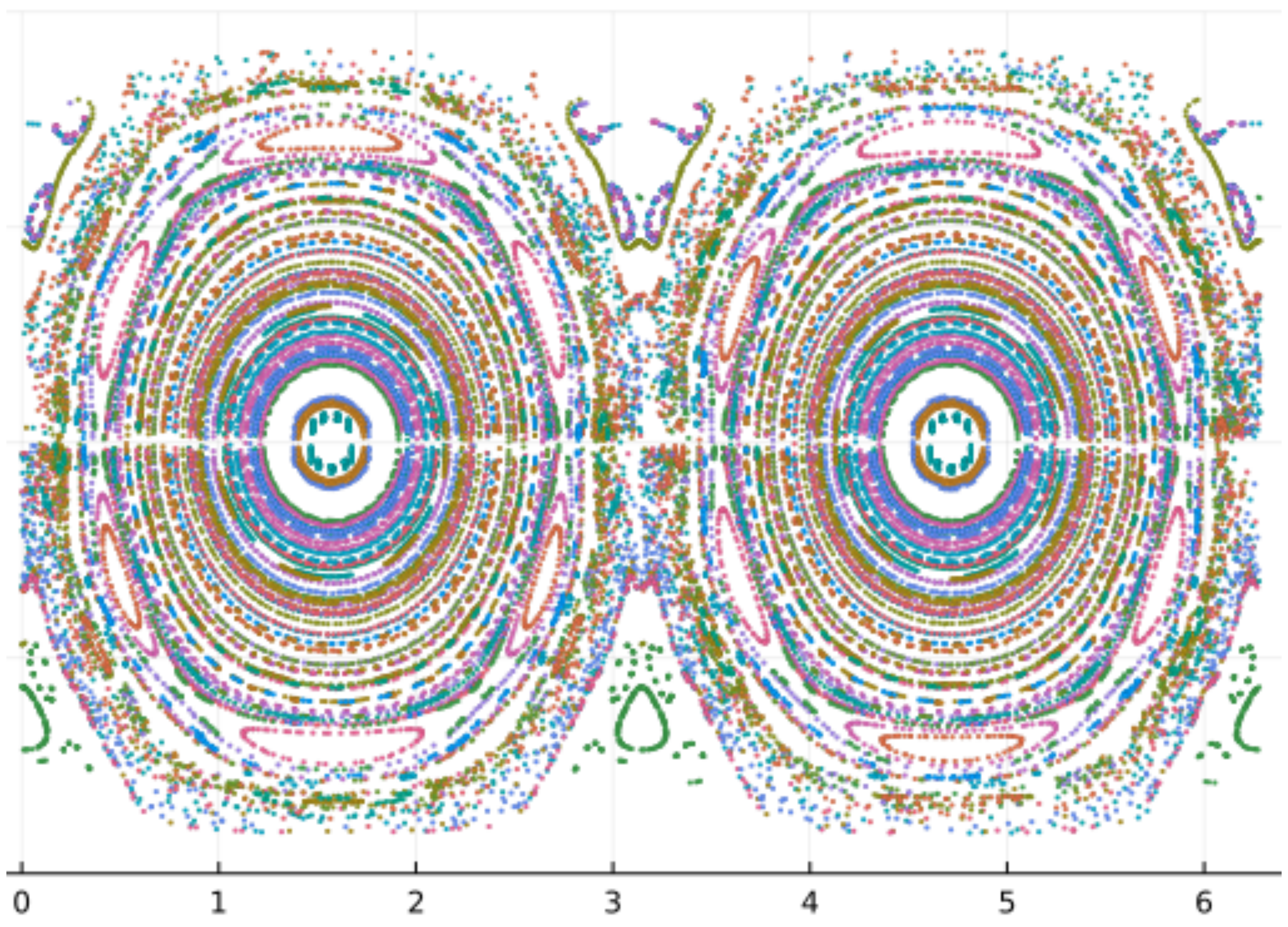


CONSTRUCT COLLISION SPACE

Let us set collision space inspired by Poincaré-Birkhoff coordinates. θ is polar angle and p_p denotes parallel momentum normalized.

