Spectral kissing and its dynamical consequences in the squeezed Kerr-nonlinear oscillator

UCONN
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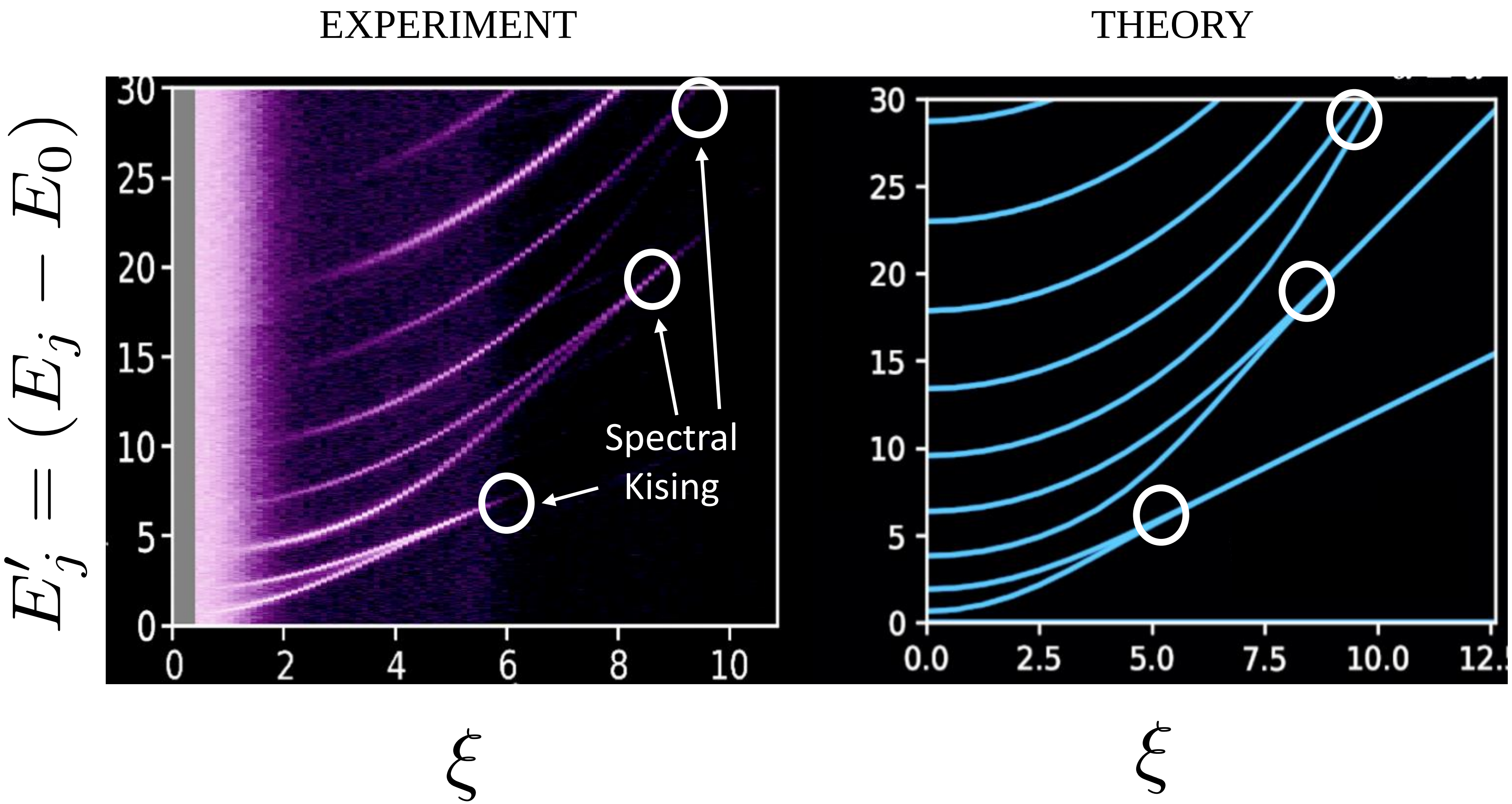
6a reunión anual del grupo de investigación en caos y termalización en sistema cuánticos de muchos cuerpos
Universidad Veracruzana
Xalapa, Veracruz, Jan 19-22, 2023

Outline

- ❑ Kerr model
    - Quantum system
    - Spectral Kissing and ESQPT
    - Density of States
    - Participation ratio
    - Husimi function of eigenstates
    - Extended Kerr model
- ❑ Classical Kerr model
    - Classical limit
- ❑ Quantum dynamics
    - Survival probability
    - FOTOC
    - Husimi entropy
- ❑ Quantum quartic oscillator
    - Stationary model
    - Driven system
    - RWA
    - Route of chaos
- ❑ Quantum Geometry

Kerr model

Kerr nonlinear oscillator



Experimental platform at Yale: superconducting circuit based on SNAIL **transmons** | arXiv:2209.03934

Spectral kissing and its dynamical consequences in the squeezed Kerr-nonlinear oscillator: | arXiv:2210.07255

Kerr model

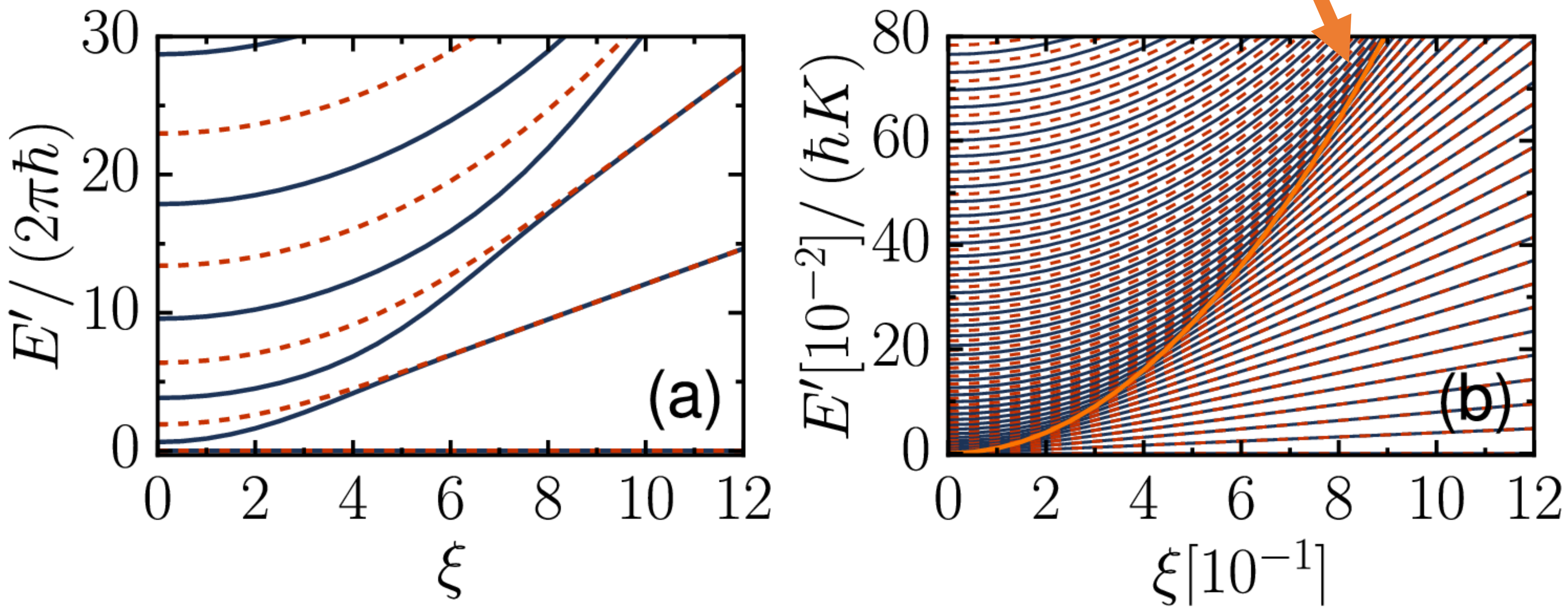
$$\frac{\hat{H}_{qu}}{\hbar K} = \underbrace{\hat{n}(\hat{n}-1)}_{\text{Kerr}} - \underbrace{\xi\left(\hat{a}^{\dagger 2} + \hat{a}^{2}\right)}_{\text{Squeezing}}$$

$\hat{n} = \hat{a}^{\dagger}\hat{a}$

$\xi = \epsilon_2/K$

$[\hat{H}_{qu}, (-1)^{\hat{a}^{\dagger}\hat{a}}] = 0$

$E'_{\mathrm{ESQPT}} = K\xi^2$



$K/(2\pi) = 0.32$ MHz

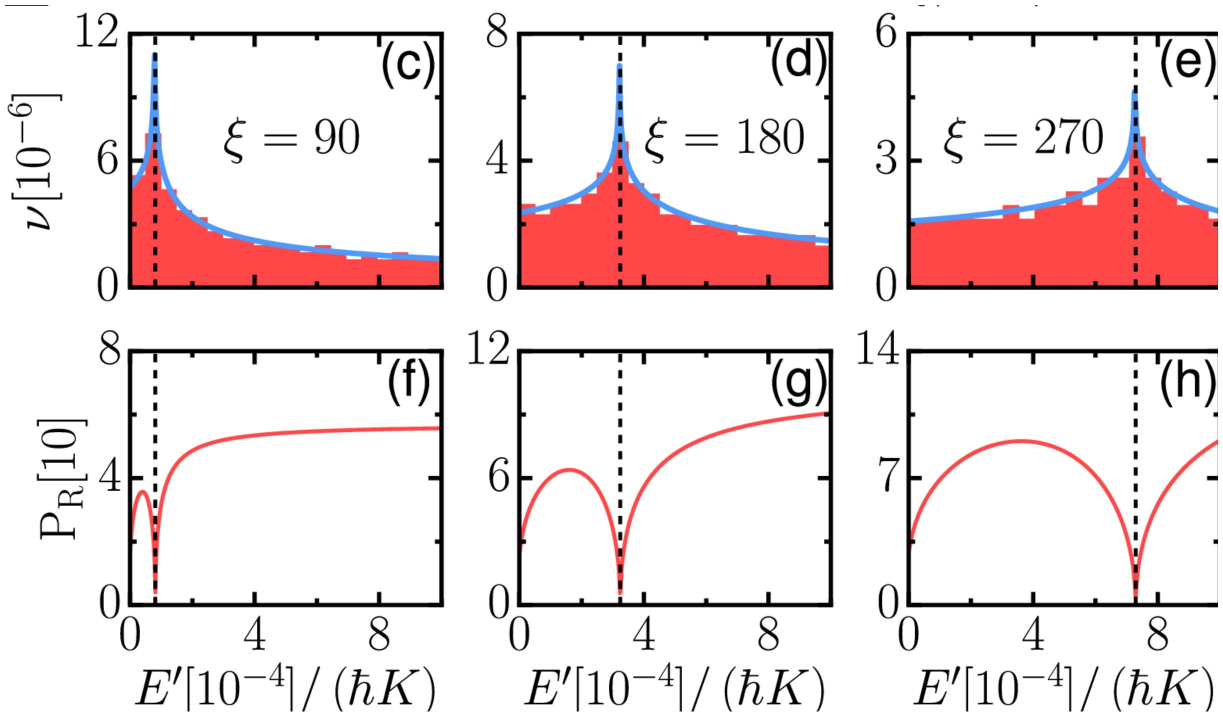
Spectral kissing and its dynamical consequences in the squeezed Kerr-nonlinear oscillator: arXiv:2210.07255

Kerr model

Density of states

Participation ratio

$$\mathrm{P_R} = 1/\sum_{n=0}^{\mathcal{N}-1} |C_n|^4$$



Spectral kissing and its dynamical consequences in the squeezed Kerr-nonlinear oscillator: arXiv:2210.07255

Kerr model

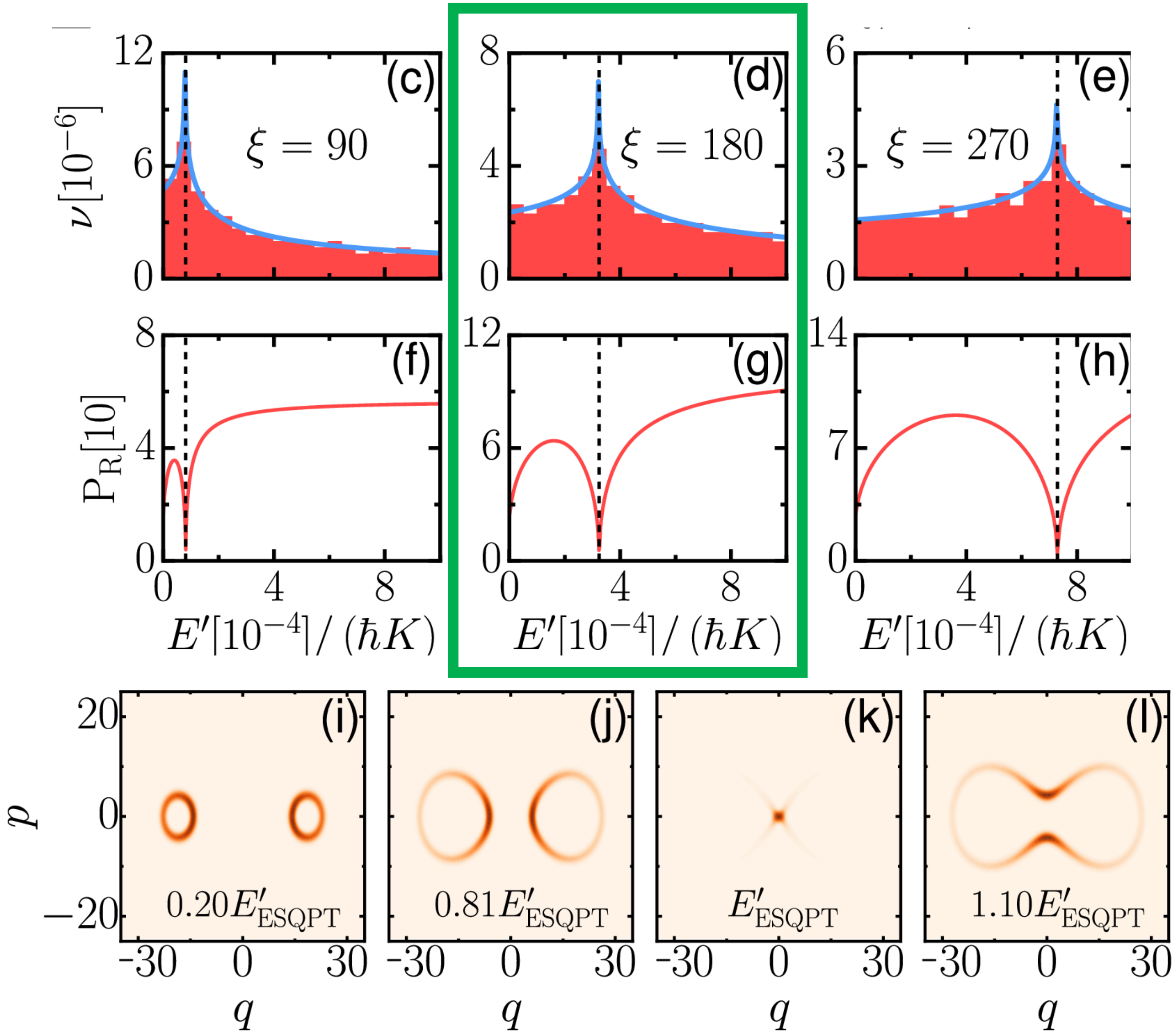
Density of states

Participation ratio

$$P_R = 1/\sum_{n=0}^{\mathcal{N}-1} |C_n|^4$$

Husimi function $\xi = 180$

$$Q^{\psi}(q,p) = \frac{1}{2\pi}\left|\sum_{n=0}^{\mathcal{N}} C_n e^{-\frac{(q^2+p^2)}{4}} \frac{(q-ip)^n}{\sqrt{2^n n!}}\right|^2$$



Spectral kissing and its dynamical consequences in the squeezed Kerr-nonlinear oscillator: arXiv:2210.07255

Classical limit

Quantum Hamiltonian with P parameters and operators that describes it: $\hat{H} = \hat{H}(\hat{o}_1, ..., \hat{o}_n, P)$

Coherent states: $|\alpha\rangle$ with a classical map in phase space $\alpha = \alpha(q, p)$

In our model $\hat{H} = \hat{H}(a, a^\dagger, P)$

Glauber coherent states $\begin{cases} a|\alpha\rangle = \alpha|\alpha\rangle \\ a^\dagger|\alpha\rangle = \alpha^*|\alpha\rangle \end{cases}$

$$h_{cl} = \lim_{\hbar \to 0} \langle\alpha|\hat{H}|\alpha\rangle$$

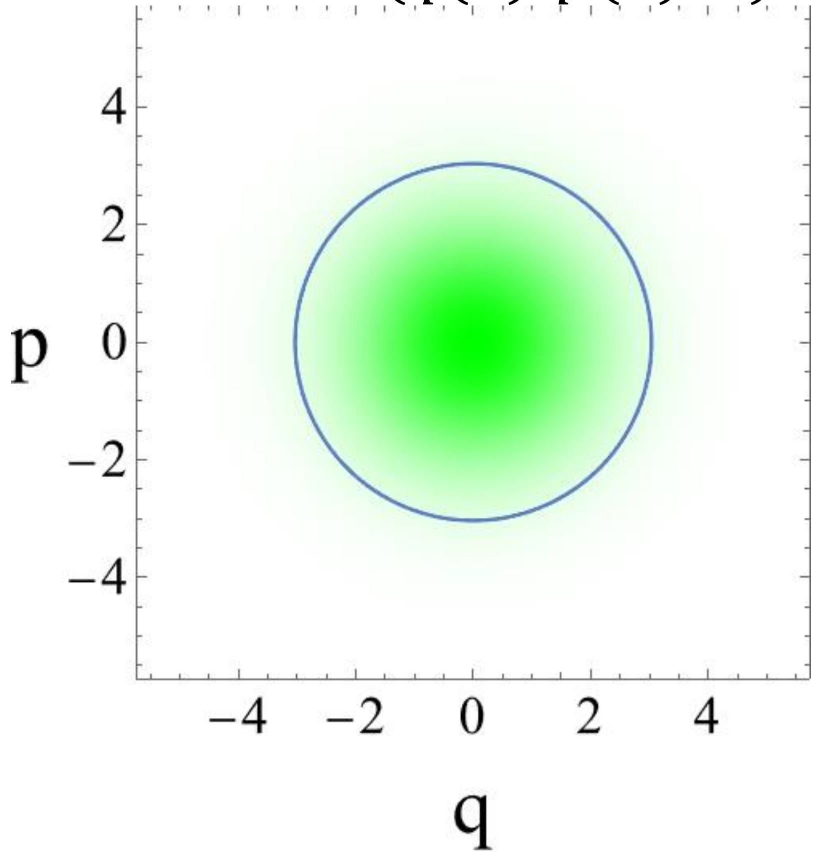
$$h_{cl} = h_{cl}(q, p)$$

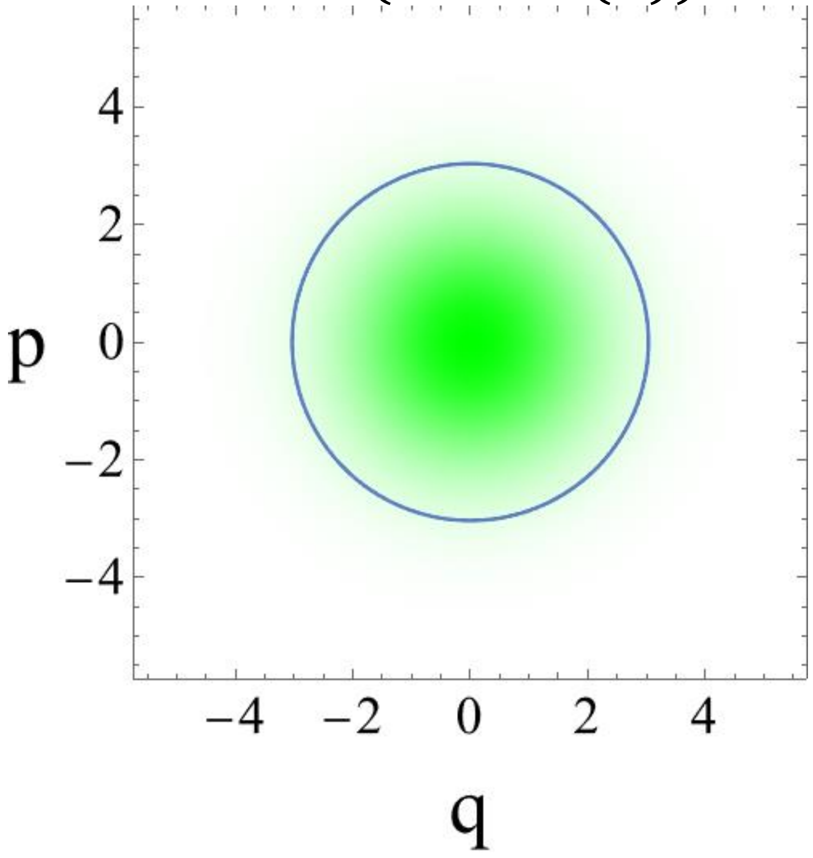
Classical limit

$$\hat{q} = \sqrt{\frac{\hbar}{2}}\left(a + a^{\dagger}\right) \qquad \hat{p} = i\sqrt{\frac{\hbar}{2}}\left(a^{\dagger} - a\right)$$

$$\hat{H} = \hat{H}(\hat{q}(\hbar), \hat{p}(\hbar), P) \qquad \hat{H} = \hat{H}(a, a^{\dagger}, P(\hbar))$$

