Modelo de Dicke de 2 fotones

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Introducción

$$H_{2\gamma} = \sum_n \frac{\omega_0}{2}\sigma_z^n + \omega b^\dagger b + g \sum_n \left((b^\dagger)^2 + b^2\right) \sigma_x^n$$

- La implementación experimental del modelo de Dicke de dos fotones (two-photon Dicke model) se realiza atrapando N iones en una cadena [1].

- Un esquema de electrodinámica cuántica de un circuito permite implementar una interacción no dipolar ultra-fuerte de dos fotones entre un flux qubit y un modo bosonic compatible con SQUID.

[1] Felicetti, S. *Spectral collapse via two-phonon interactions in trapped ions*. Phys. Rev. A **92**, 033817 *(2015)*
[2] Cheng, X. H. et al. *Nonlinear quantum Rabi model in trapped ions*. Phys. Rev. A 97, 023624 (2018)
[3] S. Stufler. Et al. *Two-photon Rabi oscillations in a single InxGa1−x/GaAs quantum dot*, Phys. Rev. B 73, 125304 (2006).

Exact isolated solutions for the two-photon Rabi Hamiltonian

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Bogoliubov transformations and exact isolated solutions for simple nonadiabatic Hamiltonians

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We present a new method for finding isolated exact solutions of a class of non-adiabatic Hamiltonians of relevance to quantum optics and allied areas. Central to our approach is the use of Bogoliubov transformations of the bosonic fields in the models. We demonstrate the simplicity and efficiency of this method by applying it to the Rabi Hamiltonian.
[DOI: 10.1063/1.1490406]

Hamiltoniano de TPI (1 átomo)

$$H_{2\gamma} = \frac{\omega_0}{2}\sigma_z + \omega b^\dagger b + g\left((b^\dagger)^2 + b^2\right)\sigma_x$$

- Introduciendo los siguientes operadores

$$K_+ = \frac{1}{2}\left(b^\dagger\right)^2, \qquad K_- = \frac{1}{2}b^2, \qquad K_0 = \frac{1}{2}b^\dagger b + \frac{1}{4}$$

  - Nuevas reglas de conmutación

$$[K_\pm, K_0] = \pm K_\pm, \qquad [K_-, K_+] = 2K_0$$

[3] Emary, C. & Bishop, R. F. *Exact isolated solutions for the two-photon quantum Rabi model*. J. Phys. A Math. Gen. 35, 8231 (2002).

Álgebra SU(1,1)

Se busca una base que diagonalice el operador $\mathbf{K_0}$, para esto se introduce la serie $D^+(k)$ sugerida en [2] , entonces se realiza el siguiente mapeo

$$M(n) :\Longrightarrow (k, m)$$

El operador $\mathbf{K_0}$ actúa sobre los estados de Bloch así:

$$\mathbf{K_0} \, | \, n \rangle = \frac{1}{2} \left(n + \frac{1}{2} \right) | \, n \rangle$$

$$\mathbf{K_0} \, | \, k, m \rangle = \left(m + k \right) | \, k, m \rangle$$

Ecuación de Eigenvalores: $\mathbf{K_0} \, | \, k, m \rangle = \left(m + k \right) | \, k, m \rangle$

[4] Perelomov, A. (1986). Generalized coherent states and their applications

¿Cuál tabla es la correcta?

Operador Casimir

$$\mathbf{C} = \mathbf{K_0}^2 - \left(\mathbf{K_0} + \mathbf{K_+}\mathbf{K_-}\right)$$

$$\mathbf{C}\,|\,k, m\rangle = k\left(k-1\right)|\,k, m\rangle$$

$$\mathbf{C}\,|\,k, m\rangle = -\frac{3}{16}\mathbb{1}\,|\,k, m\rangle$$

n: par $\quad n = 2m \quad k = \frac{1}{4}$

n: impar $\quad n = 2m + 1 \quad k = \frac{3}{4}$

n	m	k	m + k
0	0	$\frac{1}{4}$	$\frac{1}{4}$
1	0	$\frac{3}{4}$	$\frac{3}{4}$
2	1	$\frac{1}{4}$	$\frac{5}{4}$
3	1	$\frac{3}{4}$	$\frac{7}{4}$
4	2	$\frac{1}{4}$	$\frac{9}{4}$
5	2	$\frac{3}{4}$	$\frac{11}{4}$

- El álgebra SU(1,1) divide el espacio de Hilbert en 2 subespacios independientes:

$$\left|\frac{1}{4}, m\right\rangle \equiv \frac{1}{(2m)!}(b^\dagger)^{2m}|0\rangle \qquad \left|\frac{3}{4}, m\right\rangle \equiv \frac{1}{(2m+1)!}(b^\dagger)^{2m+1}|0\rangle$$