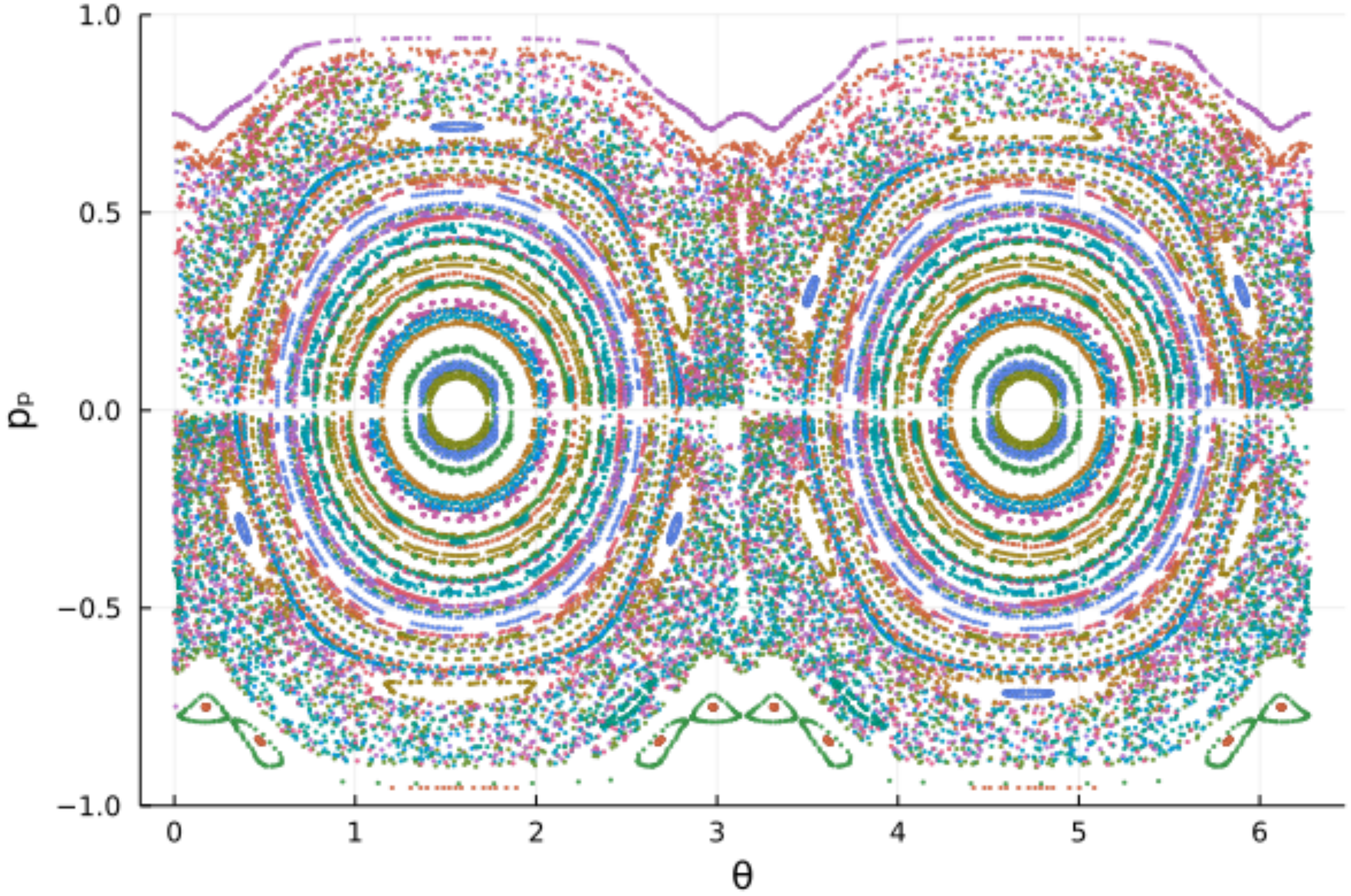


$h = 1$

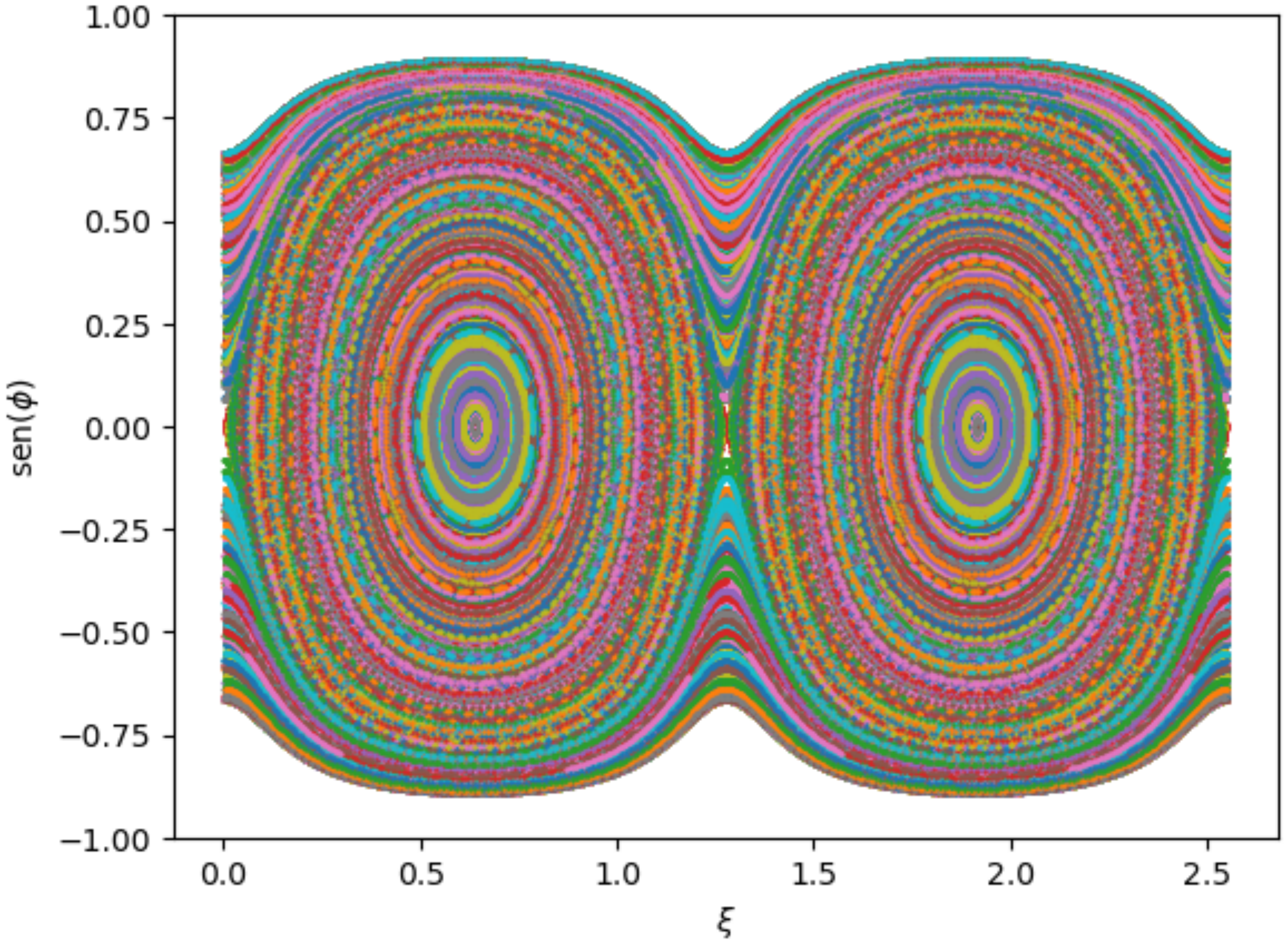


$h = 2$

CONSTRUCT COLLISION SPACE