$\hbar = 1$



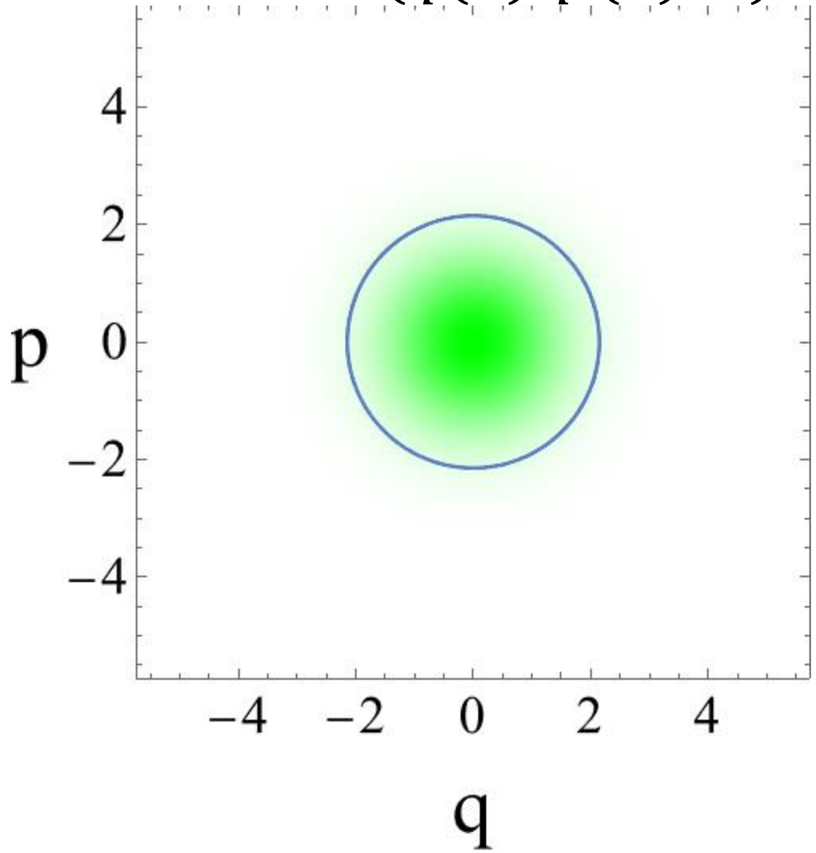


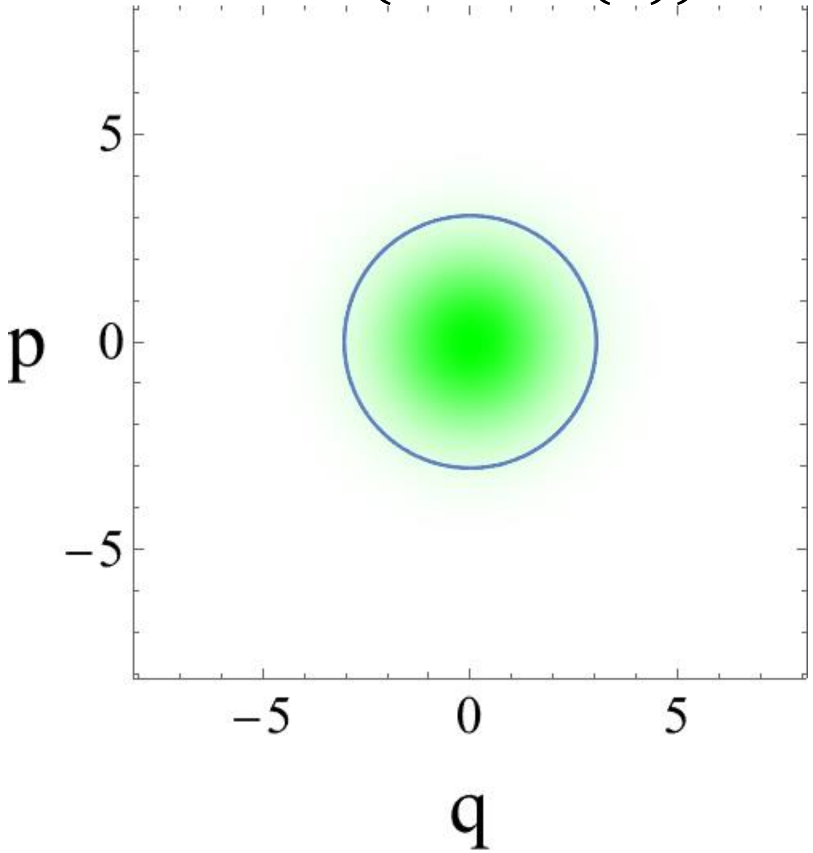
Classical limit

$$\hat{q} = \sqrt{\frac{\hbar}{2}}\left(a + a^{\dagger}\right) \qquad \hat{p} = i\sqrt{\frac{\hbar}{2}}\left(a^{\dagger} - a\right)$$

$\hbar = 0.5$

$$\hat{H} = \hat{H}(\hat{q}(\hbar), \hat{p}(\hbar), P) \qquad \hat{H} = \hat{H}(a, a^{\dagger}, P(\hbar))$$



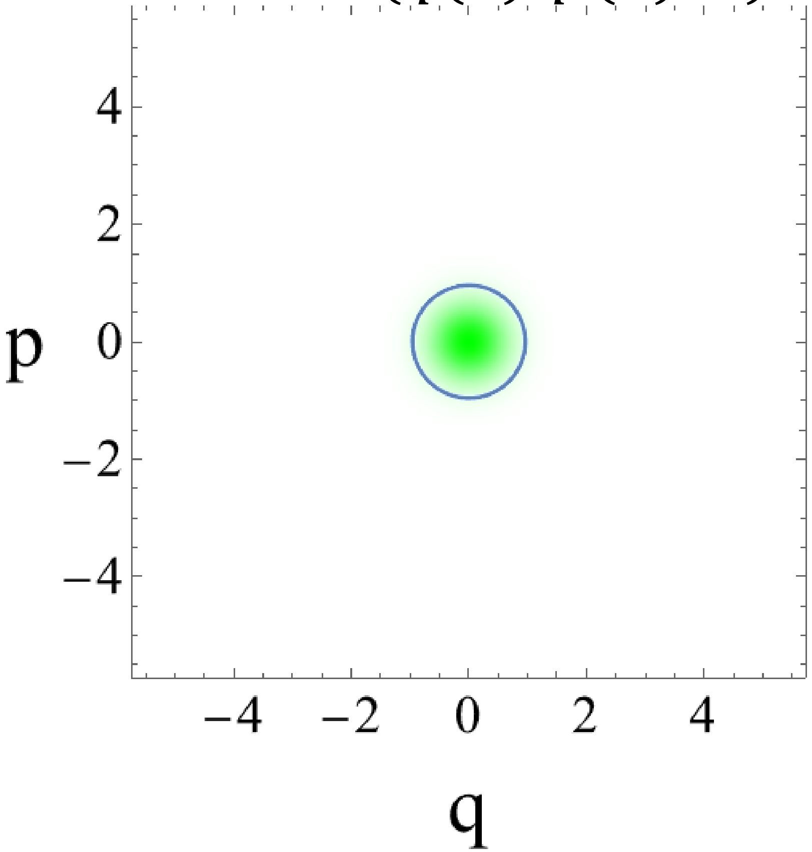


Classical limit

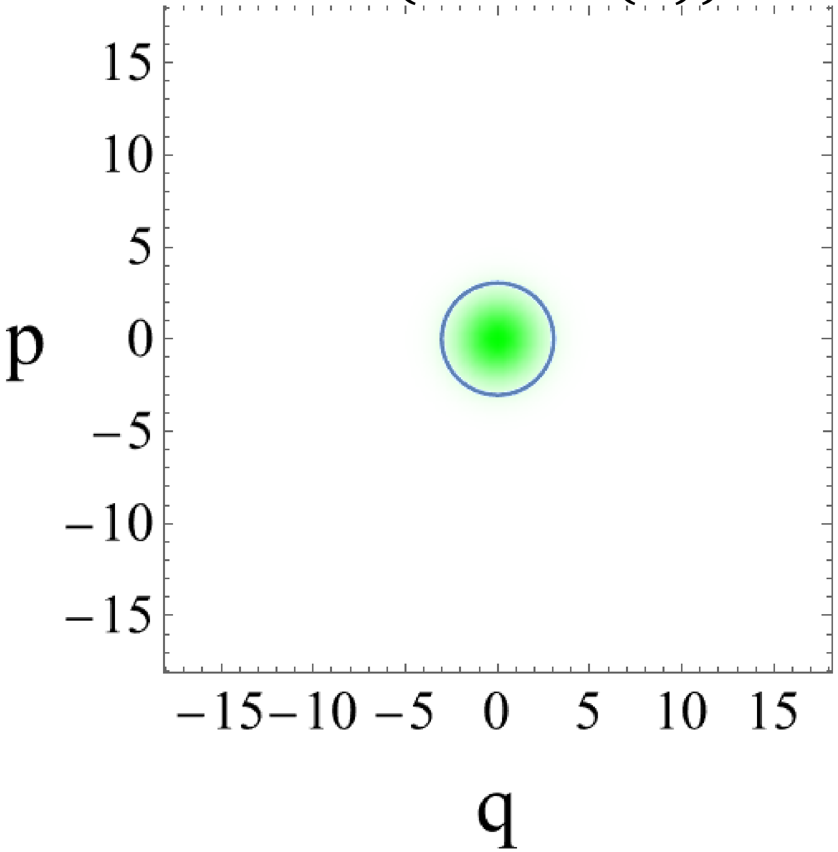
$$\hat{q} = \sqrt{\frac{\hbar}{2}}\left(a + a^\dagger\right) \qquad \hat{p} = i\sqrt{\frac{\hbar}{2}}\left(a^\dagger - a\right)$$

$\hbar = 0.1$

$\widehat{H} = \widehat{H}(\hat{q}(\hbar), \hat{p}(\hbar), P)$



$\widehat{H} = \widehat{H}(a, a^\dagger, P(\hbar))$

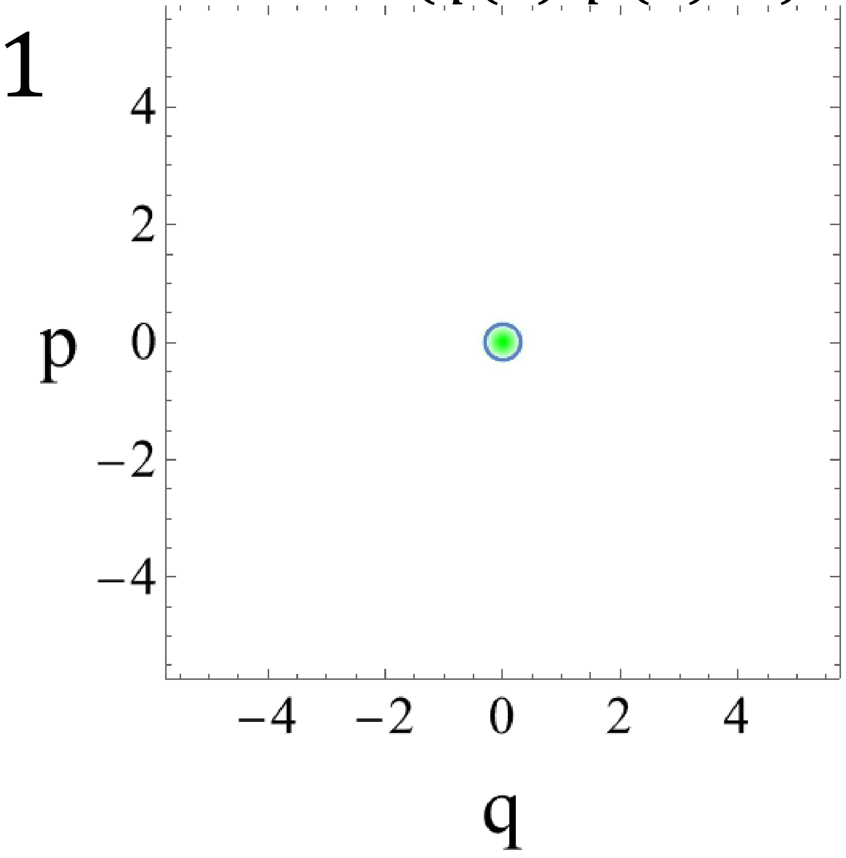


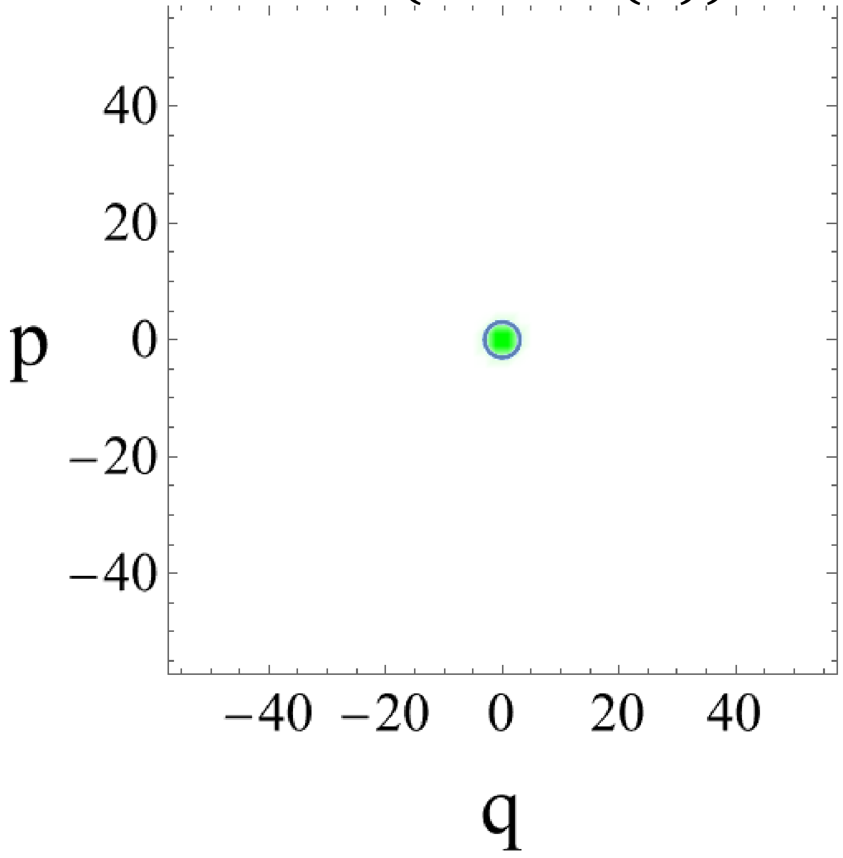
Classical limit

$$\hat{q} = \sqrt{\frac{\hbar}{2}}\left(a + a^{\dagger}\right) \qquad \hat{p} = i\sqrt{\frac{\hbar}{2}}\left(a^{\dagger} - a\right)$$

$$\hat{H} = \hat{H}(\hat{q}(\hbar), \hat{p}(\hbar), P) \qquad \hat{H} = \hat{H}(a, a^{\dagger}, P(\hbar))$$

