Instituto de Ciencias Nucleares

Función de Wigner

Mecánica cuántica en el espacio fase

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Algunos conceptos de Mecánica cuántica

Elementos

Función de onda, operadores y matriz de densidad

$$i\frac{\partial}{\partial t}\Psi(\vec{r}) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r}) + V(\vec{r})\Psi(\vec{r})$$

$$\frac{d}{dt}A_H(t) = \frac{i}{\hbar}[H_H, A_H(t)] + \left(\frac{\partial A_S}{\partial t}\right)_H$$

$$i\hbar\frac{\partial\rho}{\partial t} = [H, \rho]$$

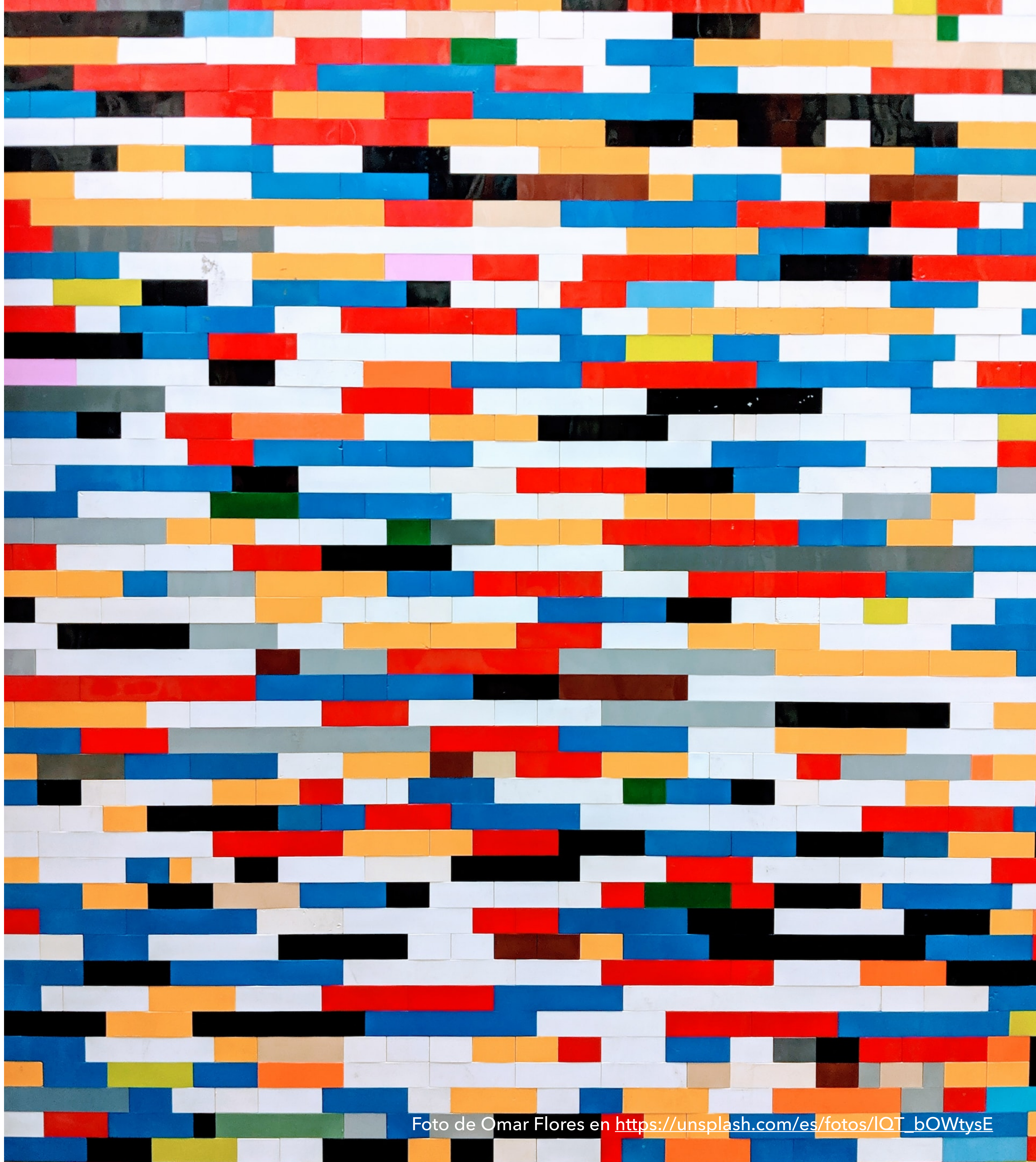

Foto de Omar Flores en https://unsplash.com/es/fotos/lQT_bOWtysE

¿Cómo visualizamos el espacio fase en mecánica cuántica?