En términos de los nuevos operadores el TPRH es:

$$H_{2\gamma} = \frac{\omega_0}{2\omega}\sigma_z + b^\dagger b + \frac{g}{\omega}\left((b^\dagger)^2 + b^2\right)\sigma_x \rightarrow H_{2\gamma} = \tilde{\omega}\sigma_z + b^\dagger b + \lambda\left((b^\dagger)^2 + b^2\right)\sigma_x$$

$$\tilde{\omega} \equiv \frac{\omega_0}{\omega}, \quad \lambda \equiv \frac{g}{\omega}$$

$$\tilde{H}_{2\gamma} = \tilde{\omega}\sigma_z + 2\left(\mathbf{K_0} - \frac{1}{4}\right) + 2\lambda\left(\mathbf{K_+} + \mathbf{K_-}\right)\sigma_x$$

Squeezing y SU(1,1)

$$S(\theta) = \exp\left[\frac{1}{2}\left(z^*\hat{a}^2 - z\hat{a}^\dagger\right)\right]$$

$$S(\rho,\theta) = \exp\left(-\frac{1}{2}\rho e^{-i\theta}\mathbf{K}_+ + \frac{1}{2}\rho e^{i\theta}\mathbf{K}_-\right)$$

- $$S(\sigma,\beta) = \exp(\sigma\mathbf{K}_+)\left(1 - |\sigma|^2\right)\exp(-\sigma^*\mathbf{K}_-)\exp\left(\beta\left(\mathbf{K_0} - \frac{1}{2}\right)\right)$$

$$S(\sigma,\beta)bS^\dagger(\sigma,\beta) = \mathrm{e}^{-\mathrm{i}\beta}(1-|\sigma|^2)^{-1/2}(b-\sigma b^\dagger) \equiv c$$

$$S(\sigma,\beta)b^\dagger S^\dagger(\sigma,\beta) = \mathrm{e}^{\mathrm{i}\beta}(1-|\sigma|^2)^{-1/2}(b^\dagger-\sigma^* b) \equiv c^\dagger.$$

Los operadores $c^\dagger$ y c se conocen como operadores bósonicos de compresión.

[3] G. Dattoli, M. Richetta, and A. Torre . Phys. Rev. A **37**, Operator disentanglement (1988).

Niveles atómicos degenerados

$$\omega_0 = 0 = \tilde{\omega}$$

- Hamiltoniano cuando $\tilde{\omega} = 0$ $\quad \tilde{h}_{2\gamma} = b^\dagger b \pm \lambda \left((b^\dagger)^2 + b^2 \right)$

$$c_\pm^\dagger \equiv \frac{b^\dagger \pm \sigma b}{\sqrt{1-\sigma^2}}, \quad c_\pm \equiv \frac{b \pm \sigma b^\dagger}{\sqrt{1-\sigma^2}}$$

$$H_S = \frac{1}{(1-\sigma^2)} \left\{ [-\sigma + \lambda + \lambda \sigma^2](c^2 + c^{\dagger 2}) + (\sigma^2 + 1 - 4\lambda\sigma)\left(c^\dagger c + \frac{1}{2} \right) \right\}$$

$$-\sigma + \lambda + \lambda\sigma^2 = 0 \quad \Rightarrow \quad \sigma = \frac{1 \pm \sqrt{1-4\lambda^2}}{2\lambda},$$

$$H \equiv \Omega \left(c_\pm^\dagger c_\pm + \frac{1}{2} \right) - \frac{1}{2}$$ tiene la forma de un oscilador armónico.

donde $\quad \Omega \equiv \sqrt{1 - 4\lambda^2}$

- Los eigenestados de este hamiltoniano se conocen como estados comprimidos c-tipo y satisfacen la siguiente ecuación de eigenestados:

$$c_\pm^\dagger c_\pm \,|\, n; \mp \sigma \rangle = n \,|\, n; \mp \sigma \rangle$$

En la representación original, los estados tienen la forma

$$|\, n; \mp \sigma \rangle = \frac{\left(1 - \sigma^2\right)^{1/4}}{\sqrt{n!}} \left[\frac{b^\dagger \pm \sigma b}{\sqrt{1 - \sigma^2}}\right]^n e^{\mp \frac{1}{2}\sigma (b^\dagger)^2} \,|\, 0 \rangle$$