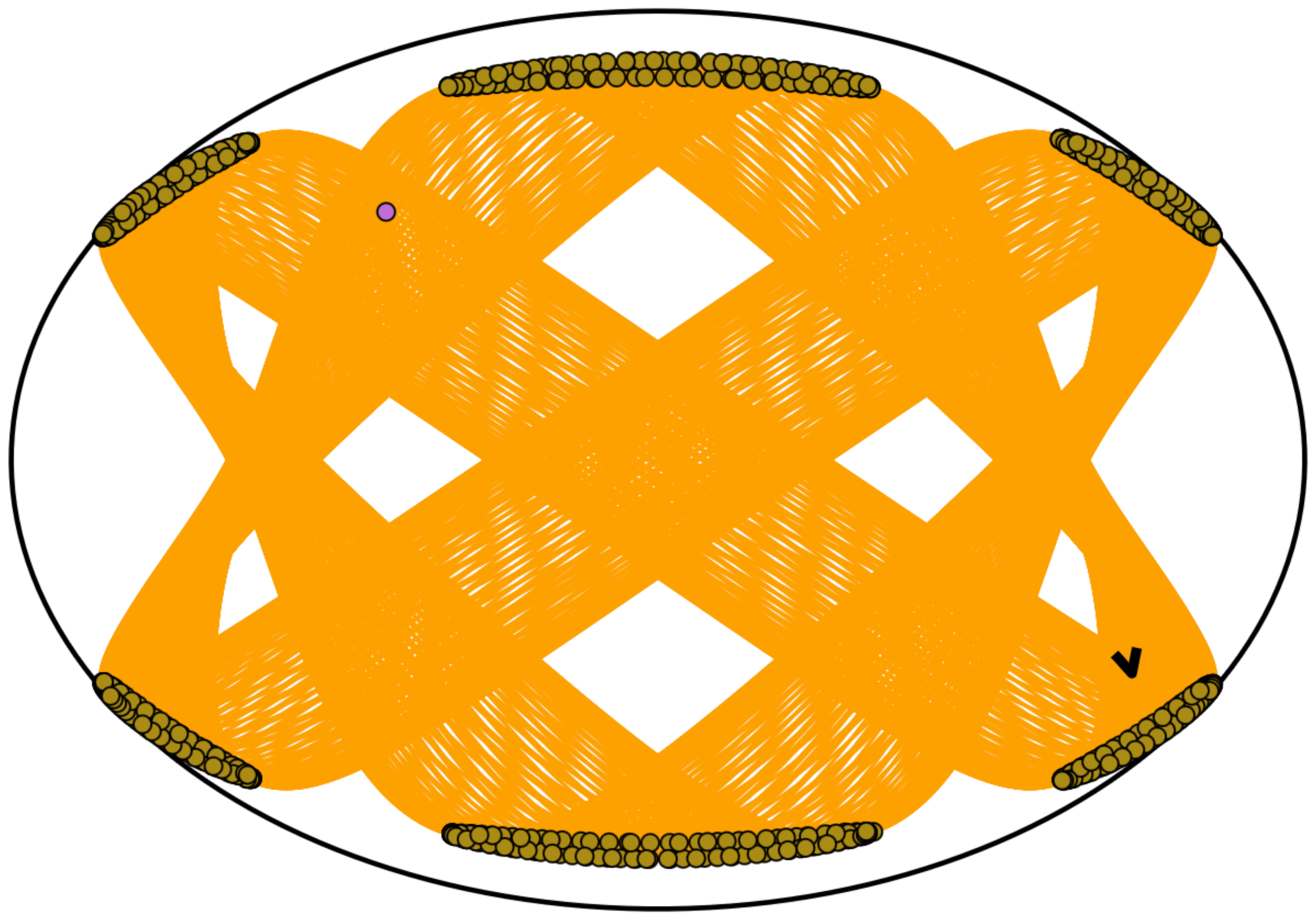


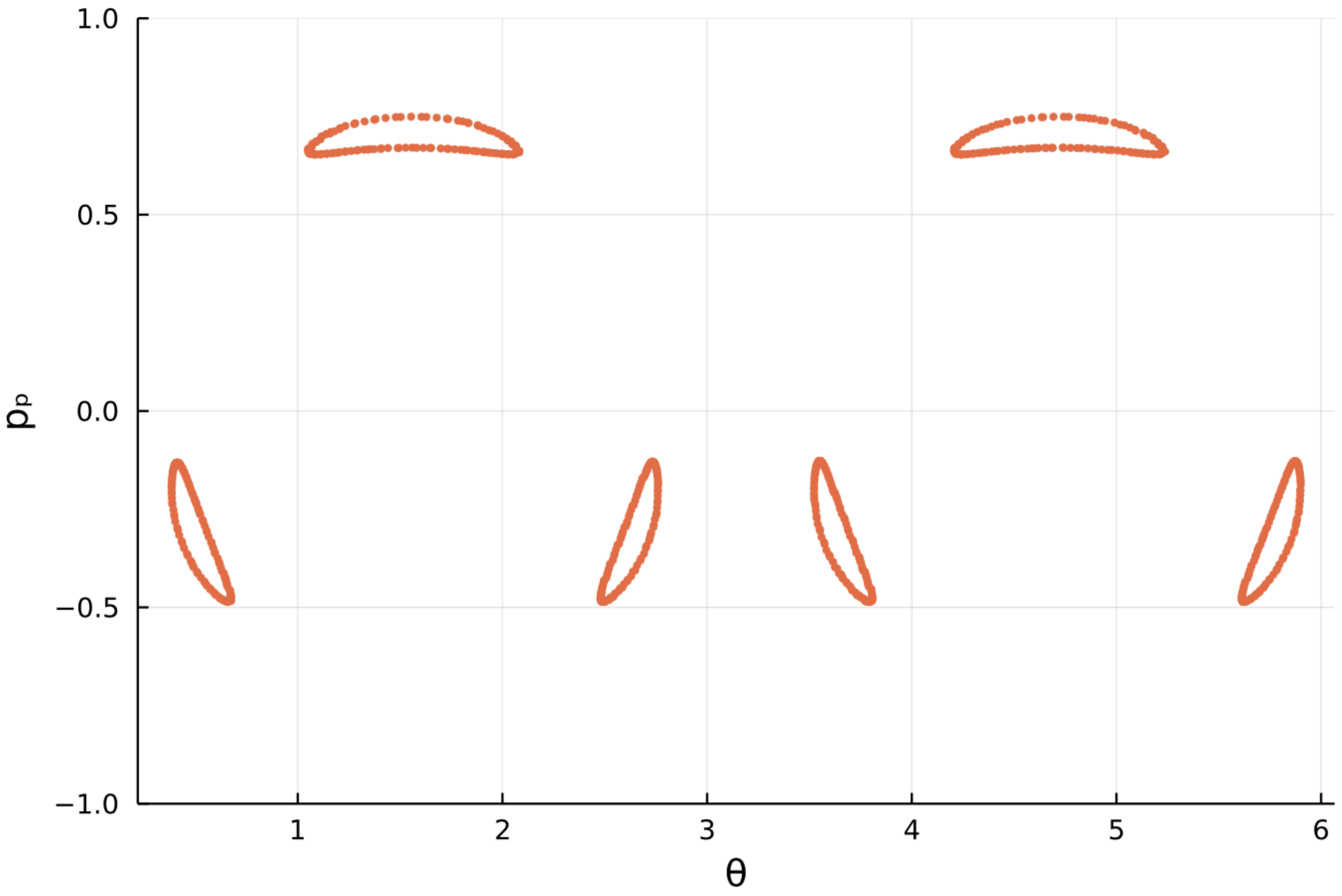
$h = 3$



$h \to \infty$