$\hbar = 0.01$



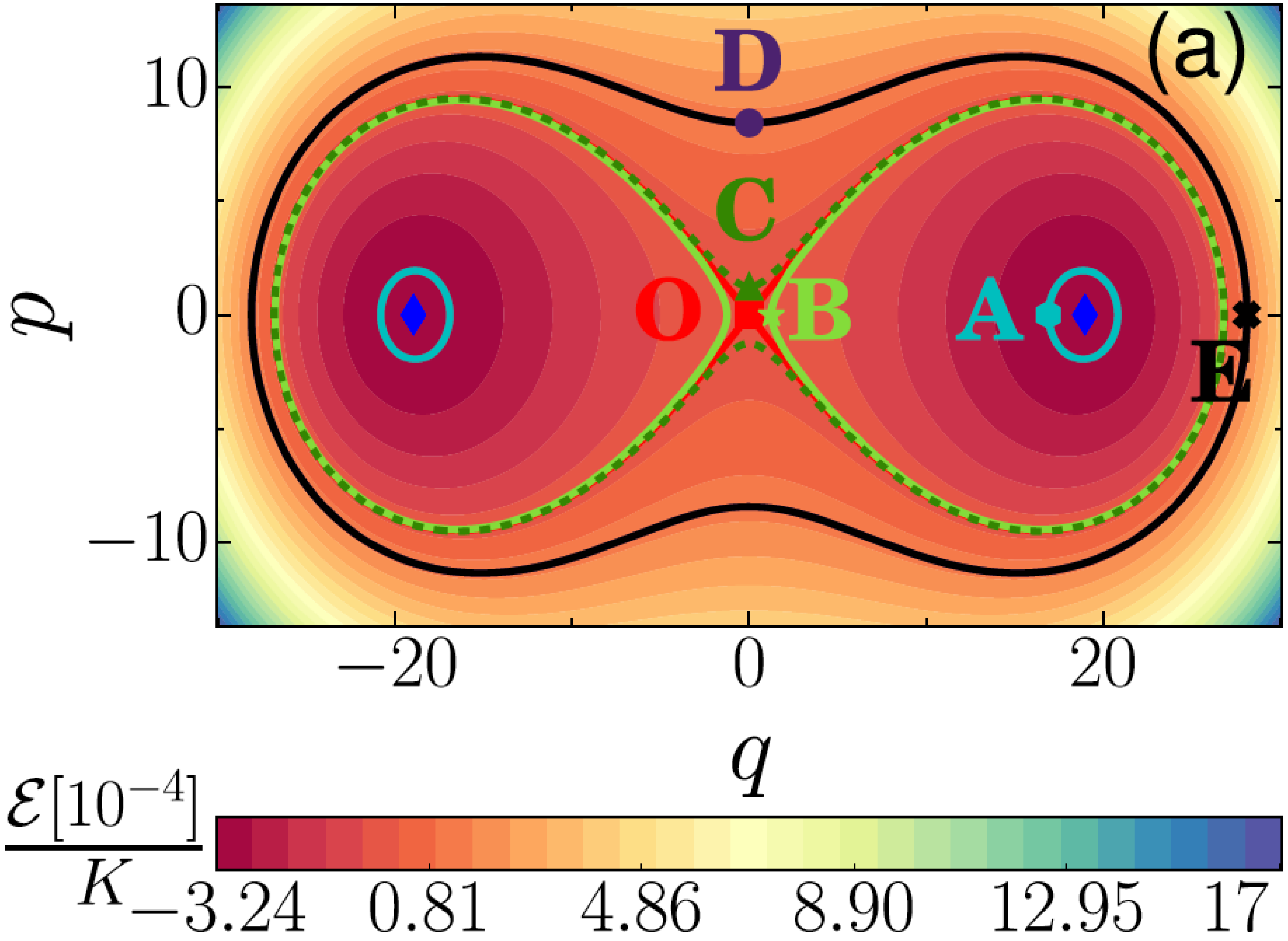


Kerr classical limit

$$\frac{H_{cl}}{K} = \frac{1}{4}(q^2 + p^2)^2 - \xi(q^2 - p^2)$$

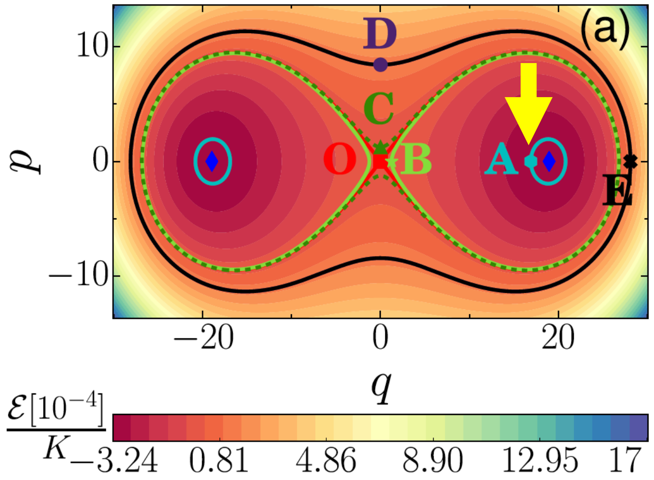
Lyapunov exponent of the unstable point

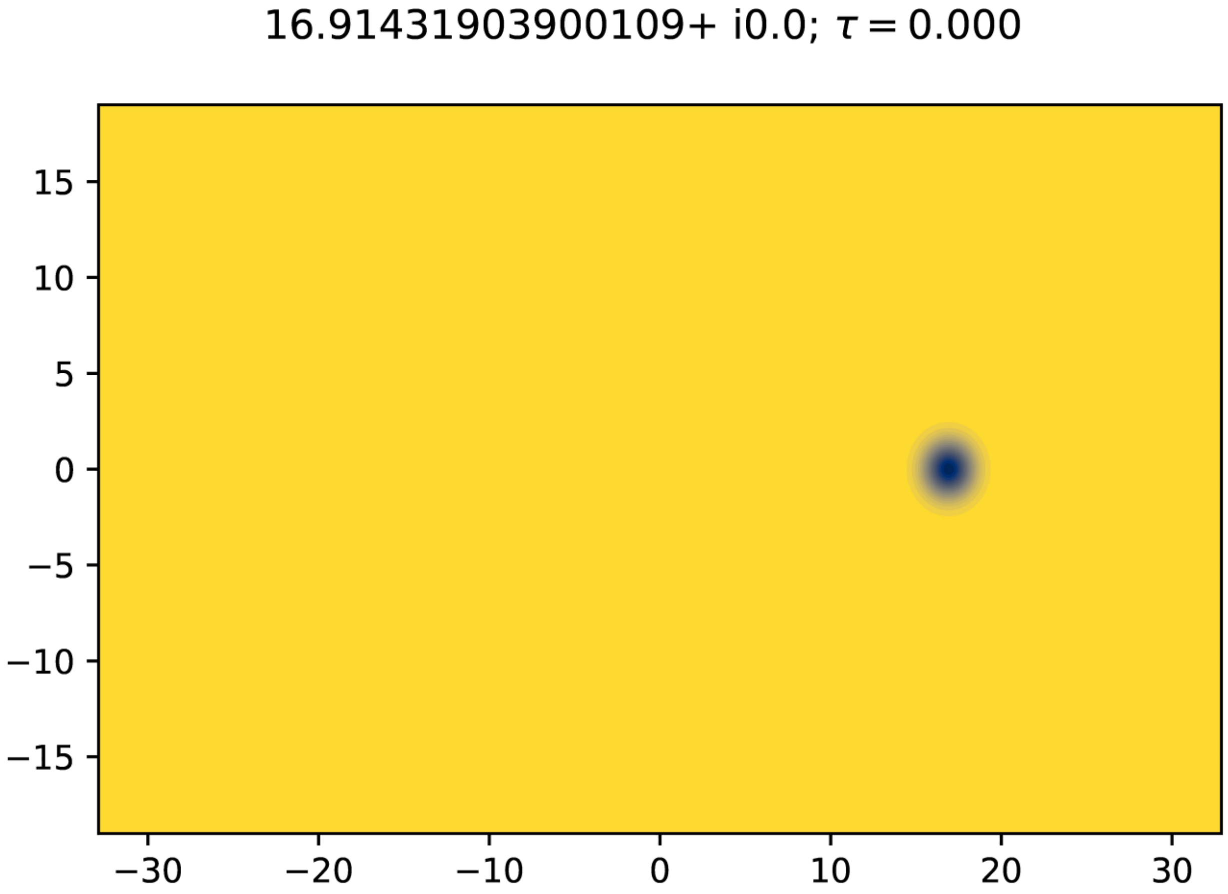
$$\lambda = 2K\xi$$



Spectral kissing and its dynamical consequences in the squeezed Kerr-nonlinear oscillator: arXiv:2210.07255

Kerr quantum dynamics





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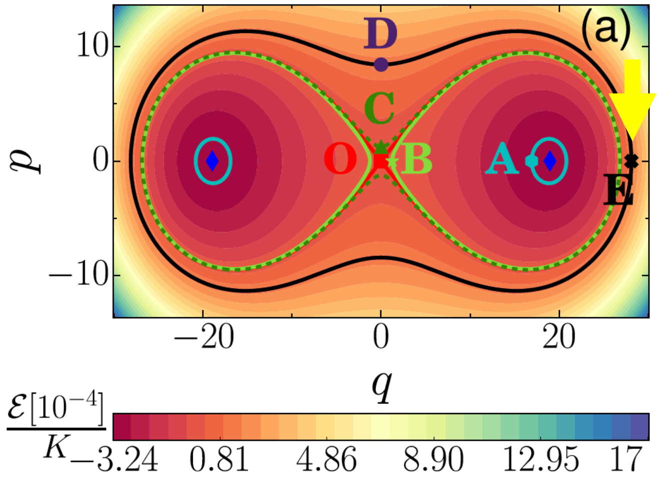

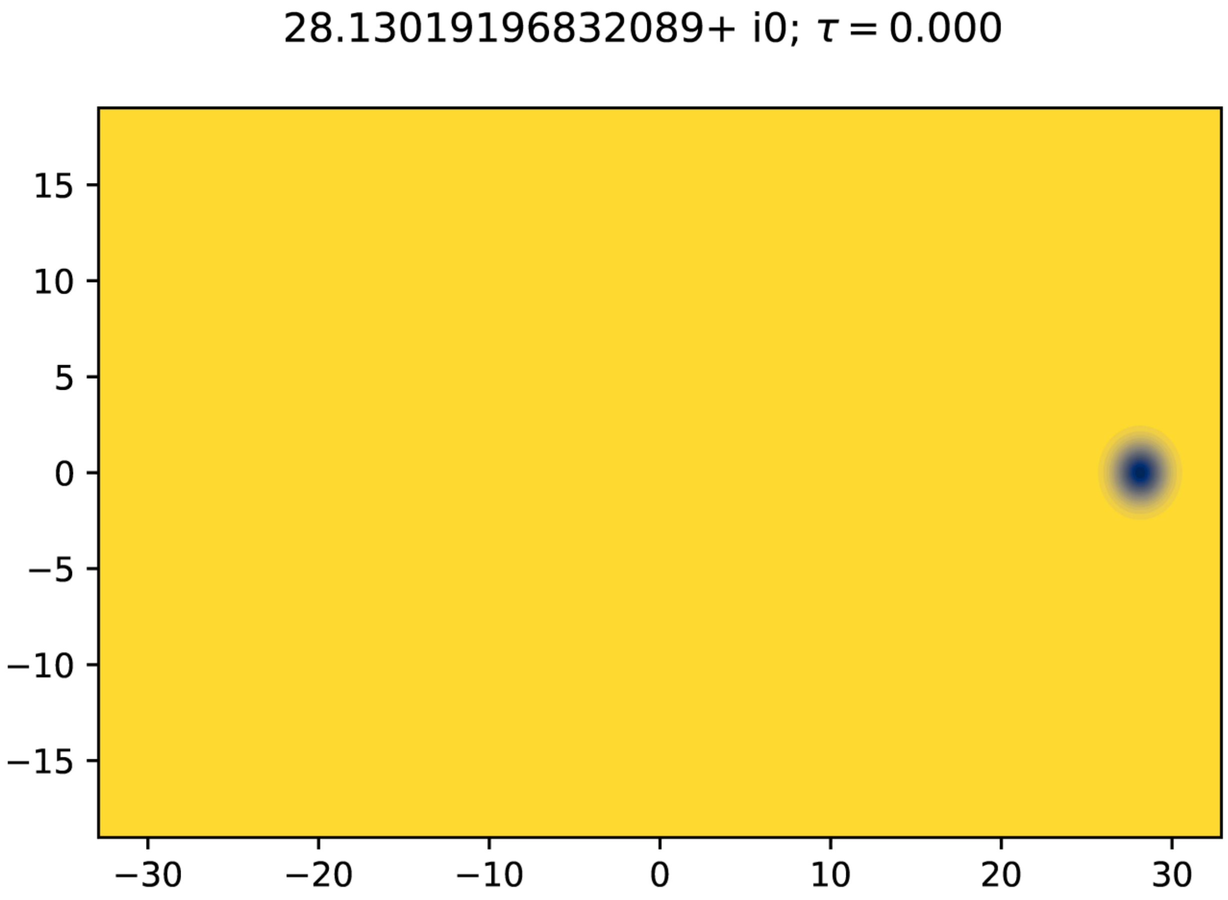

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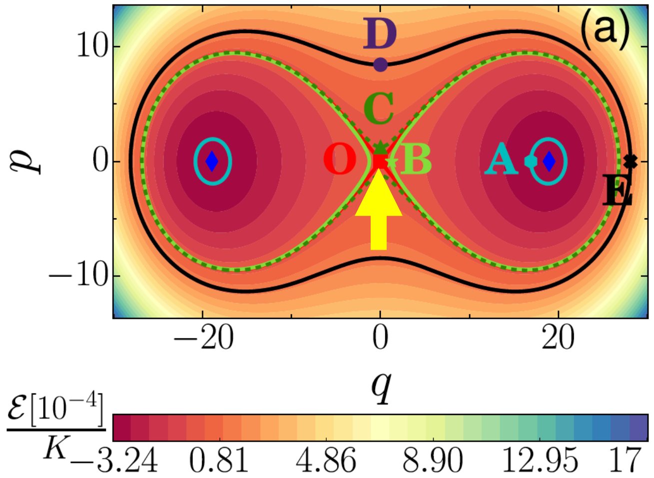

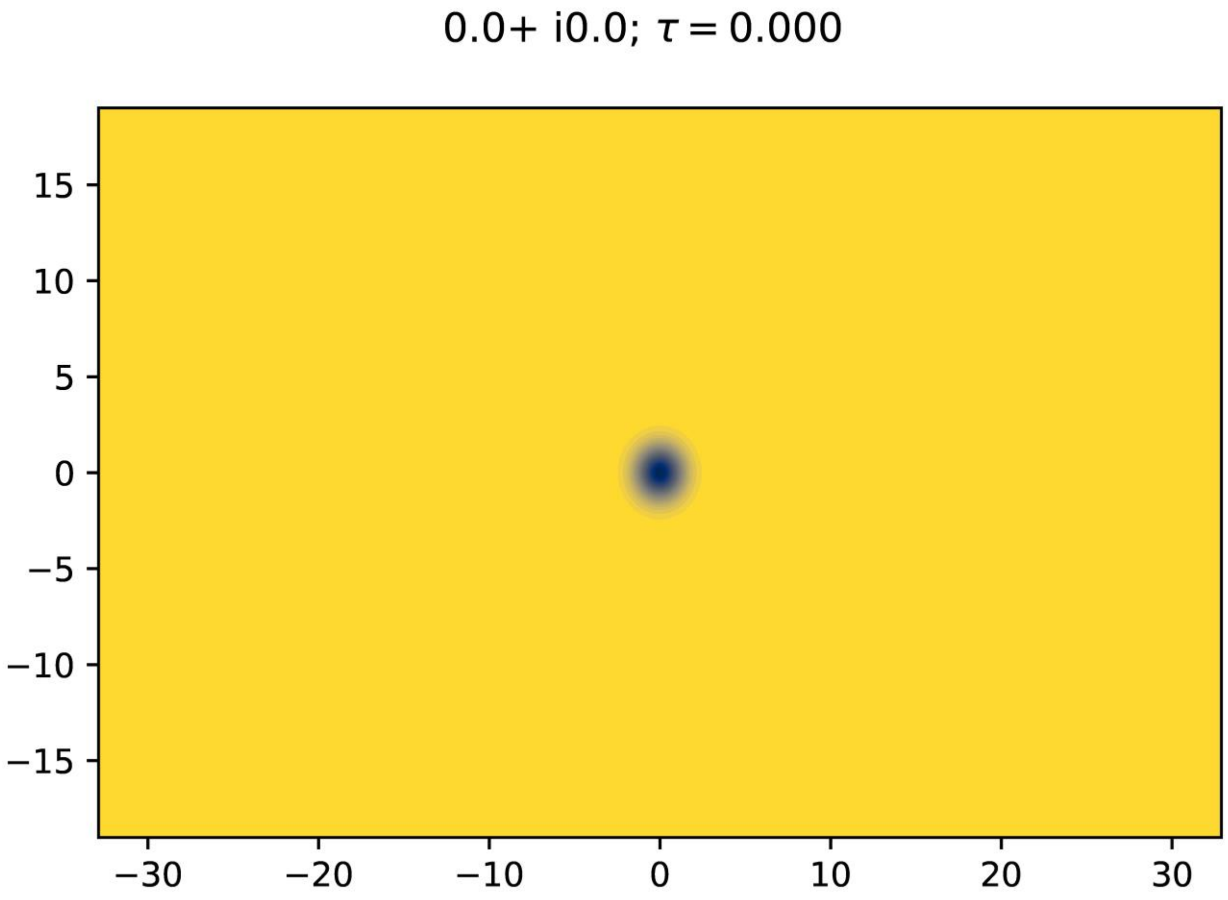

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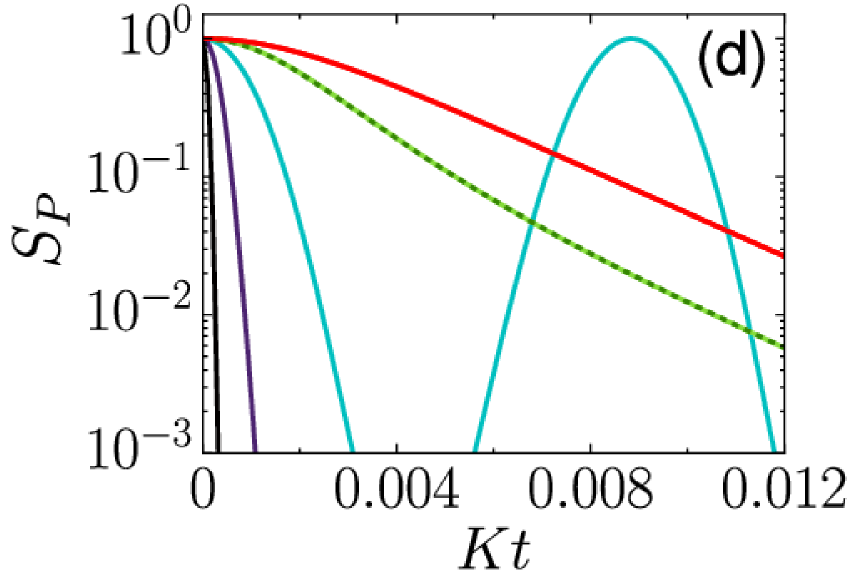
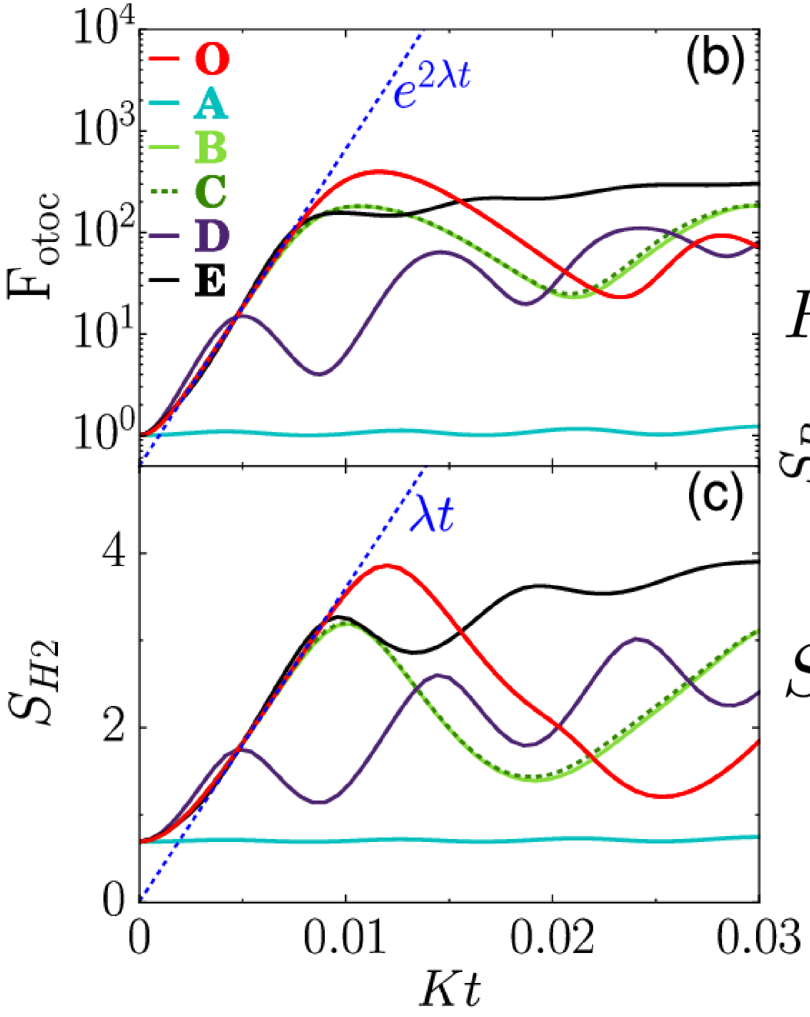
Kerr quantum dynamics