Eigenenergías del Hamiltoniano (TPRH)

$$E_n^{(\omega_0=0)} = \Omega\left(n+\frac{1}{2}\right)-\frac{1}{2}$$

$$\tilde{h}_{2\gamma}\,|n;\mp\sigma\rangle = E_n^{(\omega_0=0)}\,|n;\mp\sigma\rangle$$

Condiciones de normalización para los estados $|n;\mp\sigma\rangle$

$$|0;\mp\sigma\rangle \sim e^{\mp\frac{1}{2}\sigma(b^\dagger)^2}|0\rangle$$

Solo es normalizable para $|\sigma| < 1$ $\qquad \sigma = \dfrac{1-\sqrt{1-4\lambda^2}}{2\lambda}, \qquad |\lambda| < \dfrac{1}{2}$

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Demystifying the spectral collapse in two-photon Rabi model

C. F. Lo

We have investigated the eigenenergy spectrum of the two-photon Rabi model at the critical coupling, particularly the special feature "spectral collapse", by means of an elementary quantum mechanics approach. The eigenenergy spectrum is found to consist of both a set of discrete energy levels and a continuous energy spectrum. Each of these eigenenergies has a two-fold degeneracy corresponding to the spin degree of freedom. The discrete eigenenergy spectrum has a one-to-one mapping with that of a particle in a "Lorentzian function" potential well, and the continuous energy spectrum can be derived from the scattering problem associated with a potential barrier. The number of bound states available at the critical coupling is determined by the energy difference between the two atomic levels so that the extent of the "spectral collapse" can be monitored in a straightforward manner.

Two-photon Rabi model: analytic solutions and spectral collapse

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Spectral collapse in anisotropic two-photon Rabi model

C. F. Lo

Spectral collapse via two-phonon interactions in trapped ions

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Colapso Espectral

$$|\lambda| < \frac{1}{2}$$

- El colapso espectral es la transición de un espectro discreto a uno continuo y es característico de los modelos de tipo Rabi.
- En el espacio de fase óptico, $$\hat{H}_R = \frac{1}{2}(\alpha_+\hat{p}^2 + \alpha_-\hat{q}^2)\hat{\sigma}_0 + \frac{1}{2}\omega_0[\hat{R}_{\frac{\pi}{4}}\hat{\sigma}_+ + \hat{R}^\dagger_{\frac{\pi}{4}}\hat{\sigma}_-],$$
- El colapso espectral es similar a una transición del oscilador armónico a un oscilador invertido con el potencial de una partícula libre como un punto crítico de transición.

[5] R. J. Armenta Rico, et al. *Spectral collapse in the two-photon quantum Rabi model* Phys. Rev. A **101**, 063825 (2020)