IDENTIFICATION OF TRAJECTORIES





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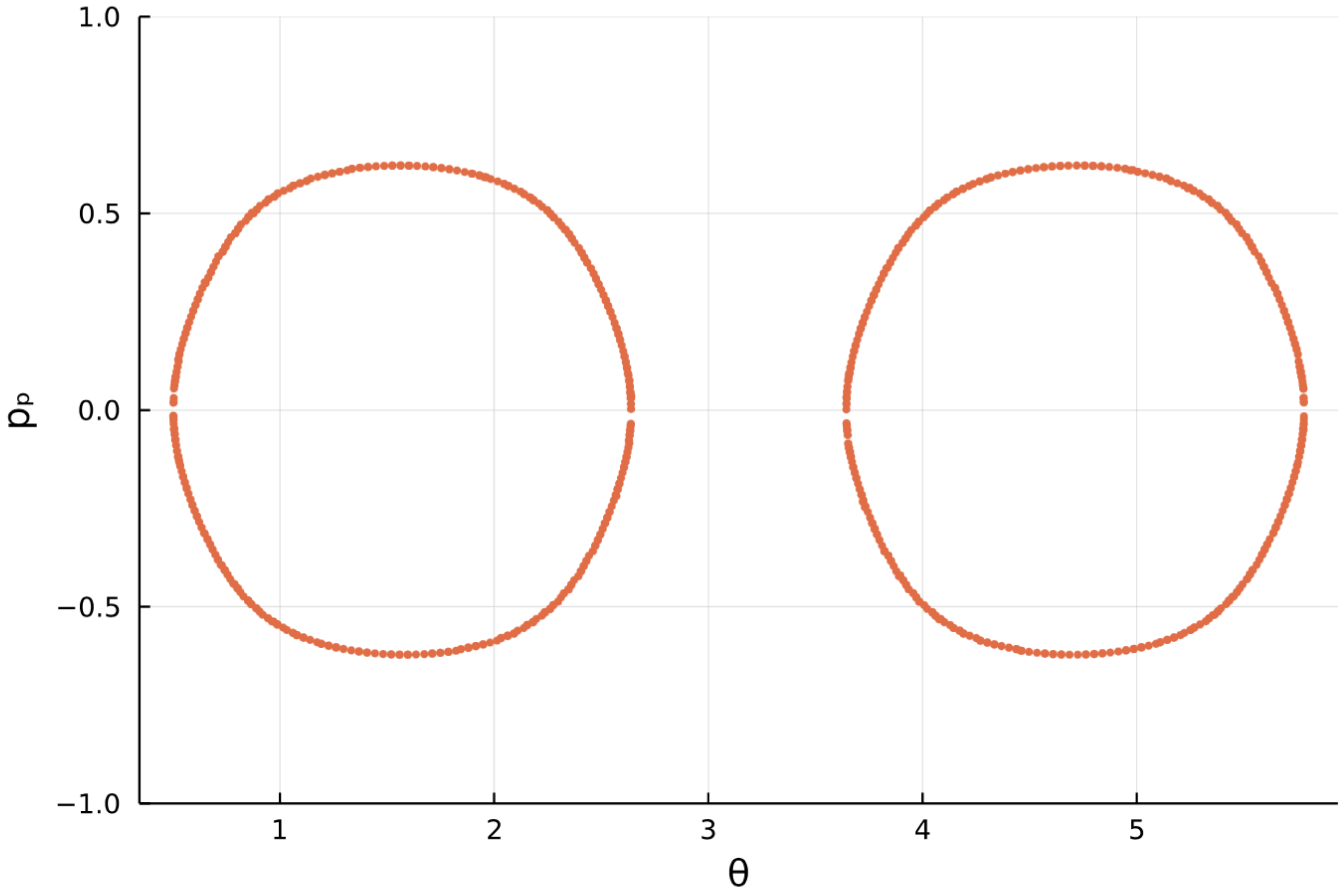