Survival Probability

$$S_p(t) = |\langle \Psi(0)|\Psi(t)\rangle|^2$$





FOTOC

$$F_{\text{otoc}}(t) = \sigma_p^2(t) + \sigma_q^2(t)$$

S_P

Husimi entropy

$$S_{\text{H2}}(t) = -\ln M_2(t),$$

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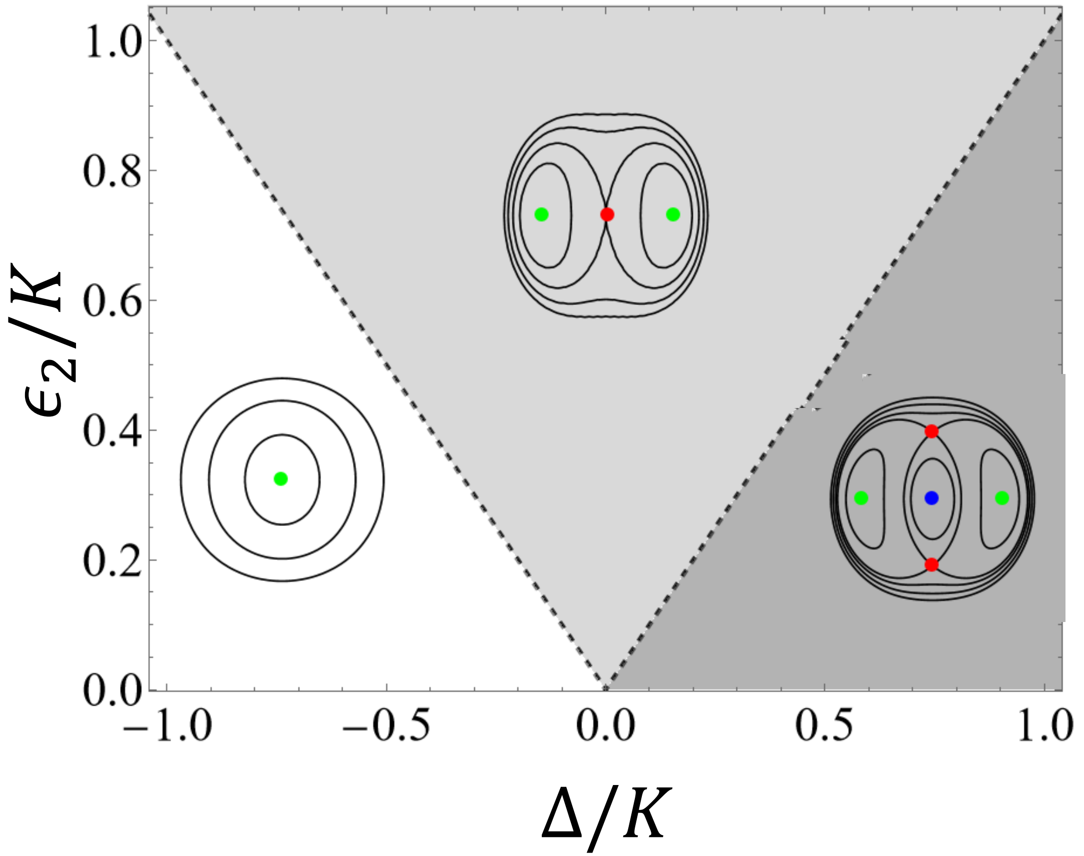
Kerr extended model*

$$\hat{H}_{eff} = \Delta a^{\dagger} a - K a^{\dagger 2} a^{2} + \epsilon_2 \left(\hat{a}^{\dagger 2} + \hat{a}^{2} \right)$$



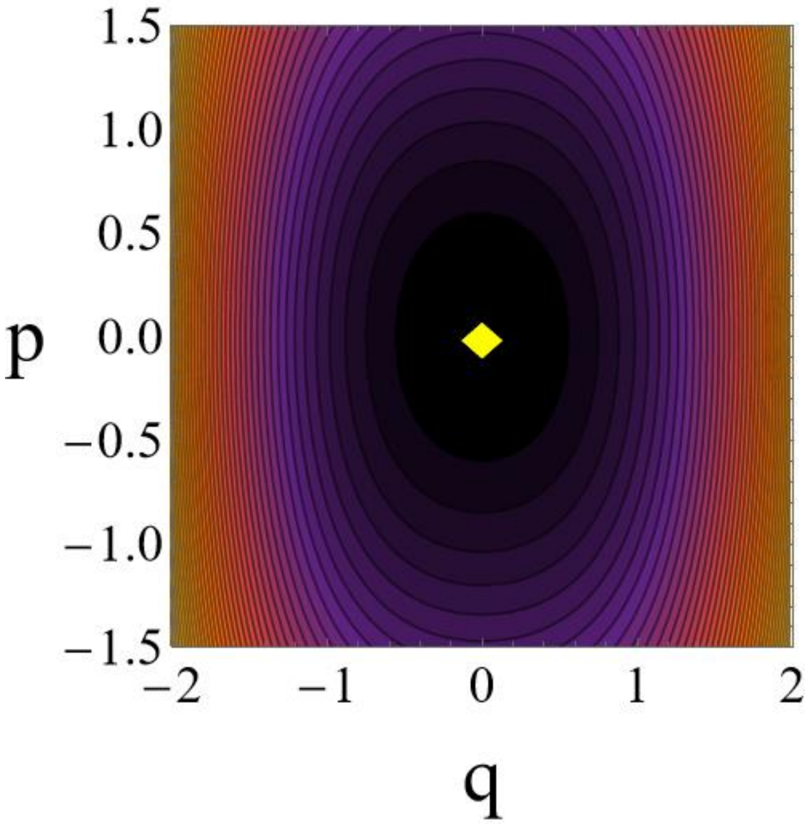
*Next talk with Miguel Prado

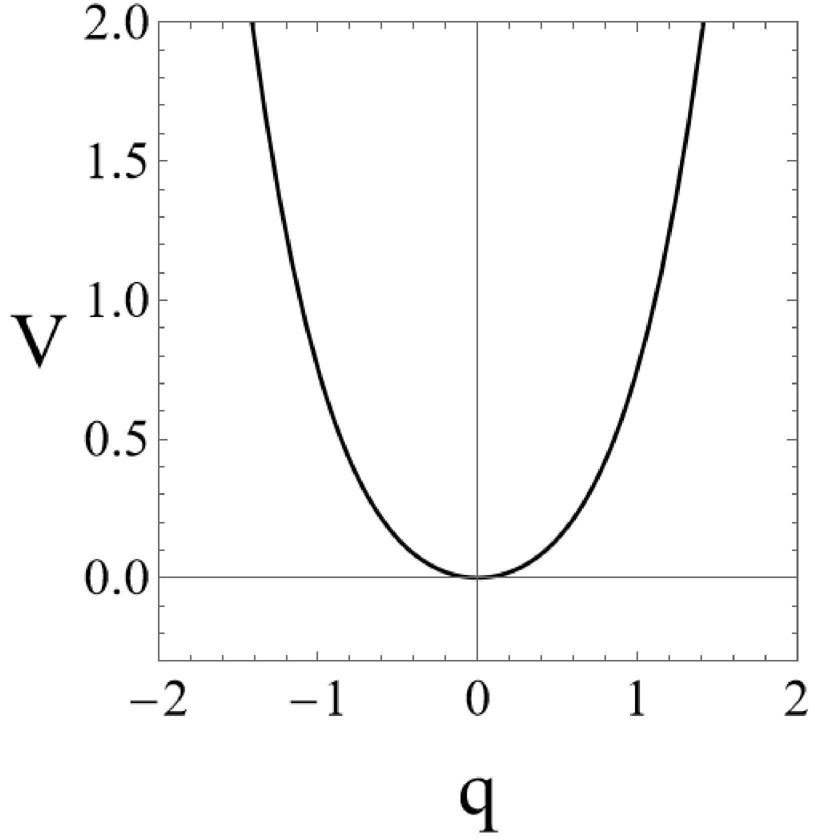
Quartic oscillator

Quartic oscillator

$$H = \frac{p^2}{2m} + \frac{k}{2}q^2 + \frac{\gamma}{4}q^4$$

$k > 0$

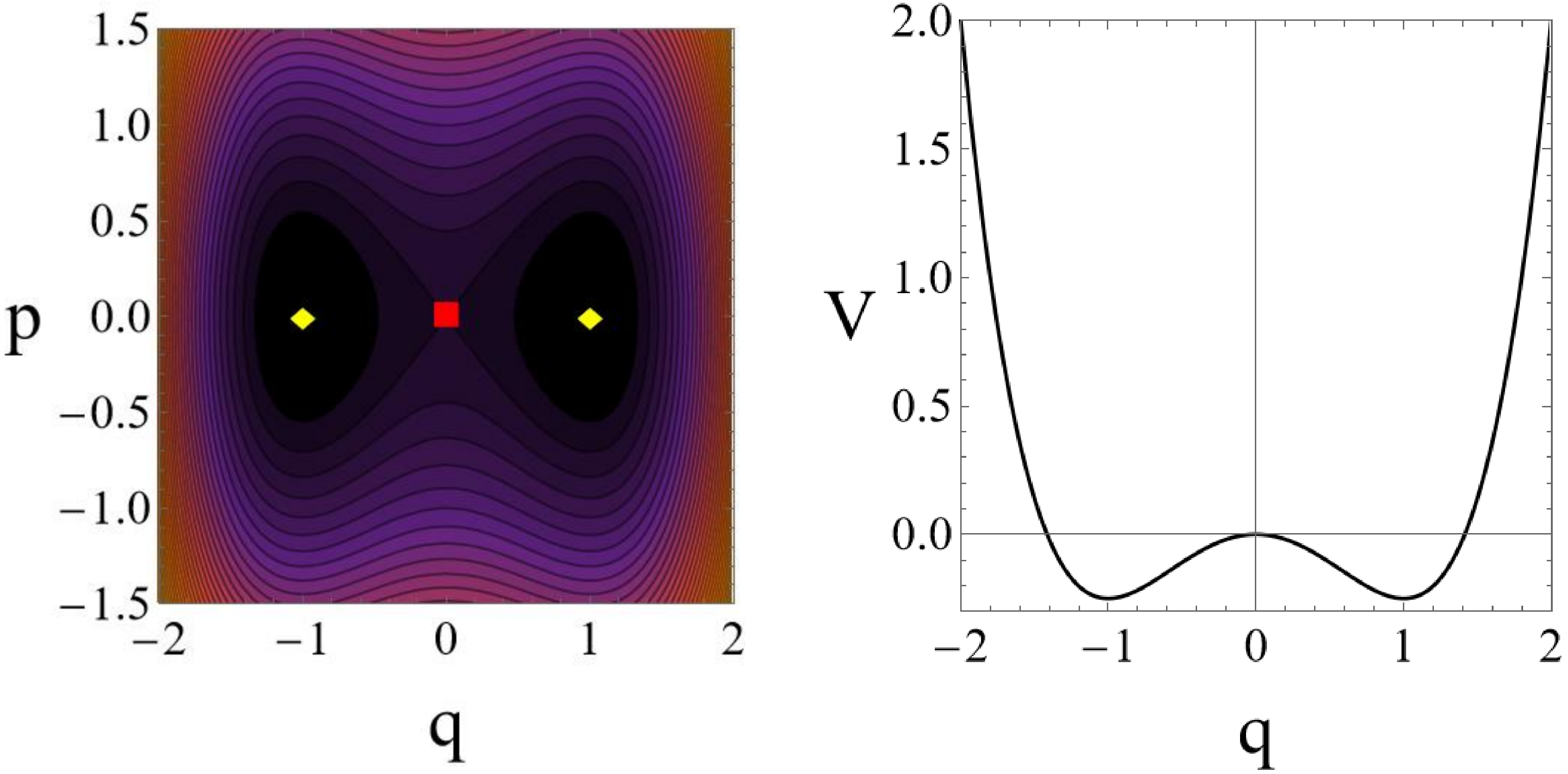




Quartic oscillator

$$H = \frac{p^2}{2m} + \frac{k}{2}q^2 + \frac{\gamma}{4}q^4$$

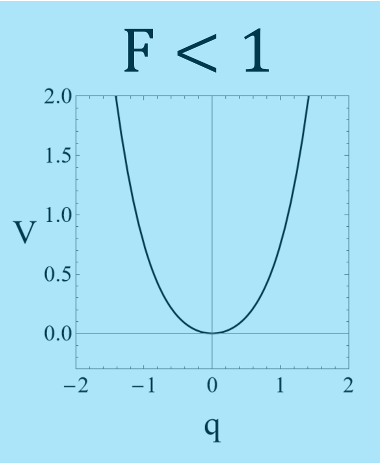
$k < 0$

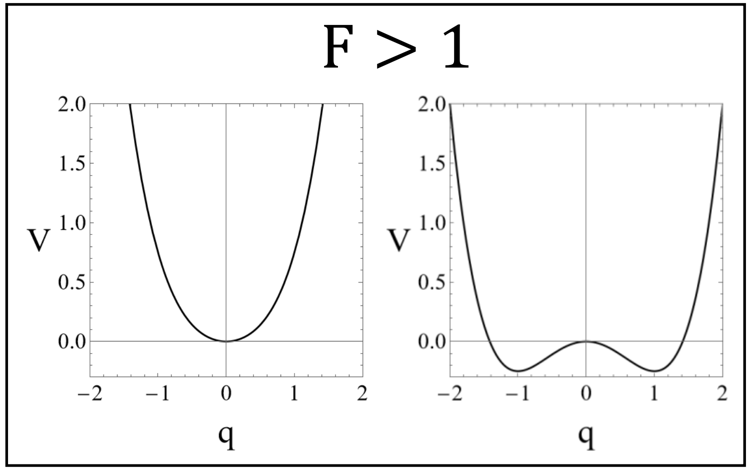


Driven quantum quartic oscillator

$$\hat{H}(t) = \frac{p^2}{2} + \frac{1}{2}\left[\omega_0^2 + F\cos(\omega_F t)\right] q^2 + \frac{\gamma}{4} q^4$$

In a classical scenario $\omega_0 = 1$





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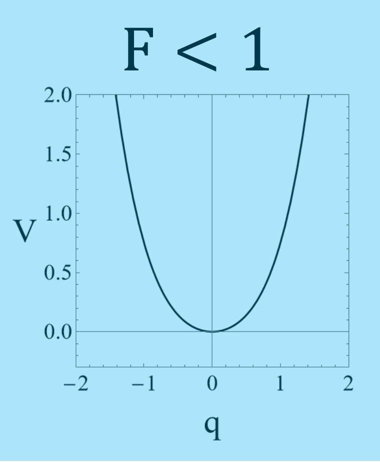
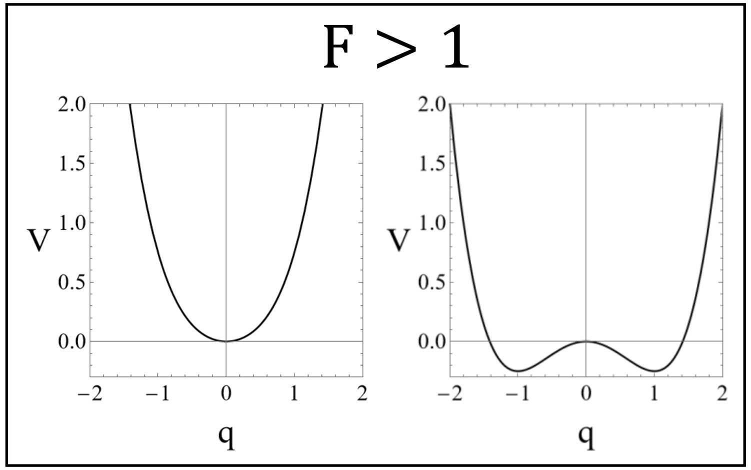
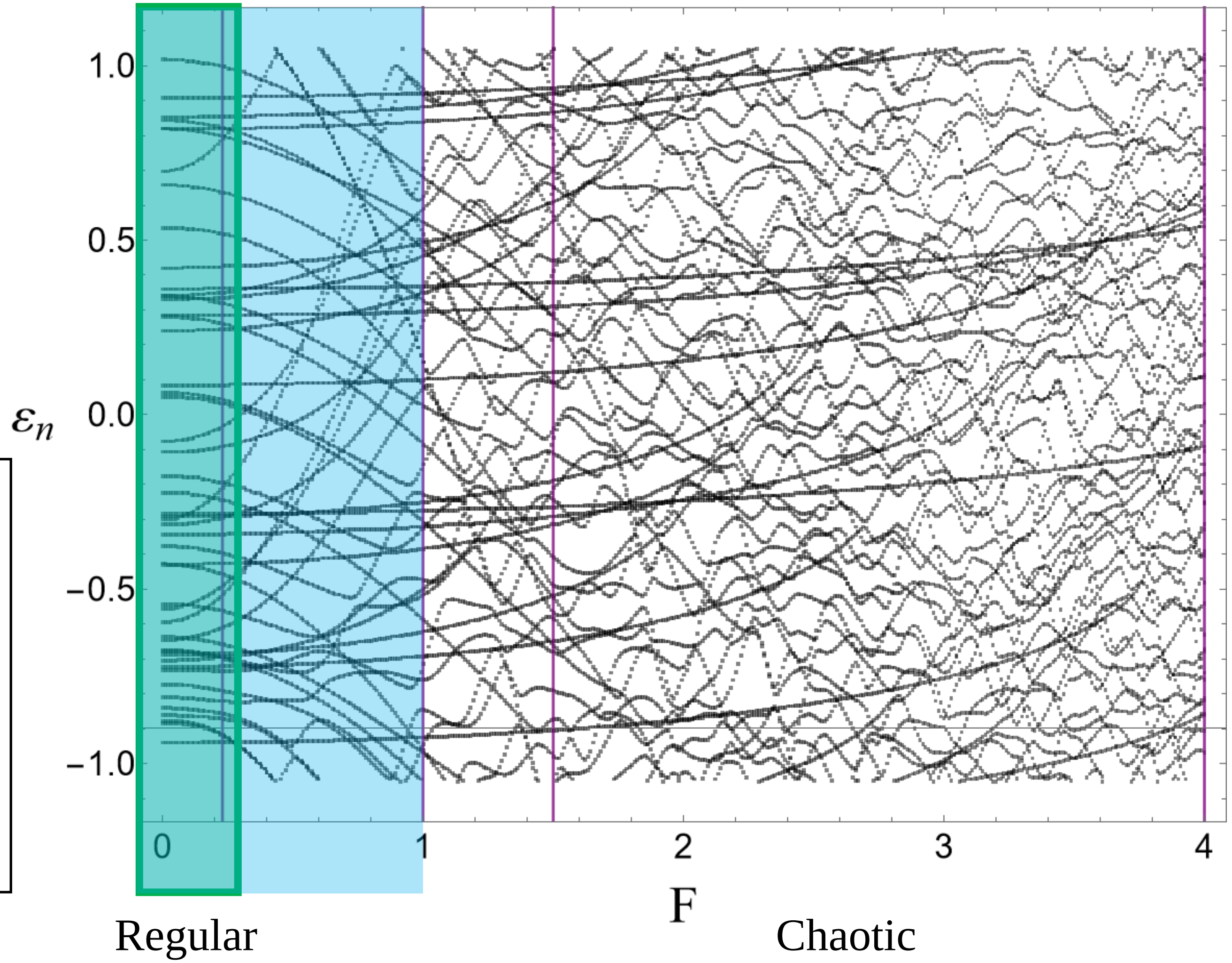