Análisis Semi-clásico

$$\hat{H}_0 = \frac{1}{2}(\alpha_+\hat{p}^2 + \alpha_-\hat{q}^2)$$

- $\alpha_- = 0$
- $g_c = \dfrac{\omega}{2}$
- $\alpha_- < 0$
-

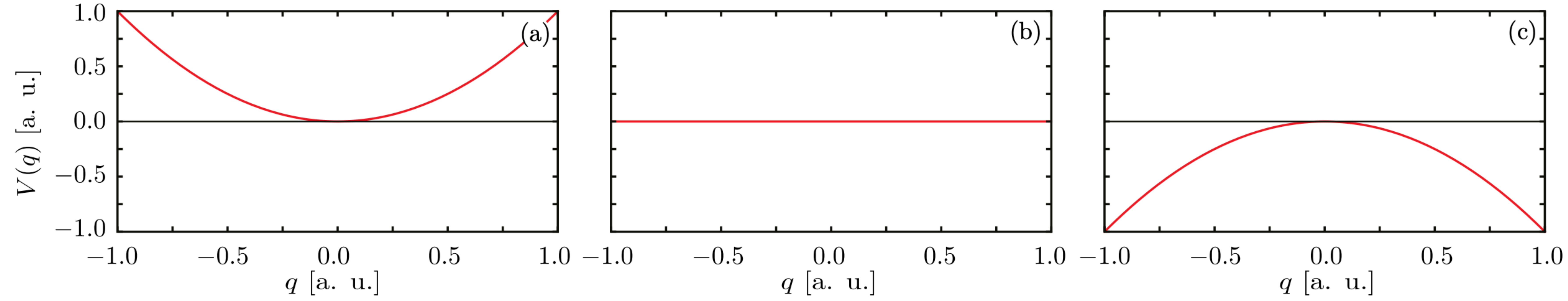
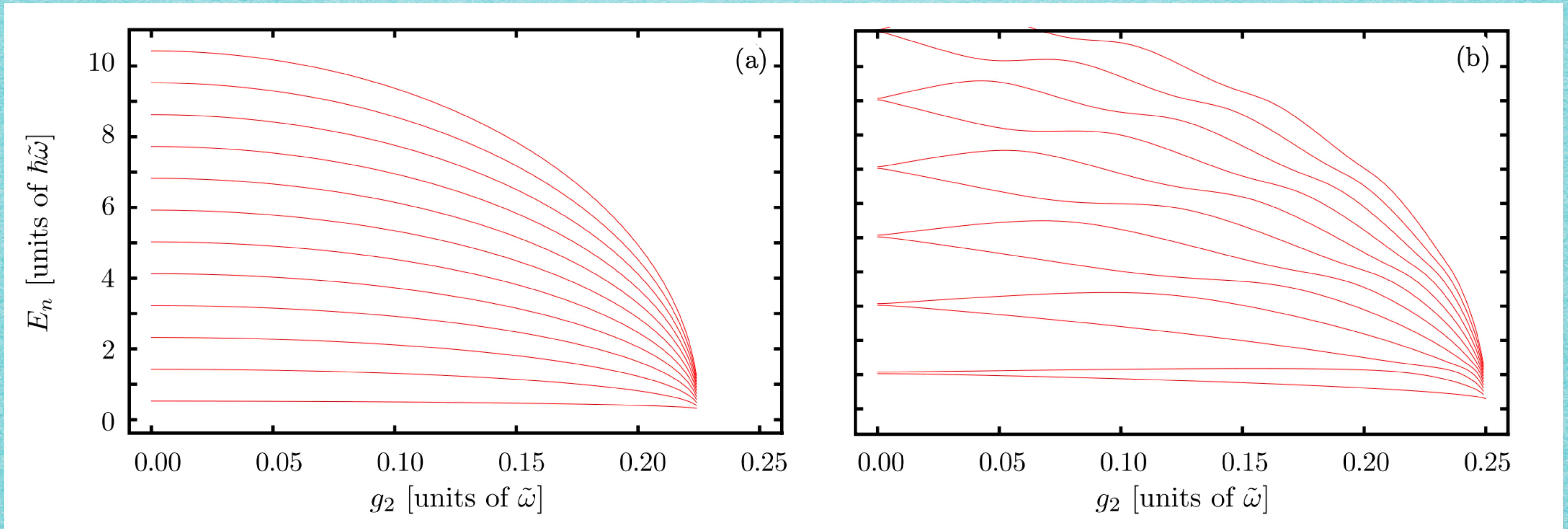


FIG. 1. Effective pseudopotential $V(\hat{q}) = (\omega - 2g_2)\hat{q}^2$ in the diagonal terms of the rotated two-photon quantum Rabi model with (a) $\omega - 2g_2 = 1$, (b) $\omega - 2g_2 = 0$, and (c) $\omega - 2g_2 = -1$.

Transición de un oscilador armónico a una partícula libre y luego a un oscilador armónico invertido.

Colapso espectral de un TPQR



(a) the degenerate qubit regime $\omega_0 = 0, \ = 0$

(b) on-resonance $\omega_0 = \tilde{\omega}$

Approximating the two-mode two-photon Rabi model

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Spectral collapse in multiqubit two-photon Rabi model

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We have shown that the smallest possible singel-qubit critical coupling strength of the N-qubit two-photon Rabi model is only 1/N times that of the two-photon Rabi model. The spectral collapse can thus occur at a more attainable value of the critical coupling. For both of the two-qubit and three-qubit cases, we have also rigorously demonstrated that at the critical coupling the system not only has a set of discrete eigenenergies but also a continuous energy spectrum. The discrete eigenenergy spectrum can be derived via a simple one-to-one mapping to the bound state problem of a particle of variable effective mass in the presence of a finite potential well and a nonlocal potential. The energy difference of each qubit, which specifies both the depth of the finite potential well and the strength of the nonlocal potential, determines the number of bound states available, implying that the extent of the incomplete spectral collapse can be monitored in a straightforward manner.

Conclusiones

- El colapso espectral en el modelo cuántico de Rabi para 2 fotones (TPRH) es análogo a una transición de un oscilador armónico a un oscilador invertido, con el punto crítico siendo análogo a una partícula libre que tiene la mitad de la frecuencia del campo bósonico. Para mostrar este mecanismo en el régimen degenerado se usa el espacio de fase óptico.

- La transformación de los bosones es fisicamente más significativo que el uso de funciones de onda en el espacio de Bargmann, especialmente da una conexión de estos bosones con los estados coherentes y estados comprimidos.

Gracias