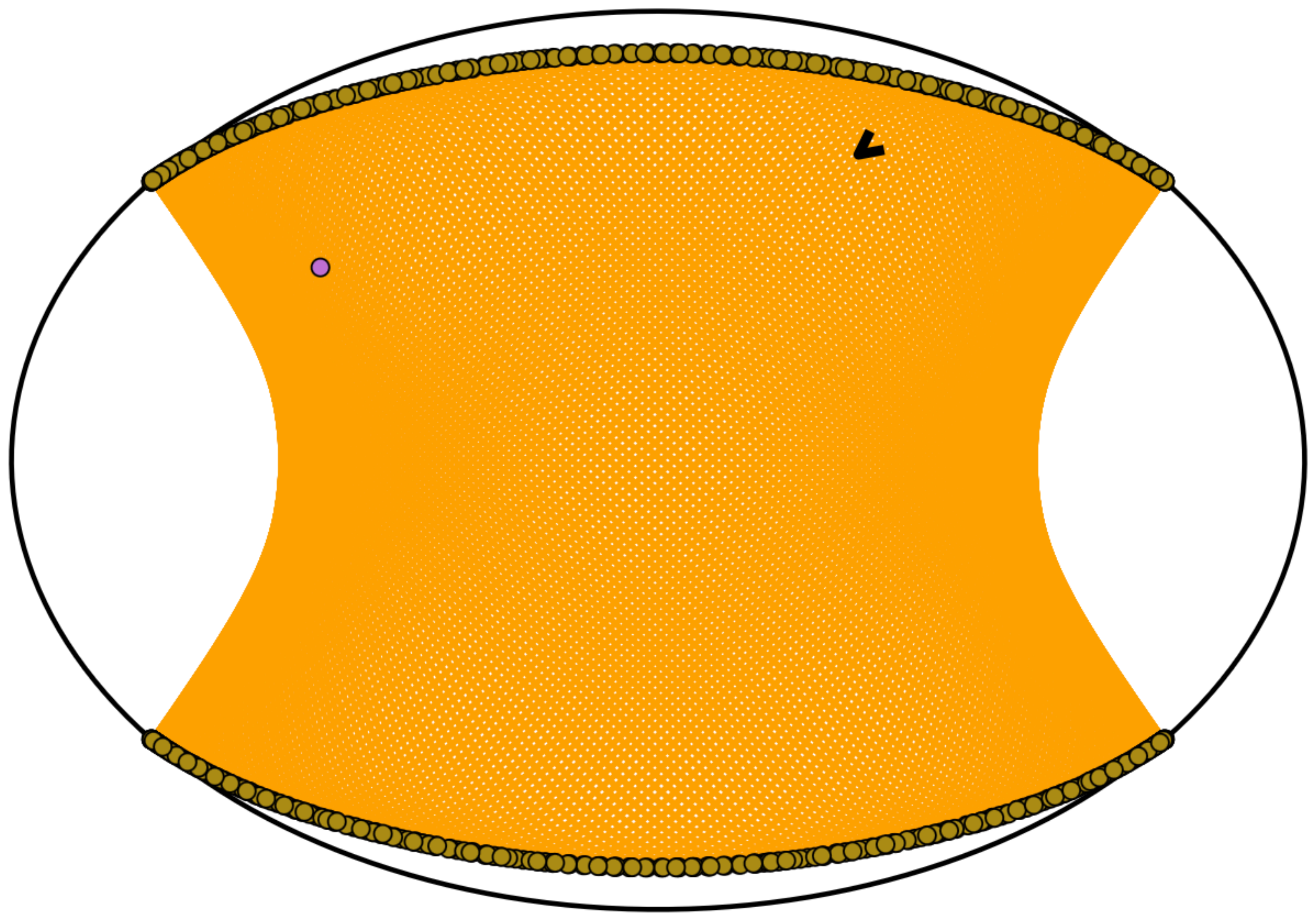

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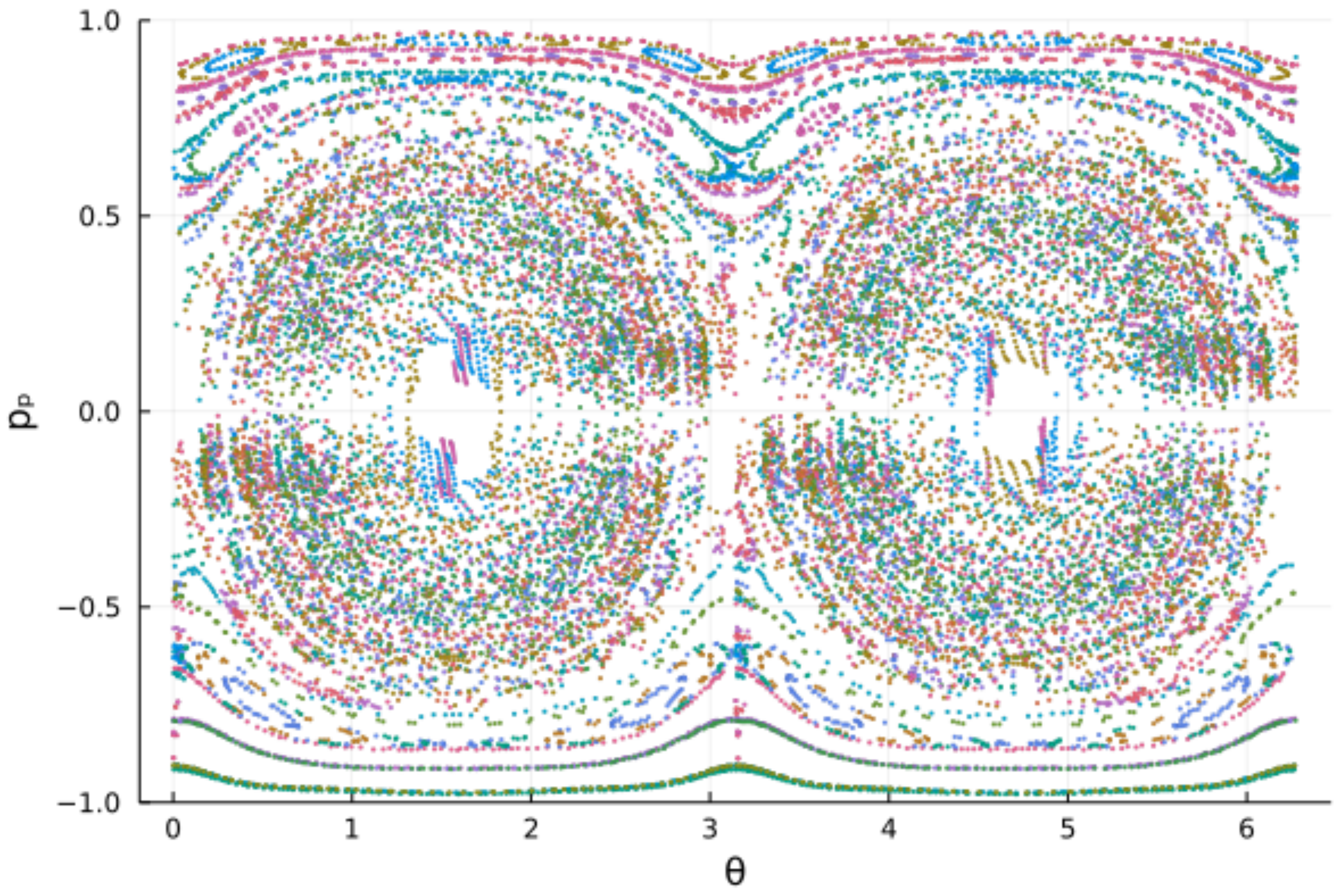
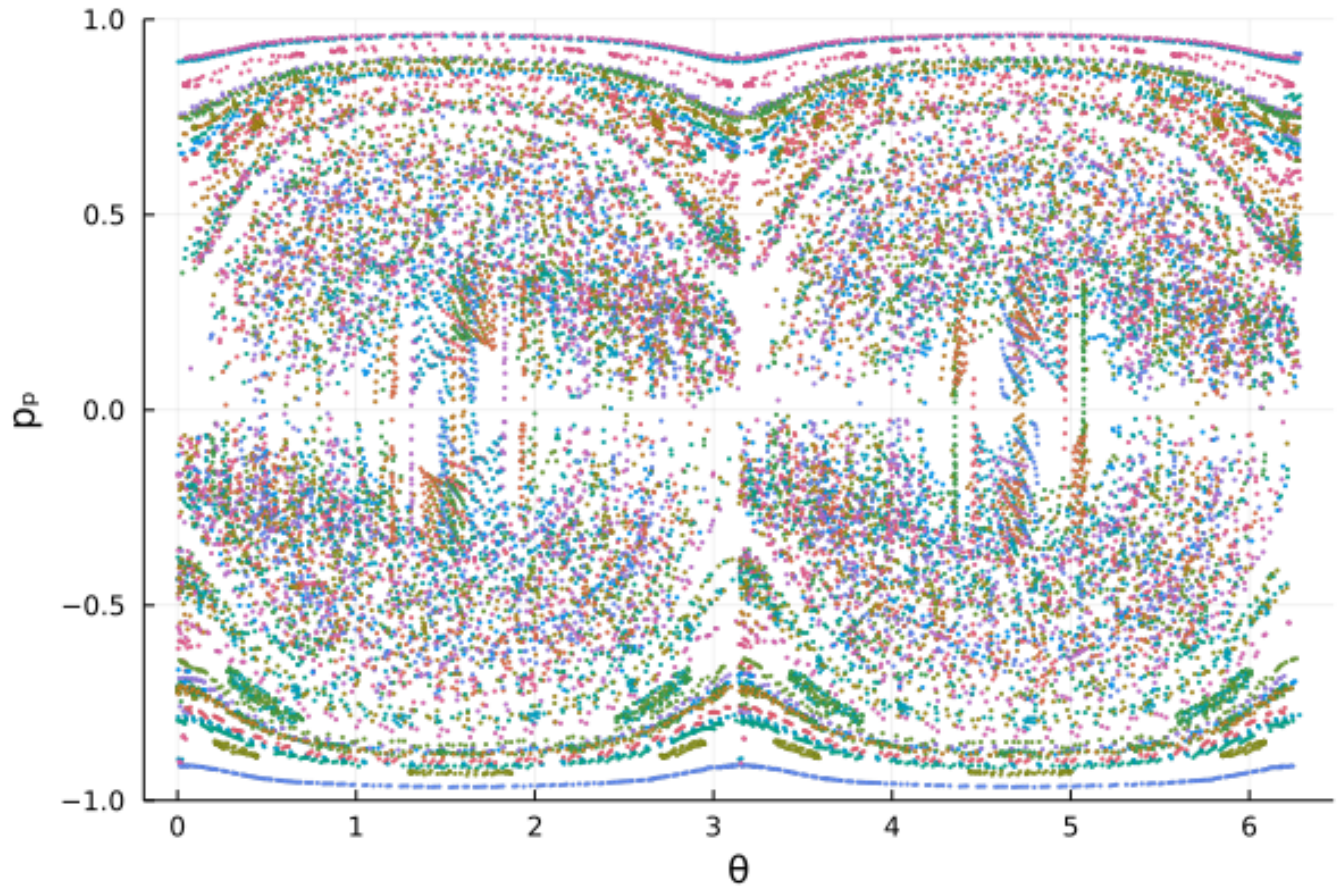