Quasienergies $\omega_0 = 1, \omega_F = 2.1, \gamma = 0.23$



Rotating Wave Approximation (RWA)

$$H(t) = H(\hat{o}_1, ..., \hat{o}_n, f(t))$$

$\hat{O}$: Passive rotating operator

$$U = U(\hat{O}(t))$$

$$U^\dagger H U - i\hbar U^\dagger \dot{U}$$

$$H_{RWA}$$

Dicke V. M. Bastidas, C. Emary, B. Regler, T. Brandes. PRL 108, 043003 (2012)

Quartic Oscillator Yaxing Zhang and M. I. Dykman, PRA **95**, 053841 (2017)

Model in the RWA

$$\hat{H}(t) = \frac{p^2}{2} + \frac{1}{2}\left[\omega_0^2 + F\cos(\omega_F t)\right] q^2 + \frac{\gamma}{4} q^4$$

If $F, \gamma\langle q^2\rangle \ll \omega_0^2$

$$\hat{H}(t) = \frac{p^2}{2} + \frac{1}{2}\omega^2(t) q^2 + \frac{\gamma}{4} q^4$$

↓

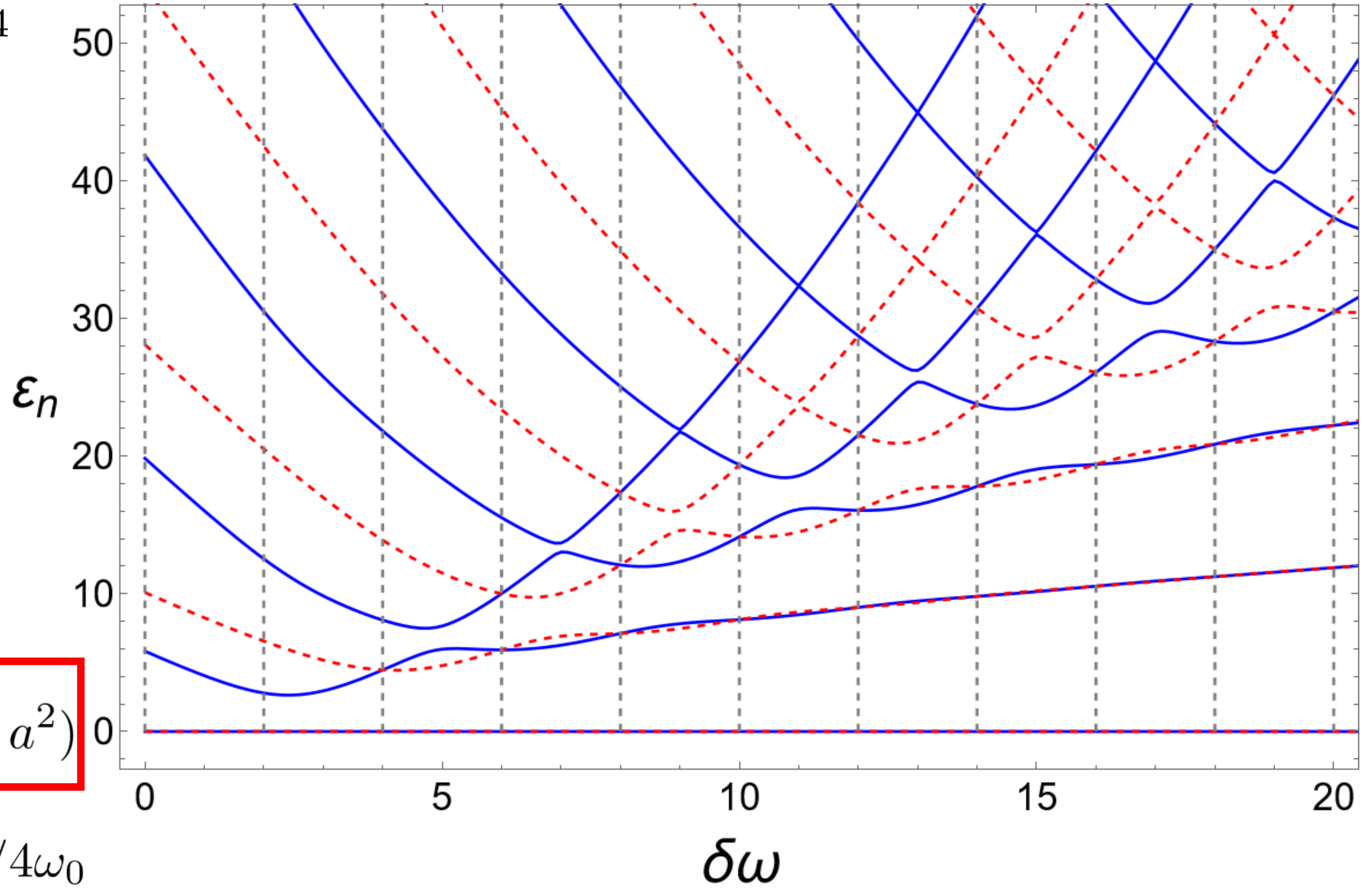
$$U(t) = \exp\left[-i\omega_F a^\dagger a t/2\right]$$

↓

$$\hat{H}_{RWA} = -\hbar\delta_{\omega_F}\hat{n} + \frac{\hbar V}{2}\left(\hat{n}^2 + \hat{n}\right) + \frac{\hbar\tilde{F}}{2}(a^{\dagger 2} + a^2)$$

Where: $\delta\omega_F = \omega_F/2 - \omega_0$ $V = 3\gamma\hbar/4\omega_0^2$ $\tilde{F} = F/4\omega_0$

Energies of the Hamiltonian in the RWA



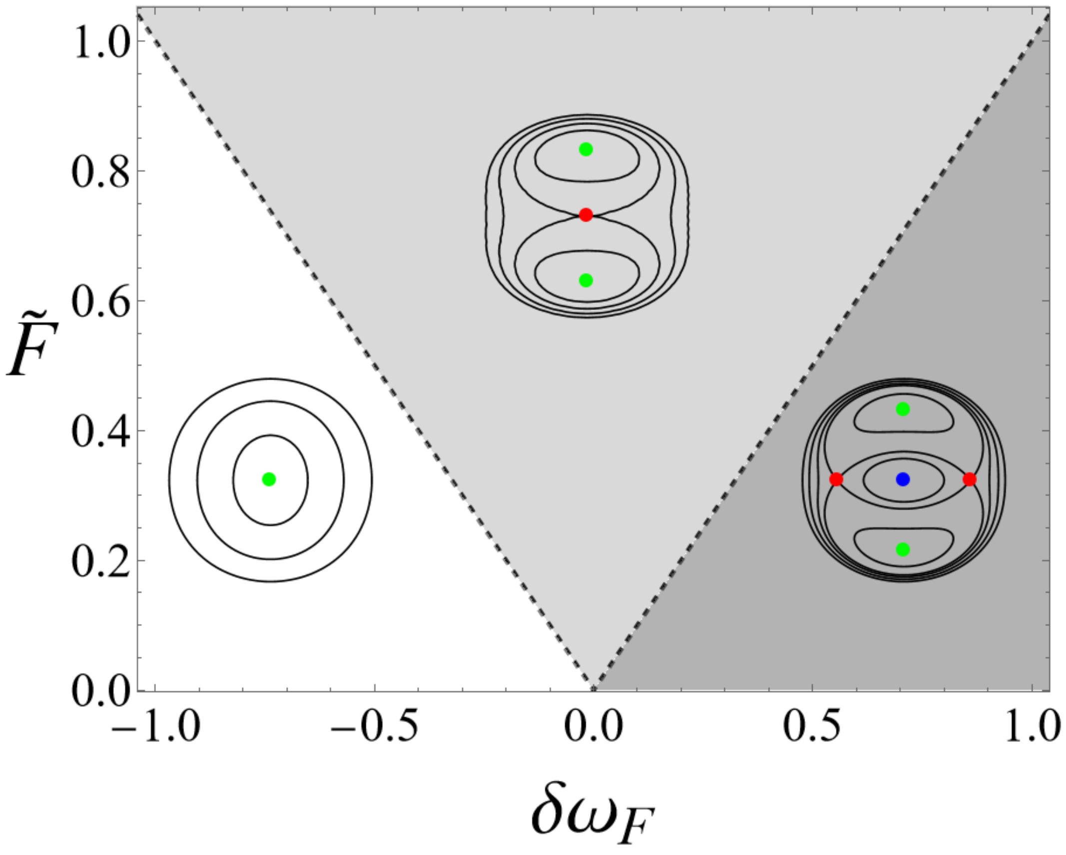
RWA: Classical Hamiltonian

$$\hat{H}_{RWA} = -\hbar\delta_{\omega_F}\hat{n} + \frac{\hbar V}{2}\left(\hat{n}^2 + \hat{n}\right) + \frac{\hbar\tilde{F}}{2}(a^{\dagger 2} + a^2)$$

$$h_{cl} = -\frac{\delta\omega_F}{2}\left(q^2 + p^2\right) + \frac{V}{8}\left(q^4 + p^4 + 2q^2p^2\right) + \frac{\tilde{F}}{2}\left(q^2 - p^2\right)$$

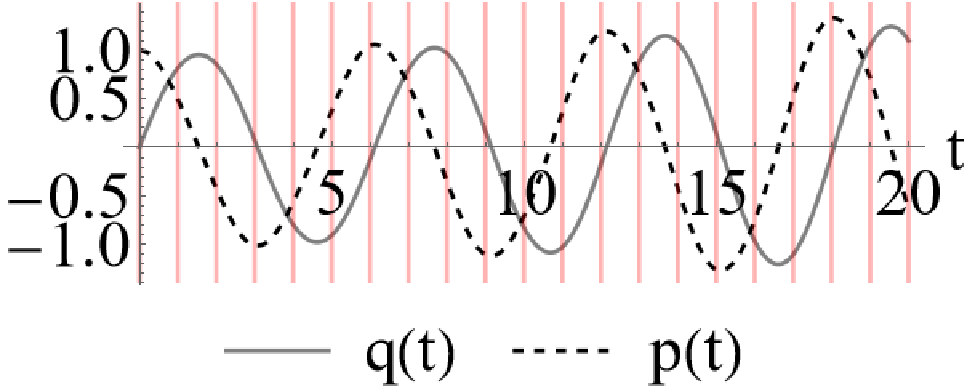
Similar to Kerr extended model

$$h_{cl} = \frac{\Delta}{2}\left(q^2 + p^2\right) - \frac{K}{4}\left(q^4 + p^4 + 2q^2p^2\right) + \epsilon_2\left(q^2 - p^2\right)$$



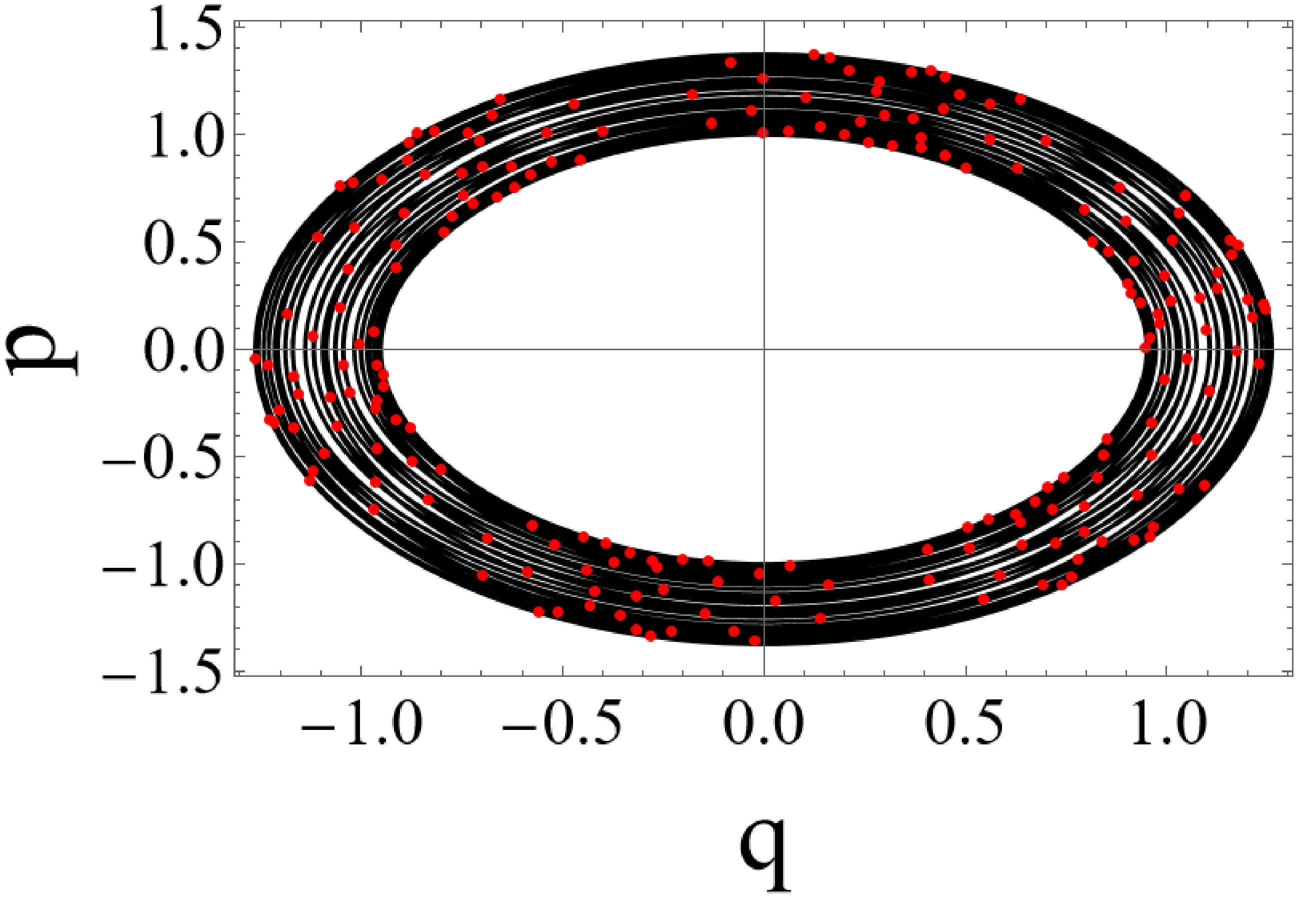
RWA: Classical interpretation

$$H(t) = \frac{p^2}{2} + \frac{1}{2}\left[\omega_0^2 + F\cos(\omega_F t)\right] q^2 + \frac{\gamma}{4} q^4$$



$\omega_0 = 1, \omega_F = 2.1, \gamma = 0.23, F = 0.23$

Poincaré Section: $t_{cut} = 1$



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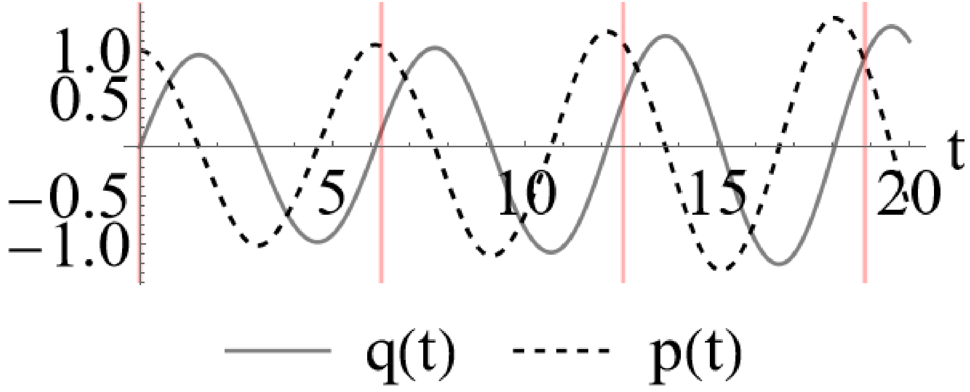
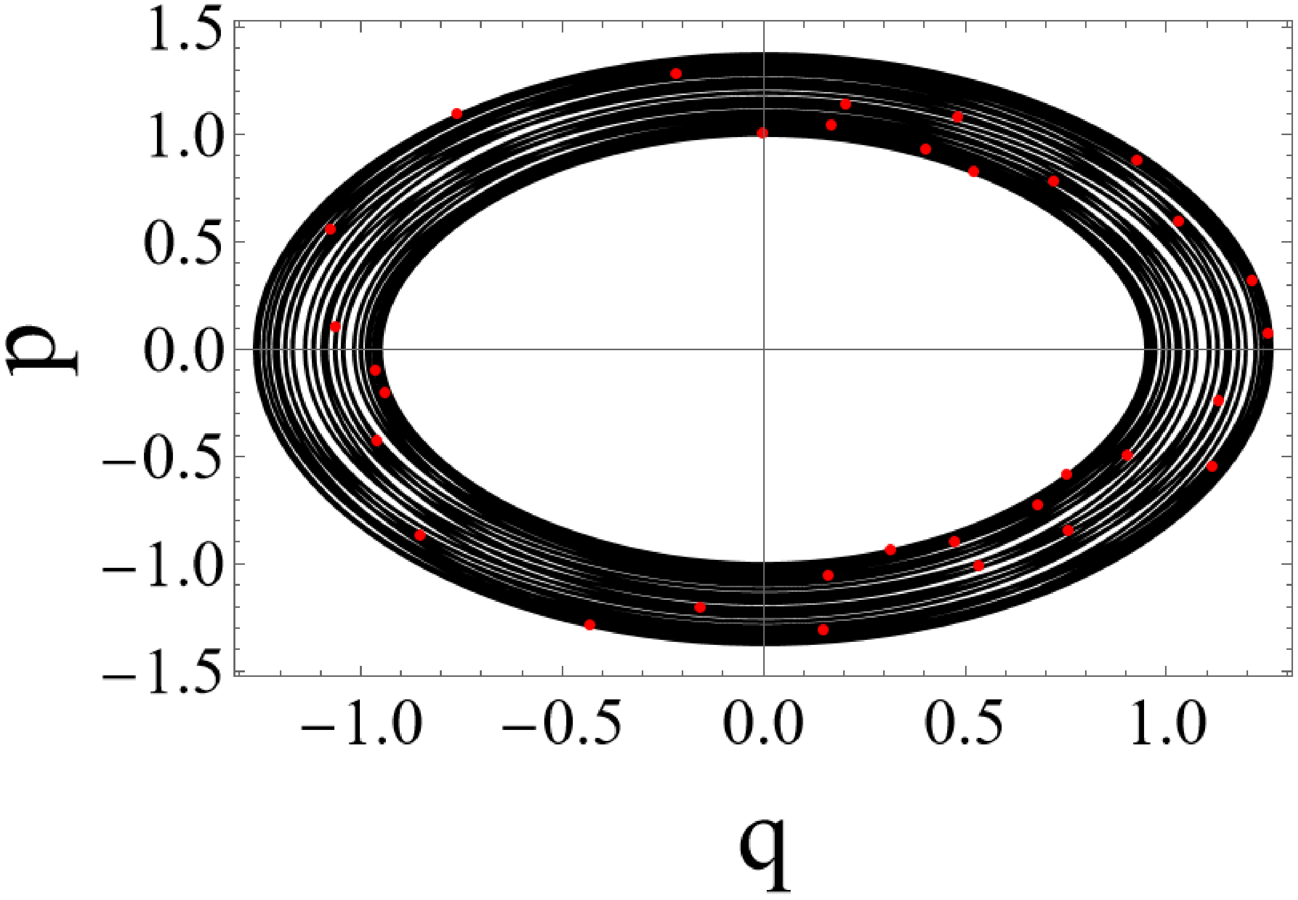