$h = 9$

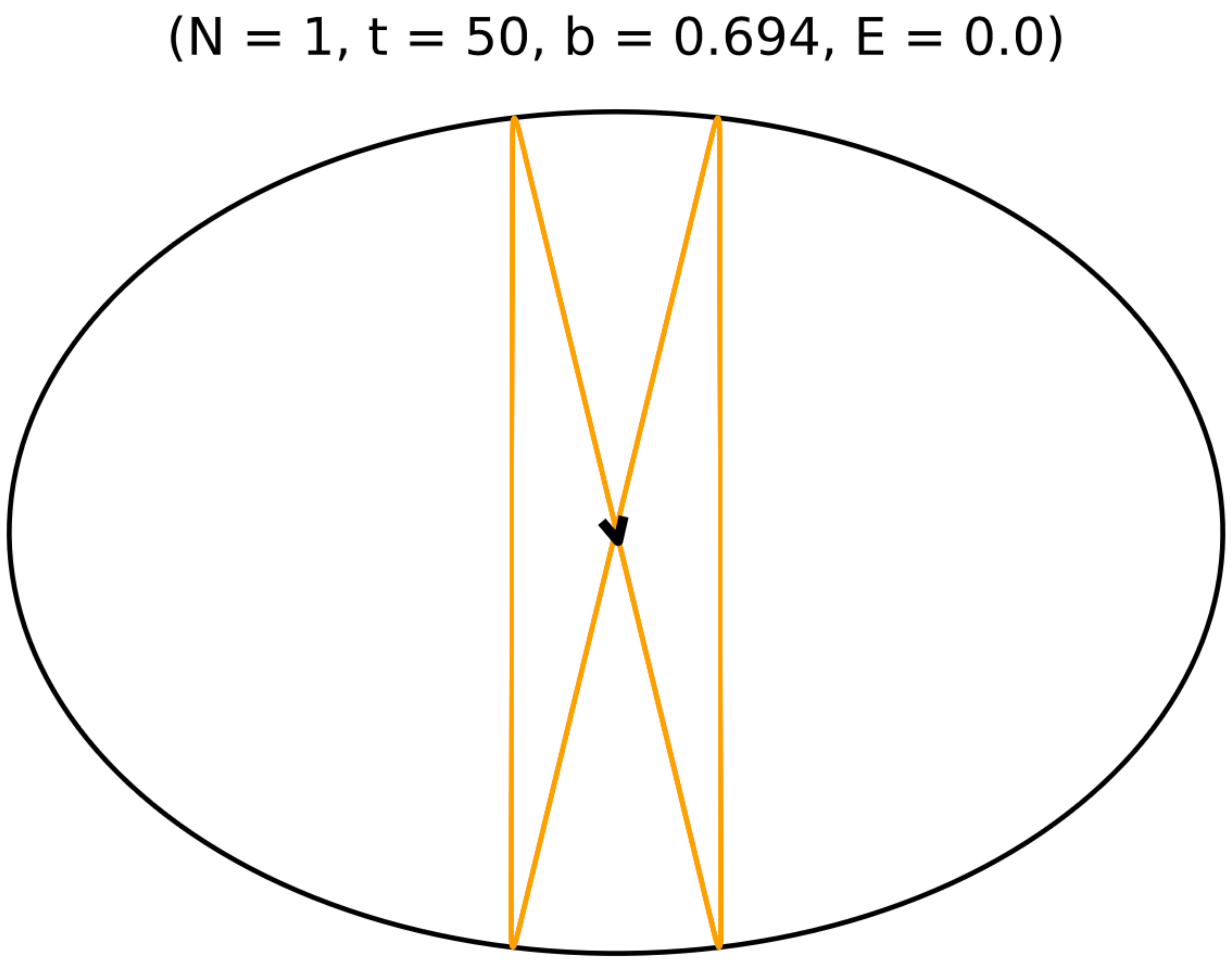


$h = 15$

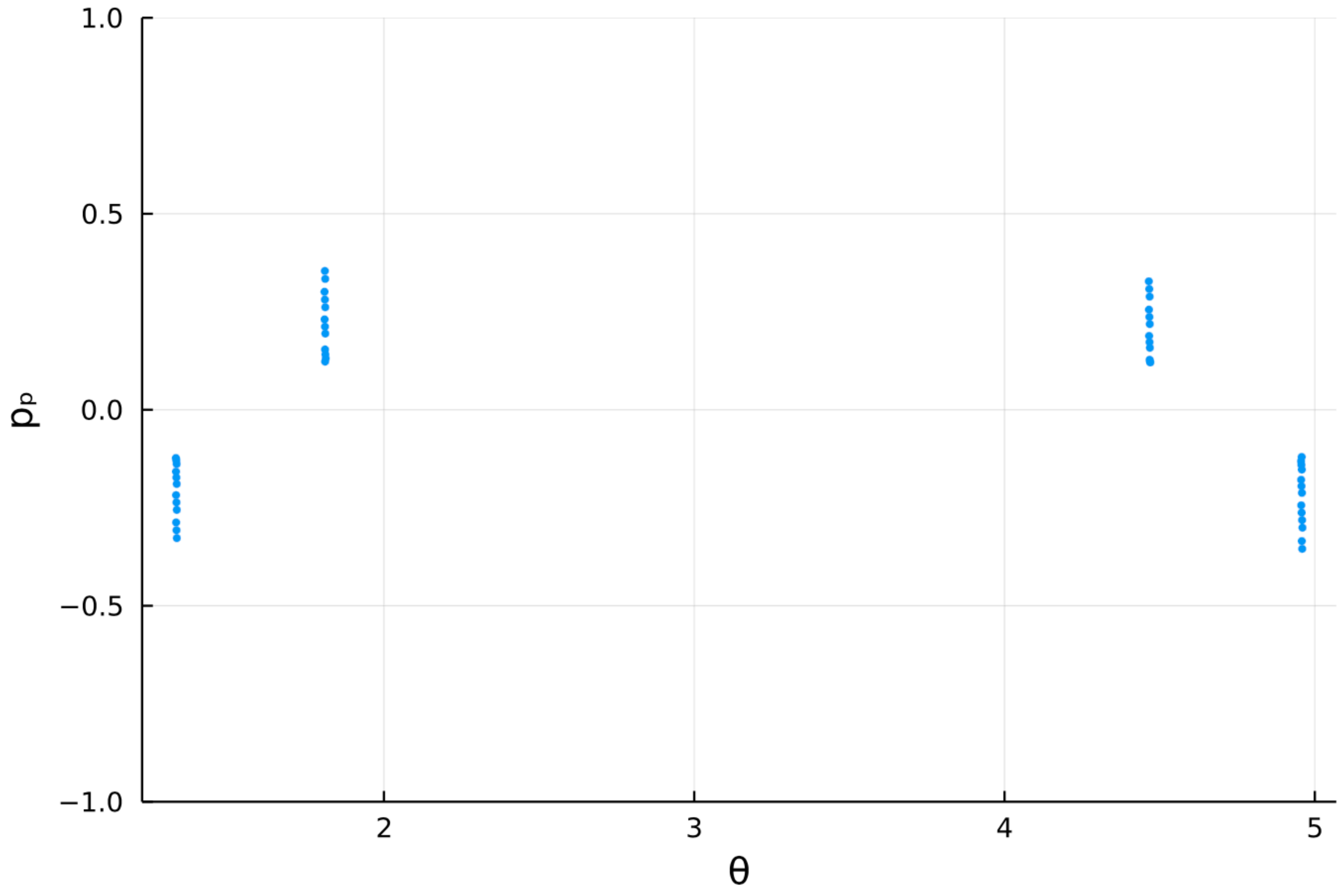
IDENTIFICATION OF TRAJECTORIES

Vertical-like collisions are problematic for the numerical algorithm.

Problem *arise* from loss of Hamiltonian character in hard-wall limit.