$\omega_0 = 1, \omega_F = 2.1, \gamma = 0.23, F = 0.23$

Poincaré Section: $t_{cut} = T_{\omega_0}$



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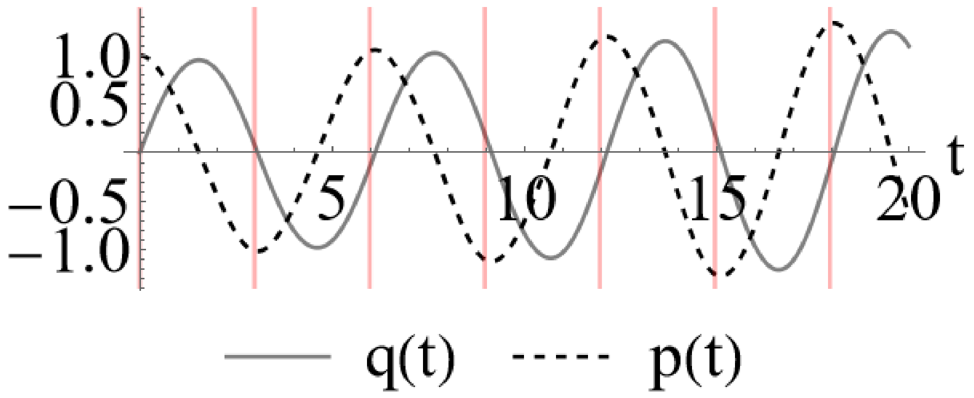
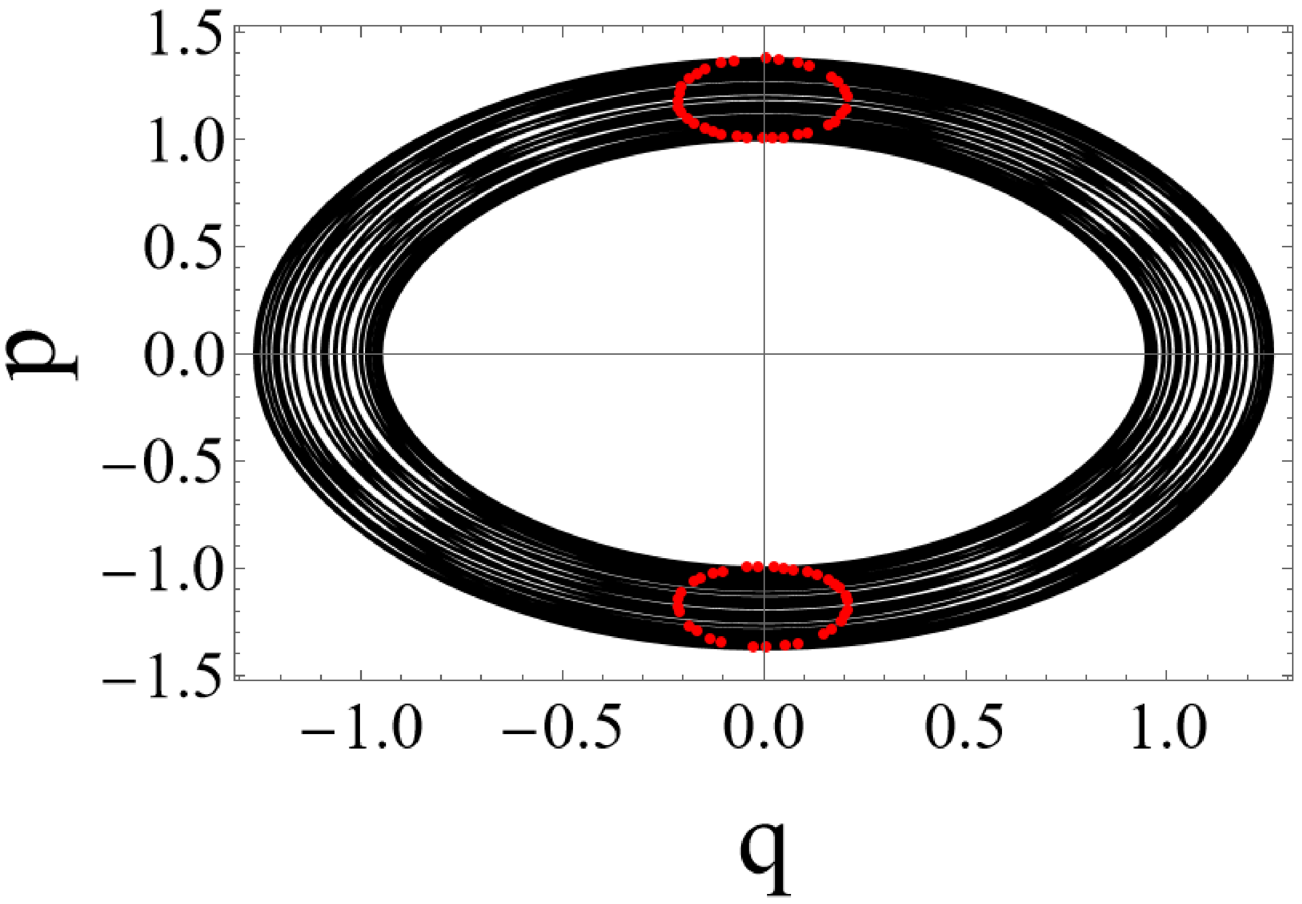
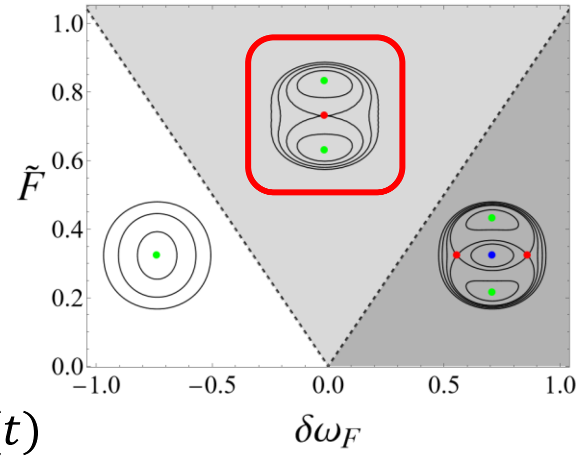