$h = 15$



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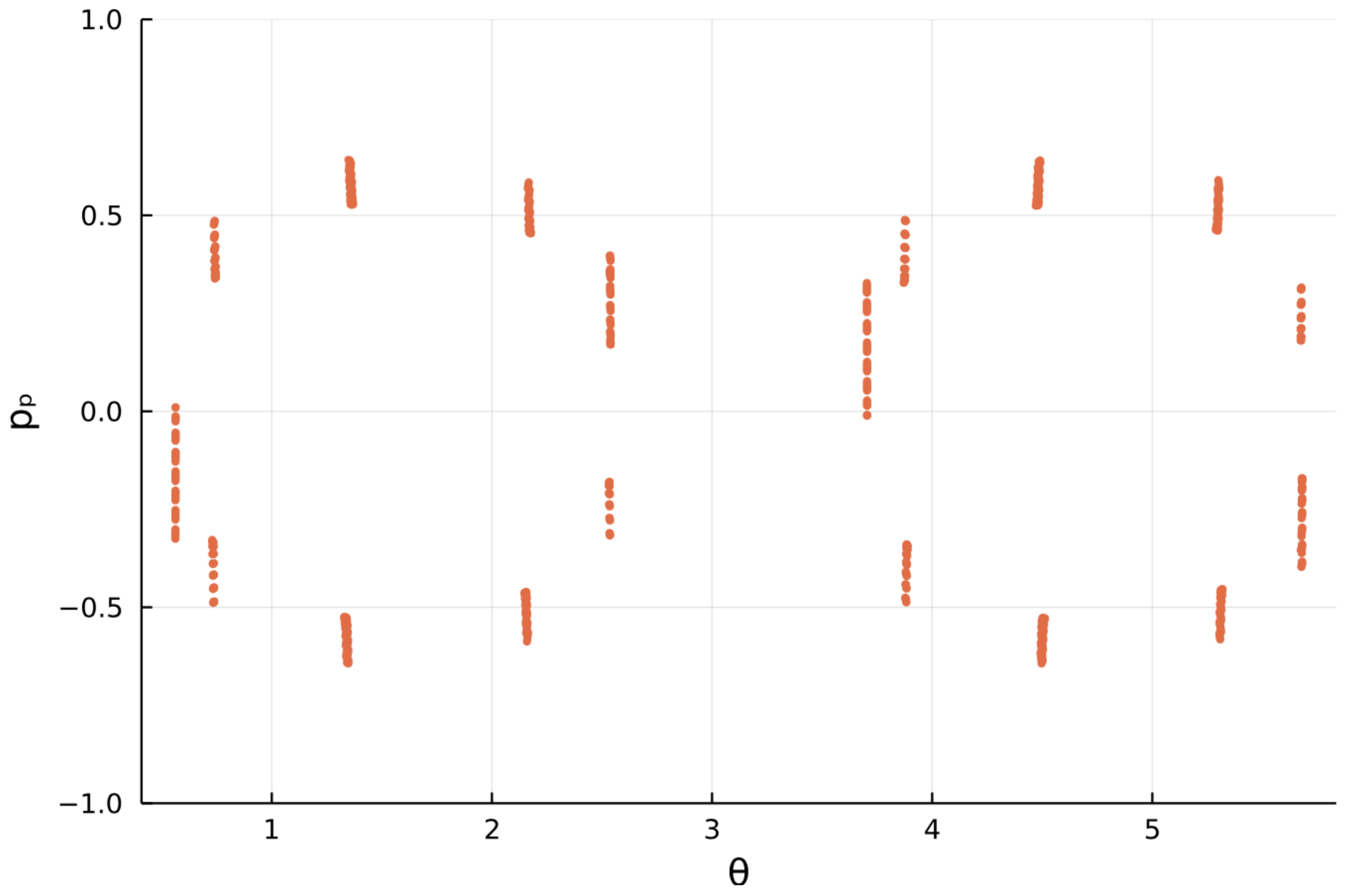
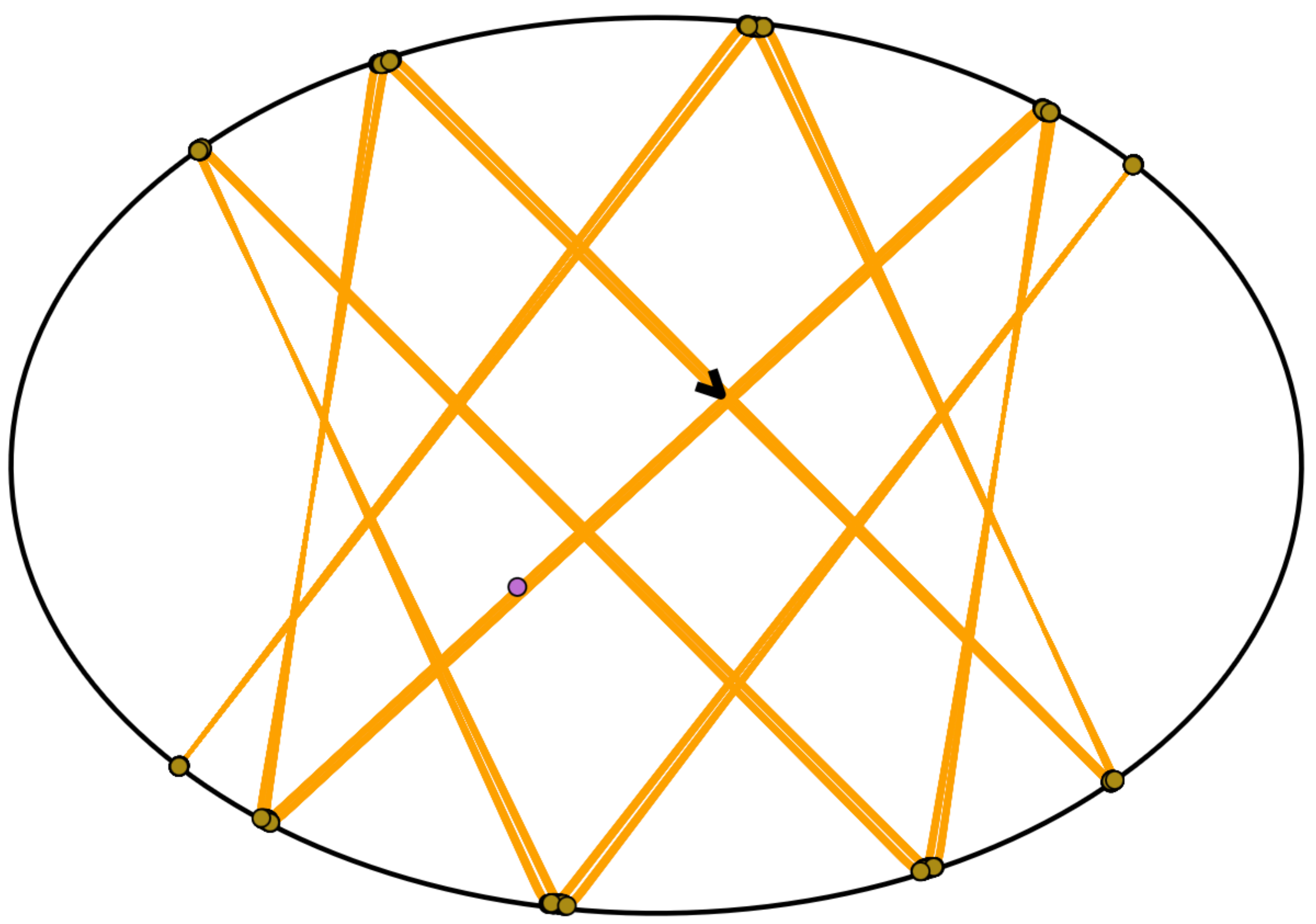
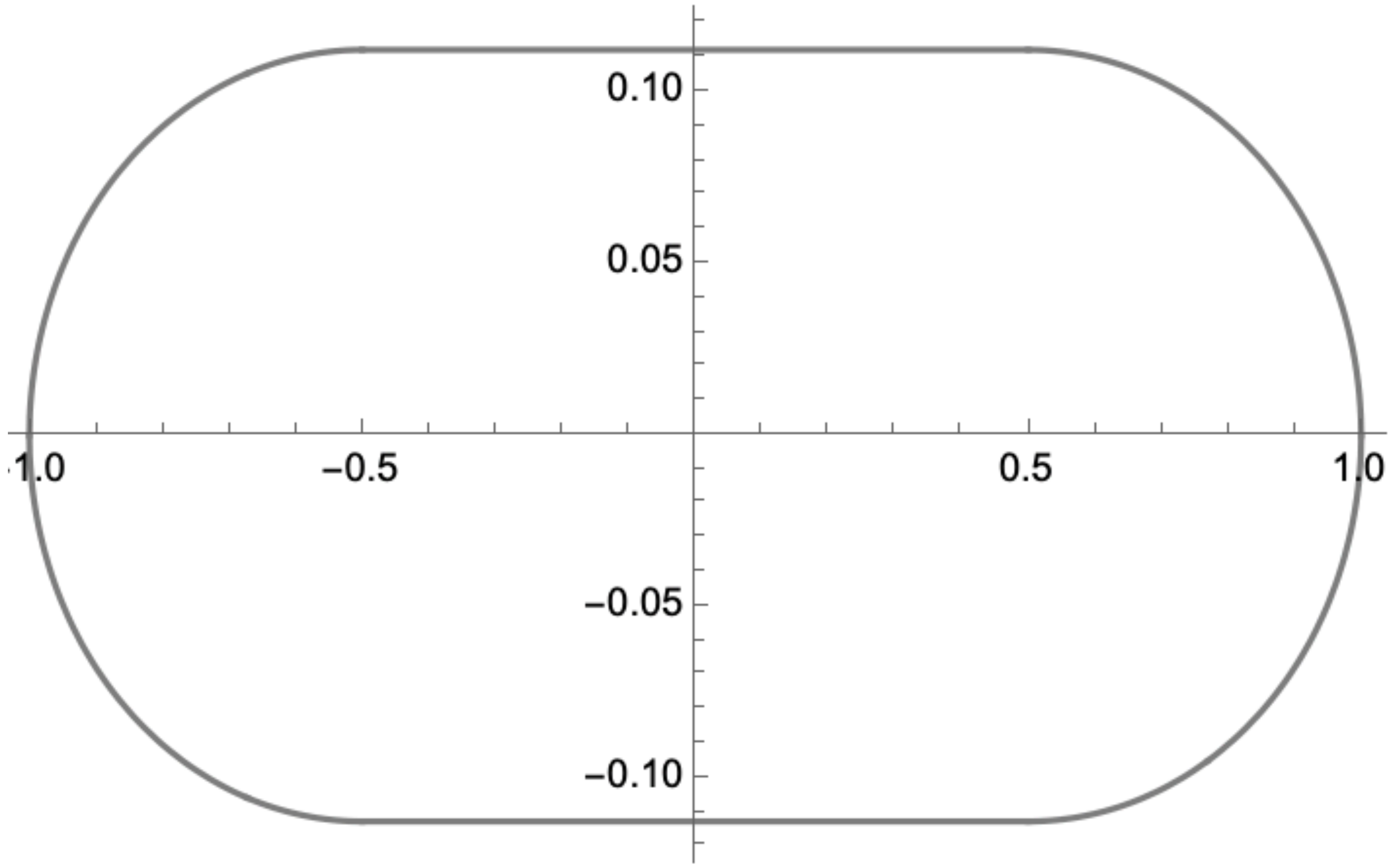




COMPARISON OF THE TOOL DEVELOPED

Looking for extend calculations in others billiards.

$$y(x) = \begin{cases} \pm\gamma & \text{if } 0 \le |x| < \delta \\ \pm\gamma\sqrt{1 - \left(\frac{|x| - \delta}{1 - \delta}\right)^2} & \text{if } \delta \le |x| \le 1. \end{cases} \qquad 0 \le \delta \le 1$$



FURTHER TOPICS FOR STUDY

- Extend the analysis between **chaos, geometry, hardness, and energy** in other billiards.
- Deepen the problem between Hamiltonian vs. non-Hamiltonian systems.
- Improving numerical implementations for stifness.
- Study **quantization** process for soft-wall billiards.
- Aim for studying **quantum chaos** in tunable systems.
- Extend our knowledge in classical soft-wall billiards for **exciton-polariton billiards**.

Thank you!

POLARITON GROUP

UAM-I

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