$\omega_0 = 1, \omega_F = 2.1, \gamma = 0.23, F = 0.23$

Poincaré Section: $\mathrm{t_{cut}} = T_{\omega_F}$

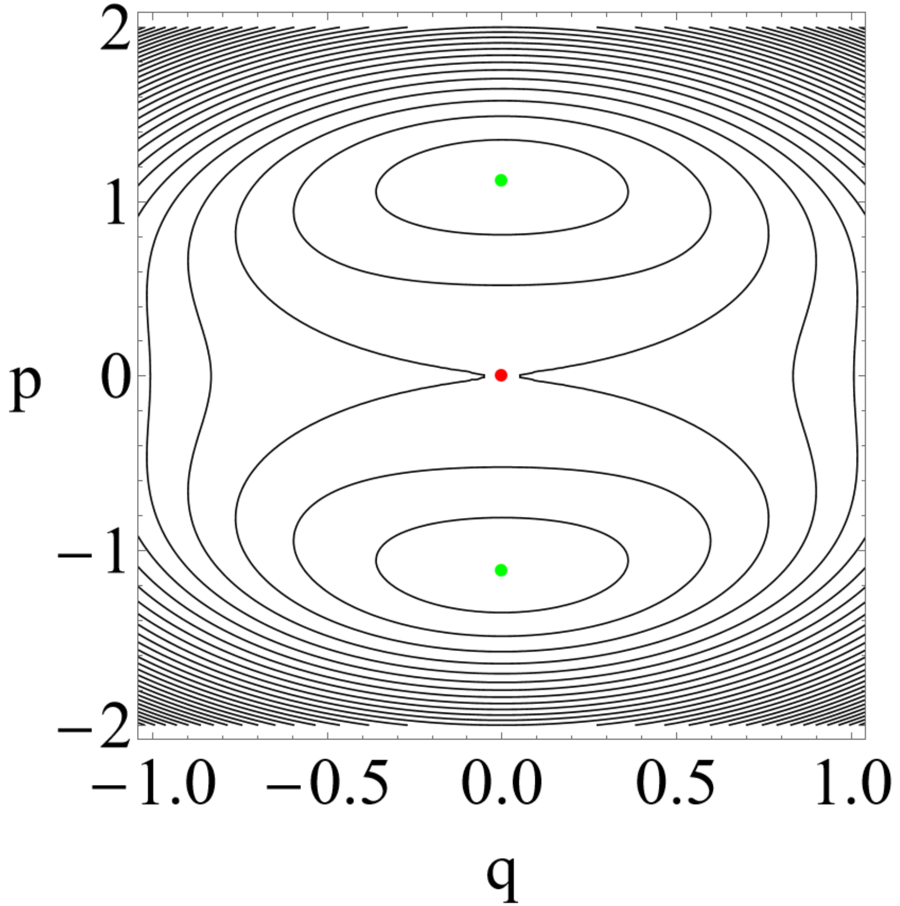


RWA and Poincaré Sections

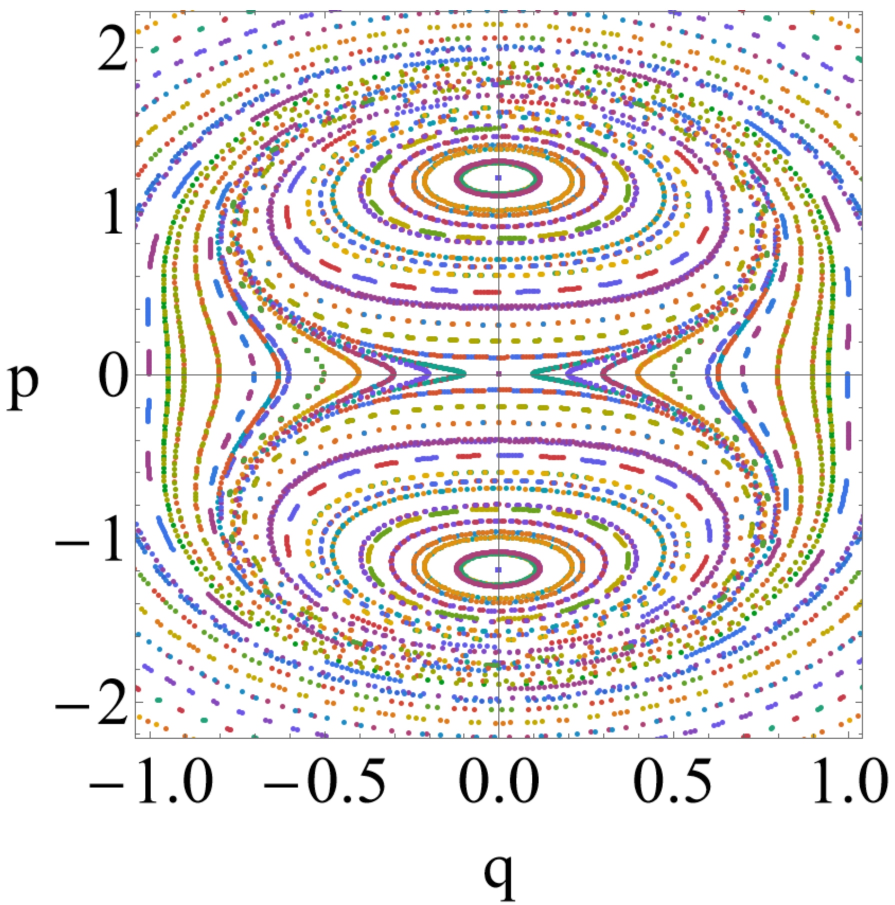


$\omega_0 = 1, \omega_F = 2.1, \gamma = 0.23, F = 0.23$

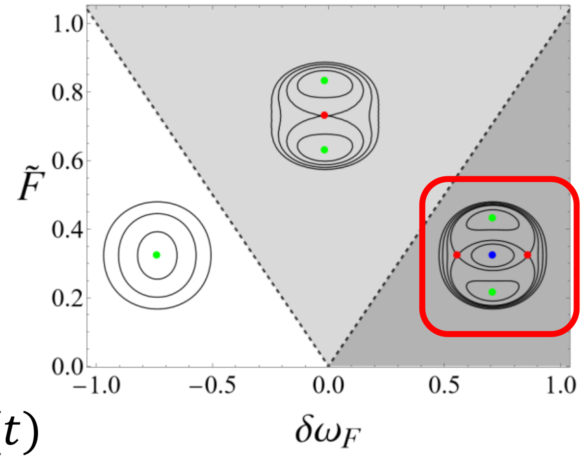
Energy contours in the H_{RWA}



Poincaré sections $H(t)$

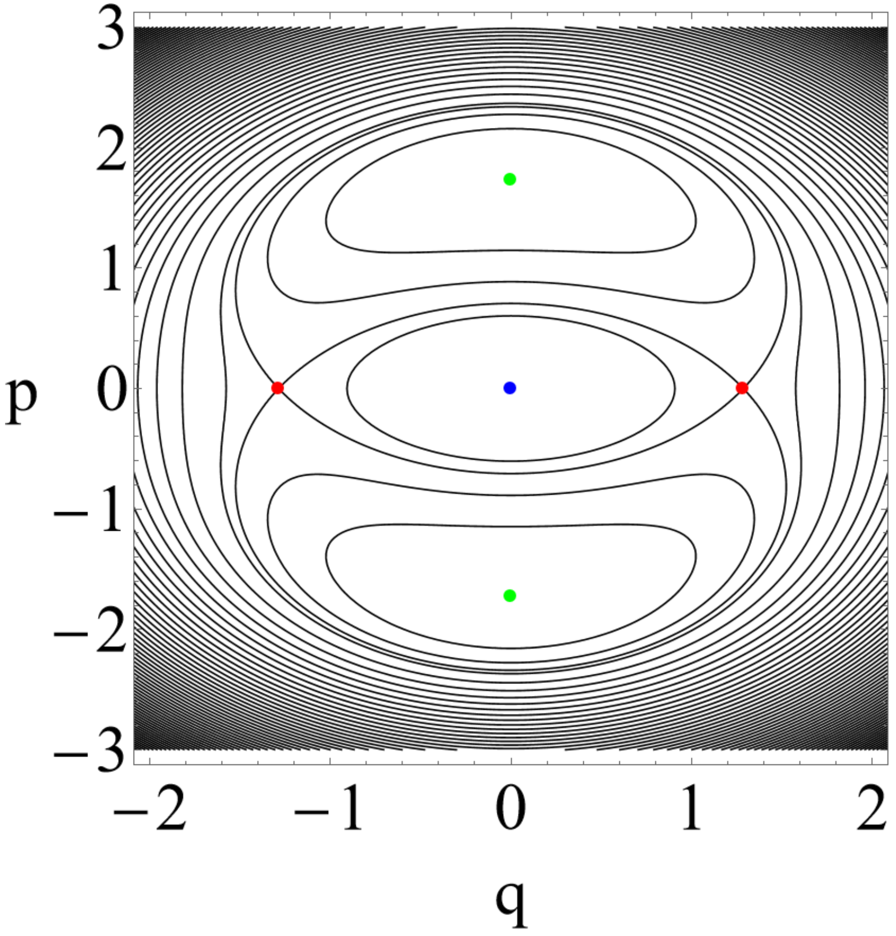


RWA and Poincaré Sections

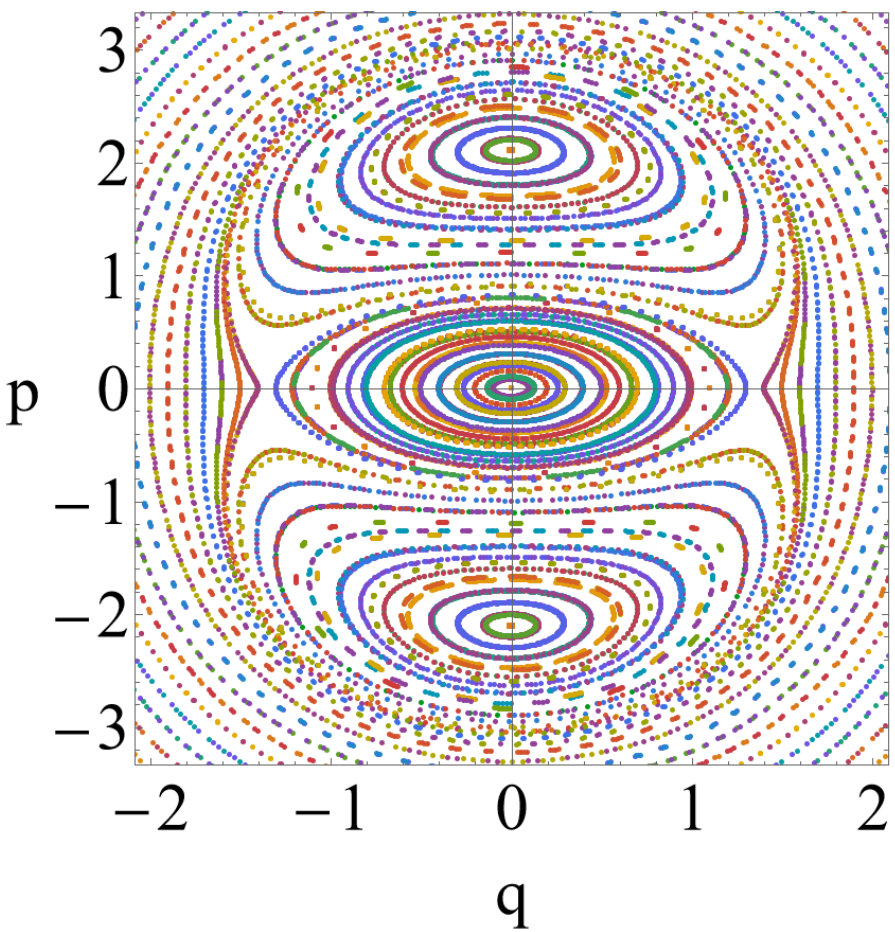


$\omega_0 = 1, \omega_F = 2.4, \gamma = 0.23, F = 0.23$

Energy contours in the H_{RWA}

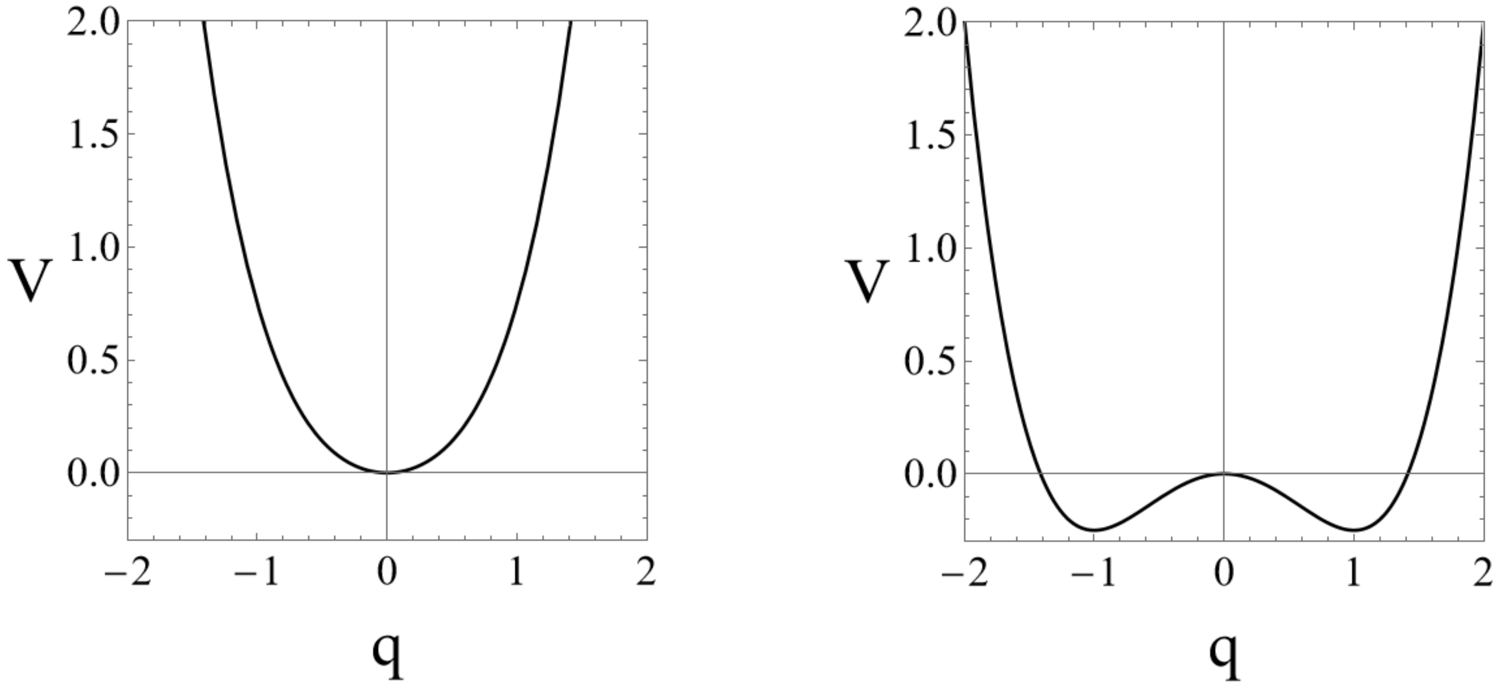


Poincaré sections $H(t)$

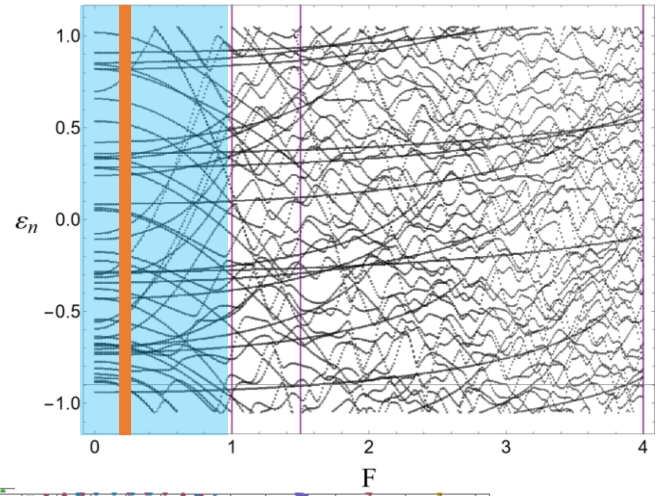


Route of chaos

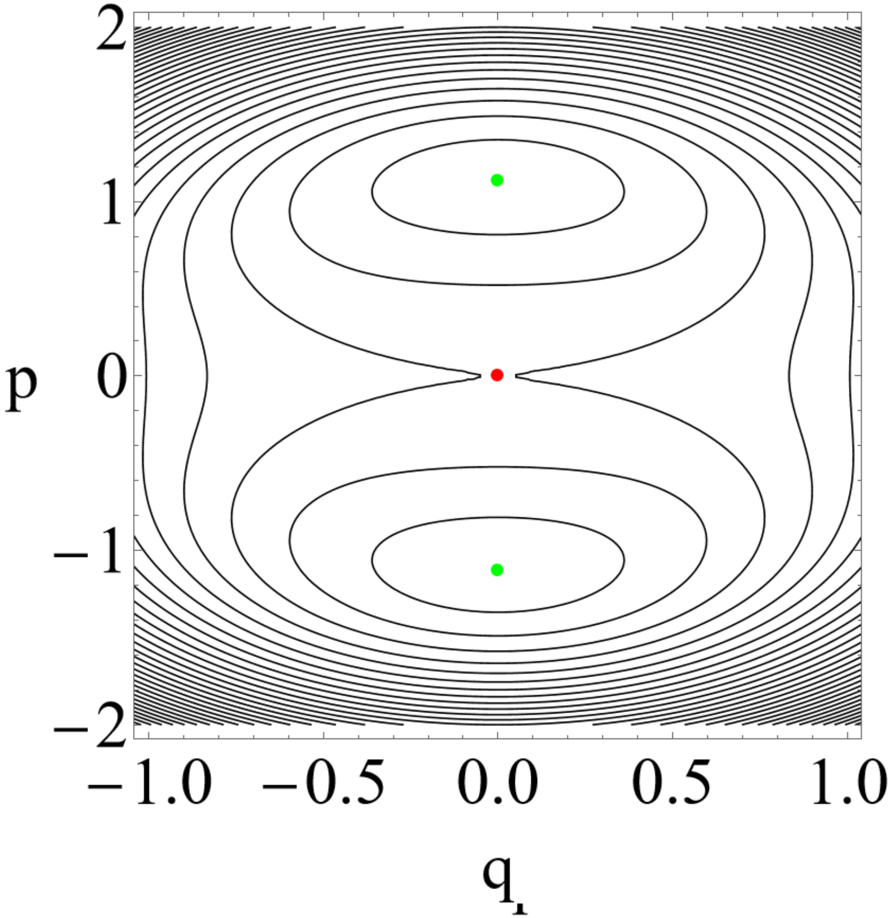
$$\hat{H}(t) = \frac{p^2}{2} + \frac{1}{2}\left[\omega_0^2 + F\cos(\omega_F t)\right] q^2 + \frac{\gamma}{4} q^4$$

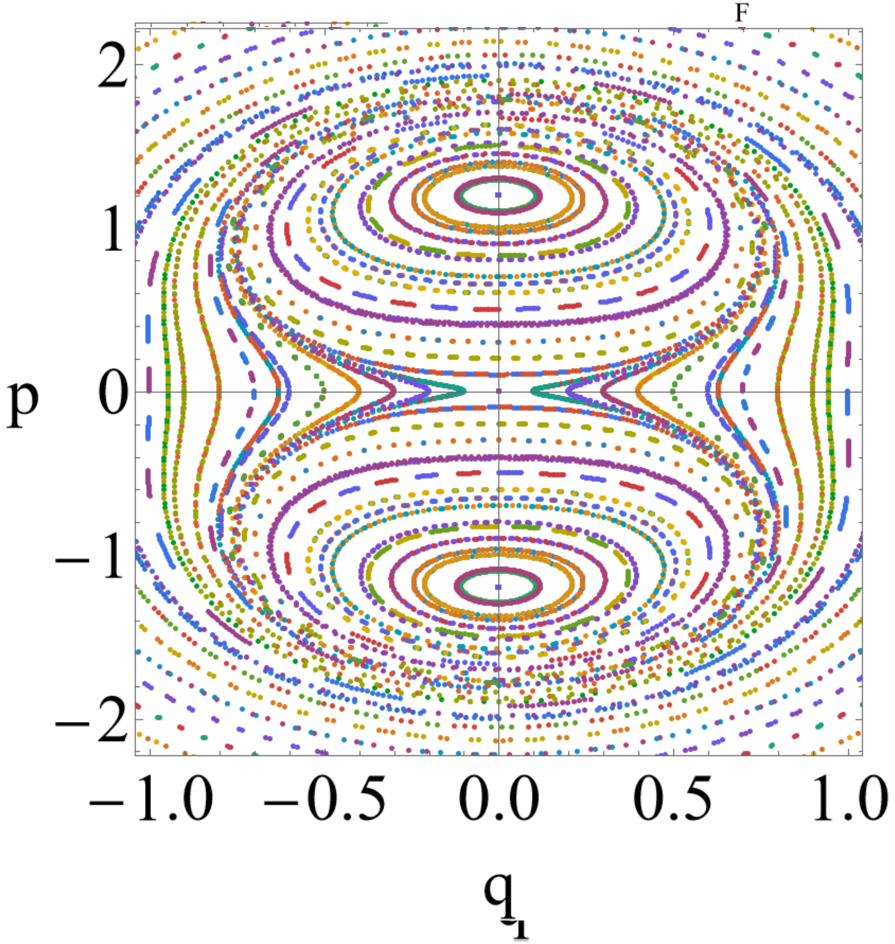


Route of chaos

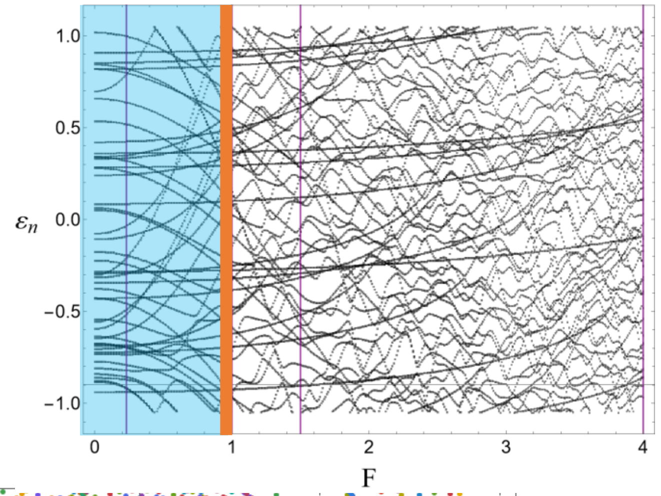


$\omega_0 = 1, \omega_F = 2.1, \gamma = 0.23, F = 0.23$





Route of chaos



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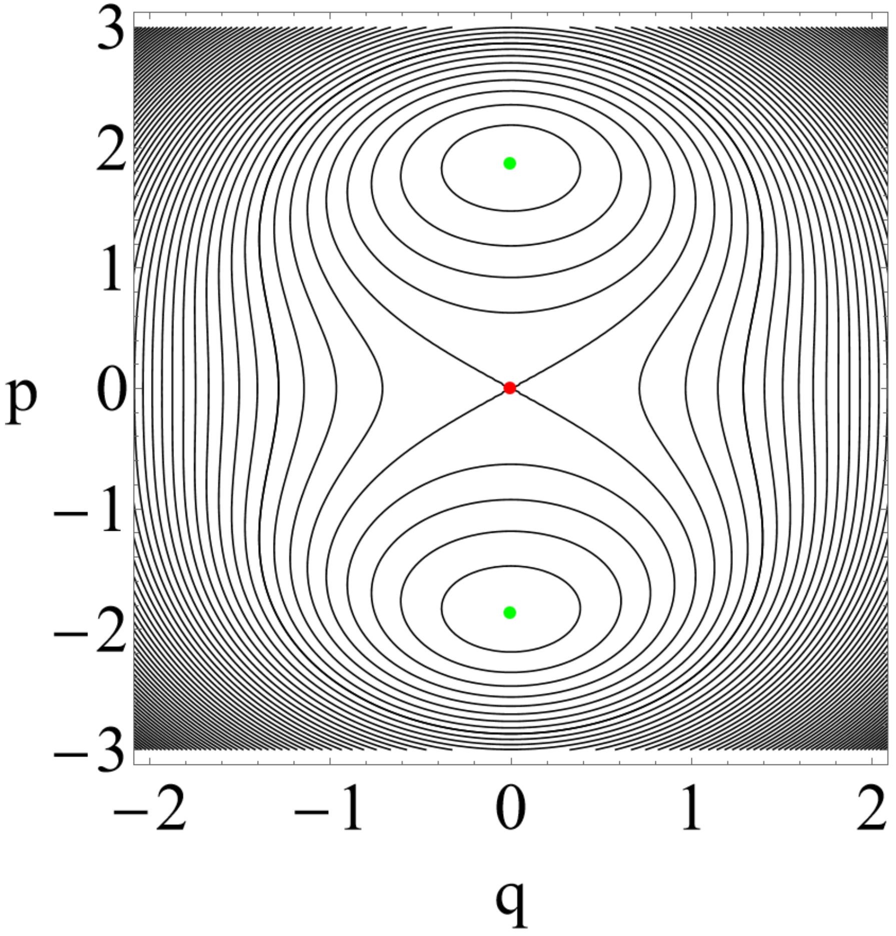
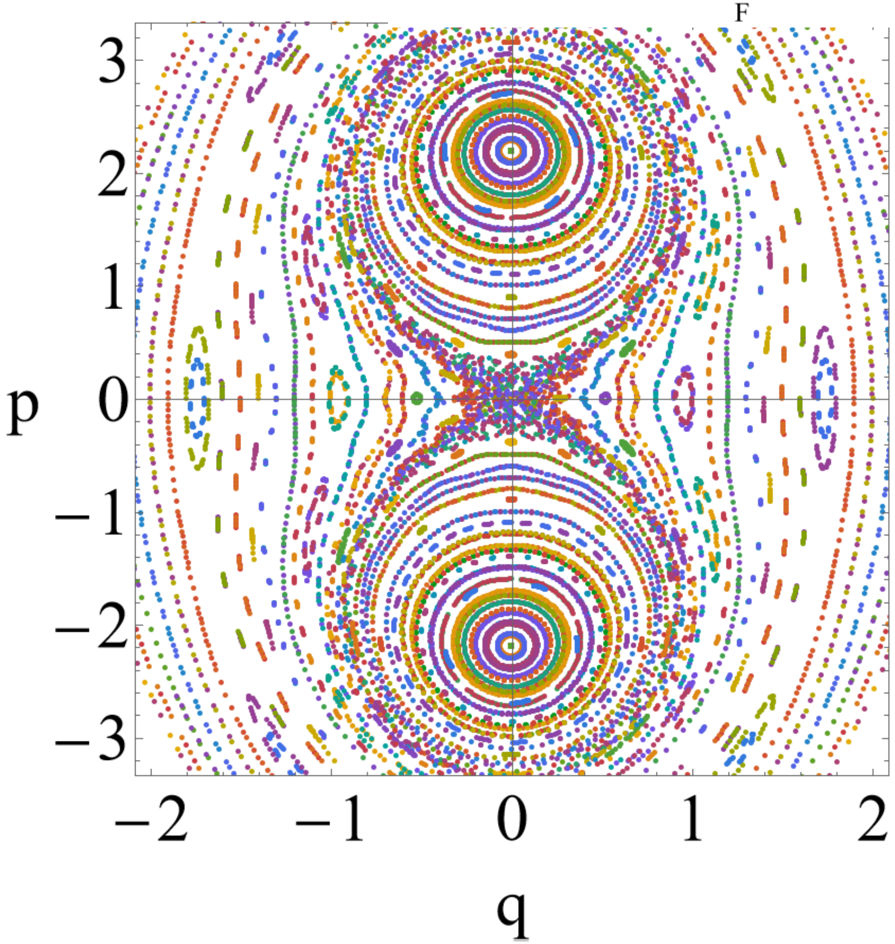
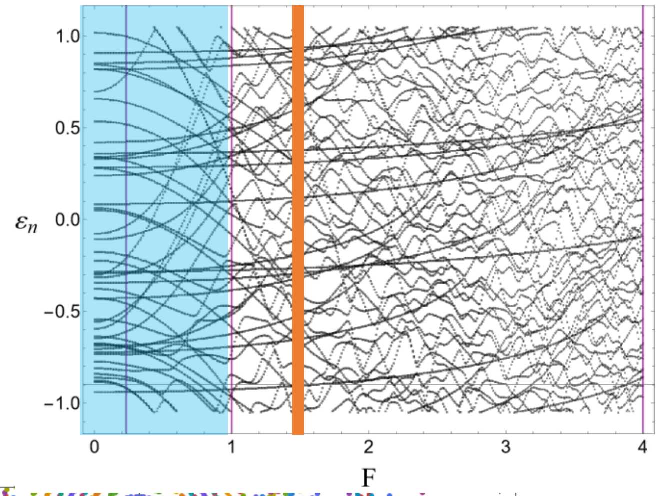
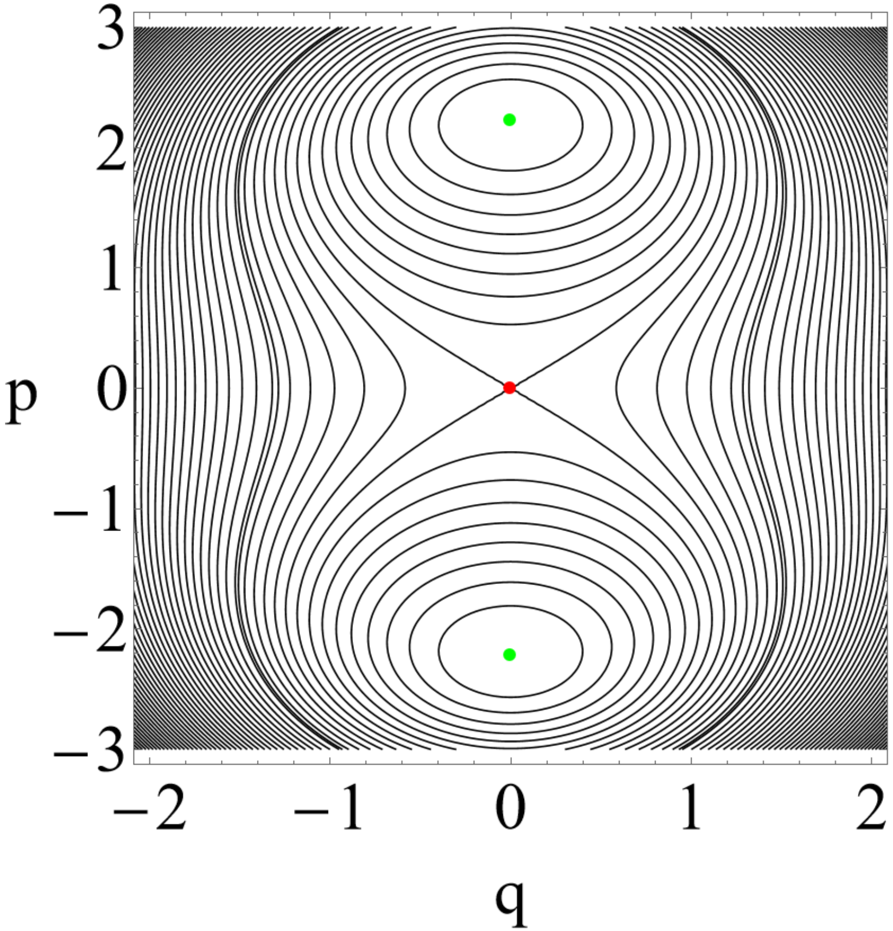


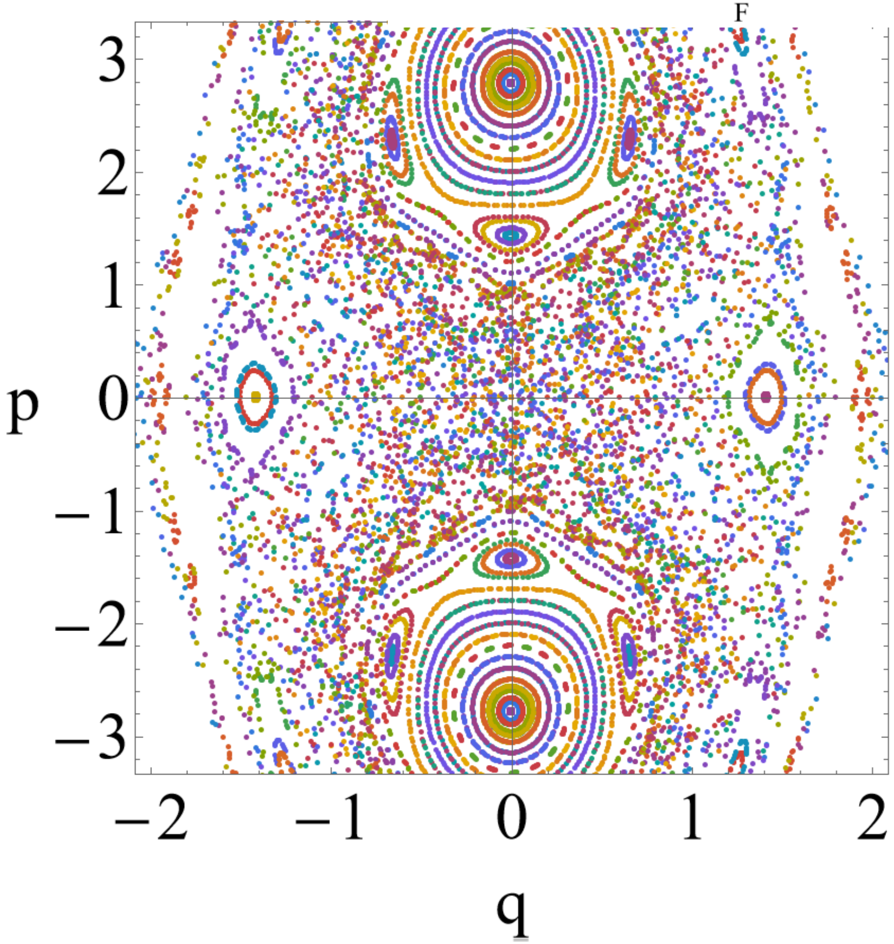


Route of chaos

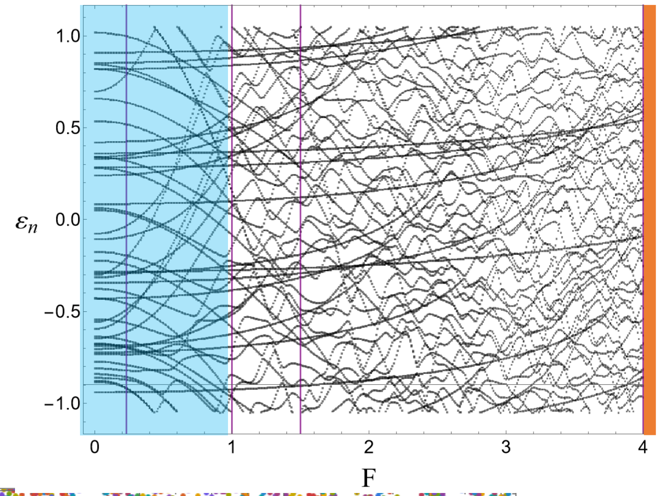


$\omega_0 = 1, \omega_F = 2.1, \gamma = 0.23, F = 1.5$

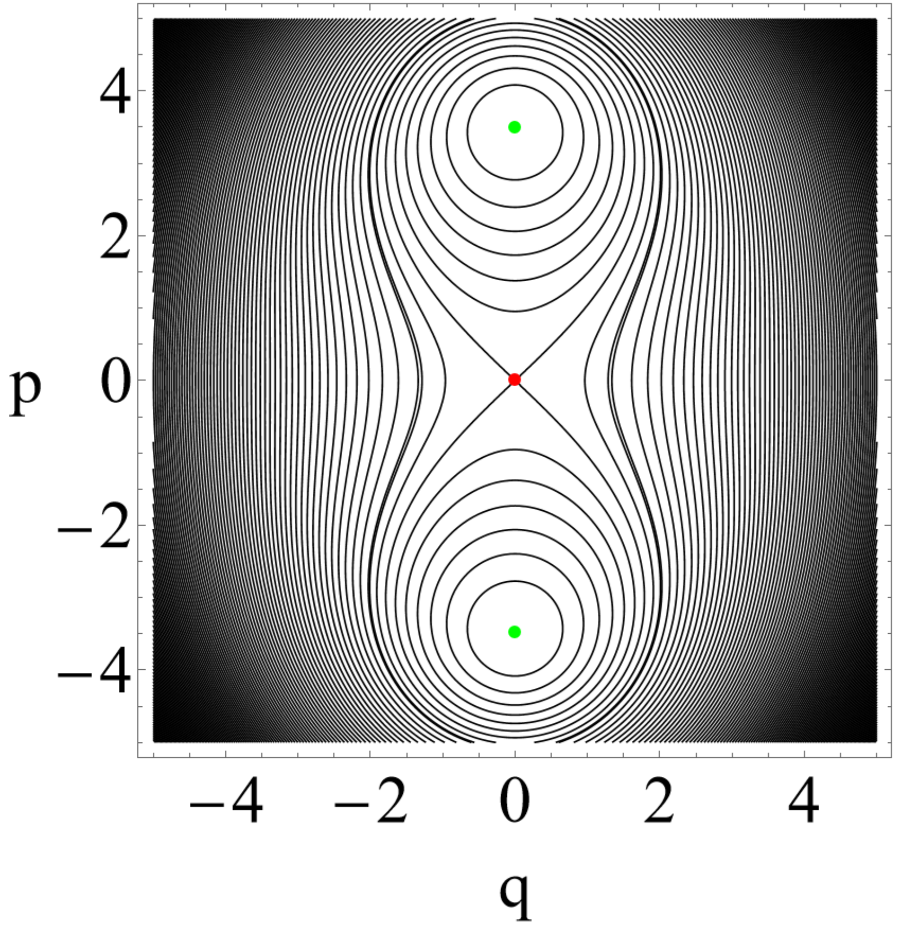


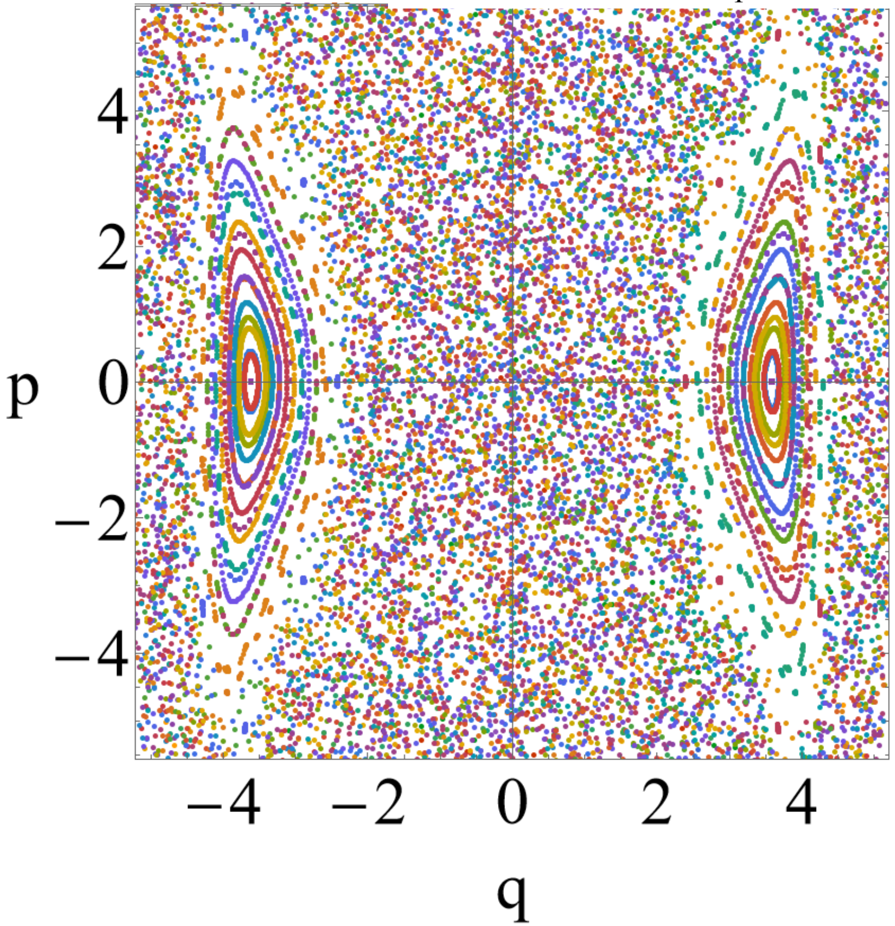


Route of chaos



$\omega_0 = 1, \omega_F = 2.1, \gamma = 0.23, F = 4$





Quantum Geometry

Quantum Geometry

Geometry of the **parameter space: Quantum Geometry Tensor**

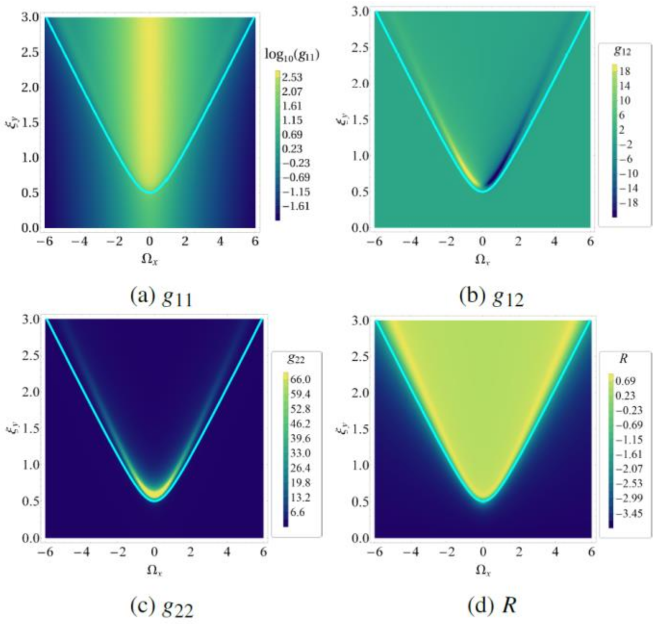
$$Q_{ij} = \langle \partial_i \psi | \partial_j \psi \rangle - \langle \partial_i \psi | \psi \rangle \langle \psi | \partial_j \psi \rangle$$

Quantum Metric Tensor

$$g_{ij} = \mathrm{Re} Q_{ij}$$

Berry curvature

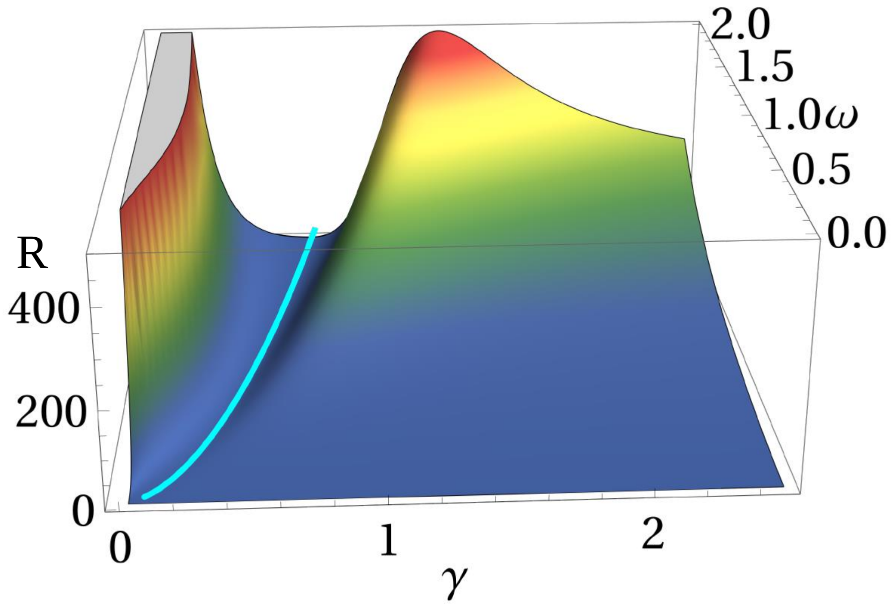
$$F_{ij} = -2 \mathrm{Im} Q_{ij}$$



Quantum Phase Transitions

Lipkin

Dicke



Quantum geometric tensor and quantum phase transitions in the Lipkin-Meshkov-Glick model	PRB **103**, 174104
Quantum metric tensor of the Dicke model: Analytical and numerical study	PRB **105**, 214106

Quantum Geometry

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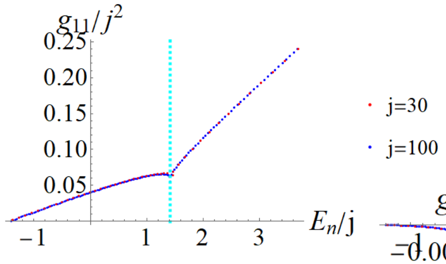
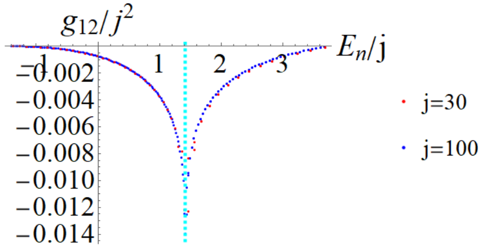
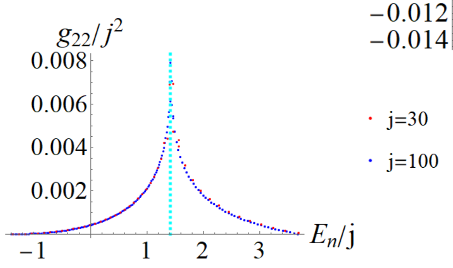


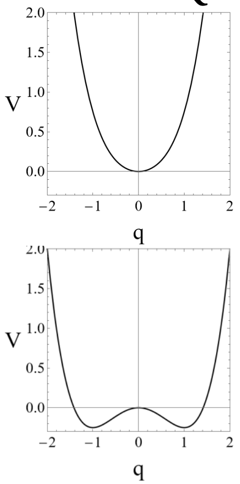
Lipkin

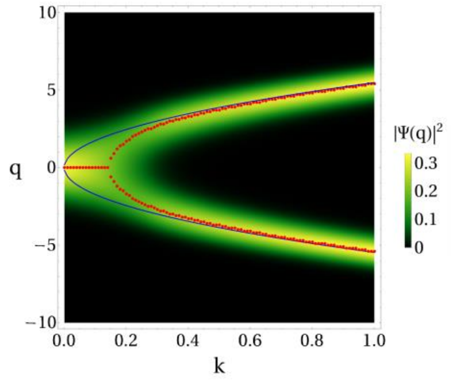
Excited Quantum Phase Transitions



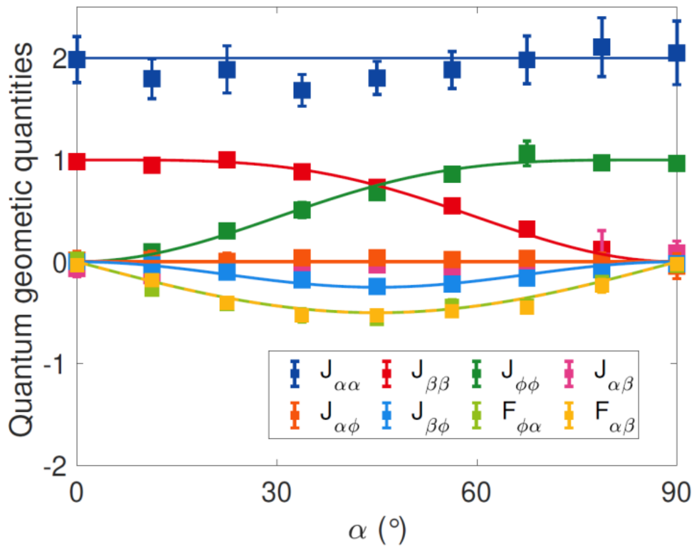


Quartic Oscillator





Experimental measures*



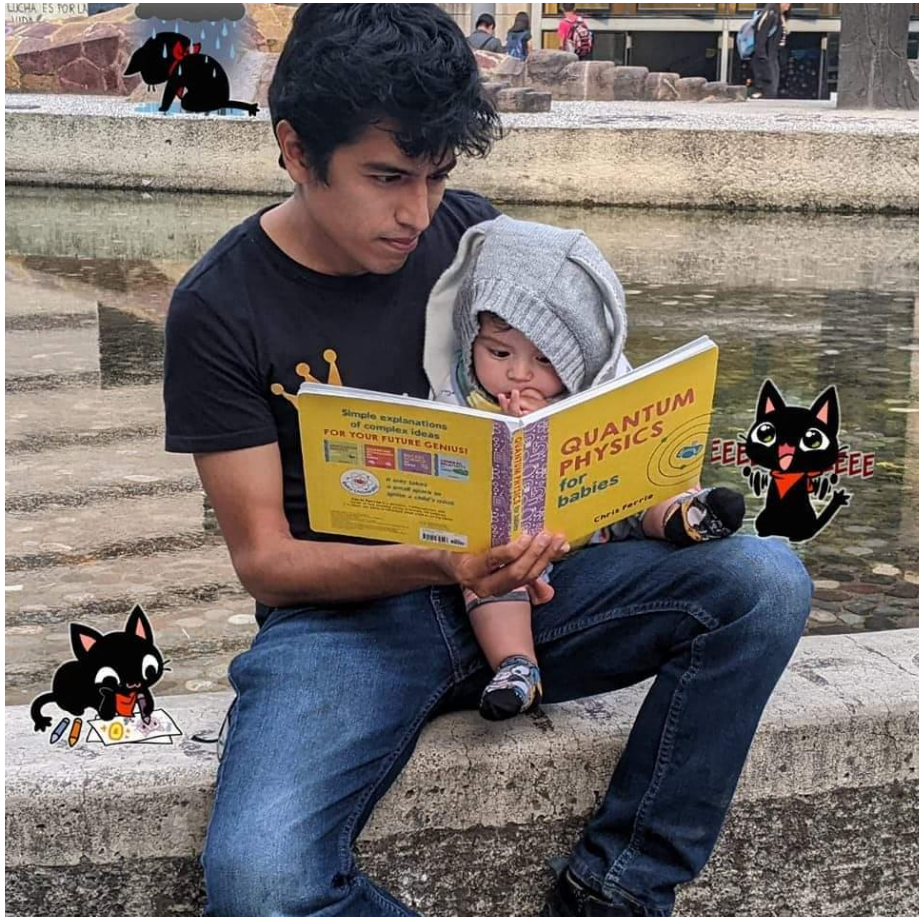
QMT and **Quartic oscillator**

In progress
arXiv:2204.13777

Experimental platform at MIT: Paola Cappellaro geometry and **metrology***

CONCLUSIONS

THANK YOU!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