Principio de indeterminación de Heisenberg

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Foto de Marcel Strauß en https://unsplash.com/es/fotos/KFSb6IhEVns

La función de Wigner

Definición

$$W(x,p) = \frac{1}{2\pi\hbar}\int dy\, \psi^*(x - \frac{y}{2})\, e^{-\frac{iyp}{\hbar}}\, \psi(x + \frac{y}{2})$$

*podría ser cualquier par de variables conjugadas

Otras definiciones

$$W(x,p) = \frac{1}{2\pi\hbar}\int dy\,\langle x-y\,|\,\hat{\rho}\,|\,x+y\rangle\, e^{-\frac{iyp}{\hbar}}$$

$$W(x,p) = \frac{1}{\pi\hbar}\int dy\,\langle x-y\,|\,\hat{\rho}\,|\,x+y\rangle\, e^{\frac{i2py}{\hbar}}$$

$$W(x,p) = \frac{1}{\pi\hbar}\int dy\,\psi^*(x+y)\psi(x-y)e^{\frac{i2py}{\hbar}}$$

Schleich, W. (2001) Quantum Optics in Phase Space. Wiley.

Propiedades

de la función de Wigner de un estado puro

- $$\int dp\, W(x,p) = |\psi(x)|^2$$
- $$\int dx\, W(x,p) = |\varphi(p)|^2$$
- $$\int dx\, dp\, W(x,p) = 1$$
- $$\langle \hat{A} \rangle = \int dp \int dx\, W(x,p)\, A(x,p)$$

Una propiedad más

Función de Wigner como una función de cuasi-probabilidad

$$-\frac{2}{\hbar} \leq W(x,p) \leq \frac{2}{\hbar}$$

Negativity of the Wigner function as an indicator of nonclassicality

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A measure of nonclassicality of quantum states based on the volume of the negative part of the Wigner function is proposed. We analyze this quantity for Fock states, squeezed displaced Fock states and cat-like states defined as coherent superposition of two Gaussian wave packets.

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I. INTRODUCTION

Analyzing pure quantum states in an infinite dimensional Hilbert space it is useful to distinguish a family of *coherent states*, localized in the classical phase space and minimizing the uncertainty principle. These quantum analogues of points in the classical phase space are often considered as 'classical' states. For an arbitrary quantum state one may pose a natural question, to what extent it is 'nonclassical' in a sense that its properties differ from that of coherent states. In other words, is there any parameter that may legitimately reflect the degree of nonclassicality of a given quantum state? This question was motivated with the first observation of nonclassical features of electromagnetic fields such as sub-poissonian statistics, antibunching and squeezing. Additionally, it is well known that the interaction of (non)linear devices with quantum states may flip from one state to another; for instance, nonlinear devices may produce nonclassical states from their interaction with the vacuum or a classical field. A systematic survey of nonclassical properties of quantum states would be worthwhile because of the nowadays ever increasing number of experiments in nonlinear optics. An earlier attempt to sheding some light on the nonclassicality of a quantum state was pioneered by Mandel [1], who investigated the radiation fields and introduced a parameter **q** measuring the deviation of the photon number statistics from the Poissonian distribution, characteristic of coherent states.

In general, to define a measure of nonclassicality of quantum states one can follow several different approaches [2]. Distinguishing a certain set $\mathcal{C}$ of states (e.g. the set of coherent states $|\alpha\rangle$), one looks for the distance of an analyzed pure state $|\psi\rangle$ to this set, by minimizing a distance $d(|\psi\rangle, |\alpha\rangle)$ over the entire set $\mathcal{C}$. Such a scheme based on the trace distance was first used by Hillery [3, 4], while other distances (Hilbert-Schmidt distance [5, 6] or Bures distance [7, 8]) were later used for this purpose. The same approach is also applicable to characterize mixed quantum states: minimizing the distance of the density ρ to the set of coherent states is related [6, 9] to the search for the

Ejemplos

Oscilador armónico

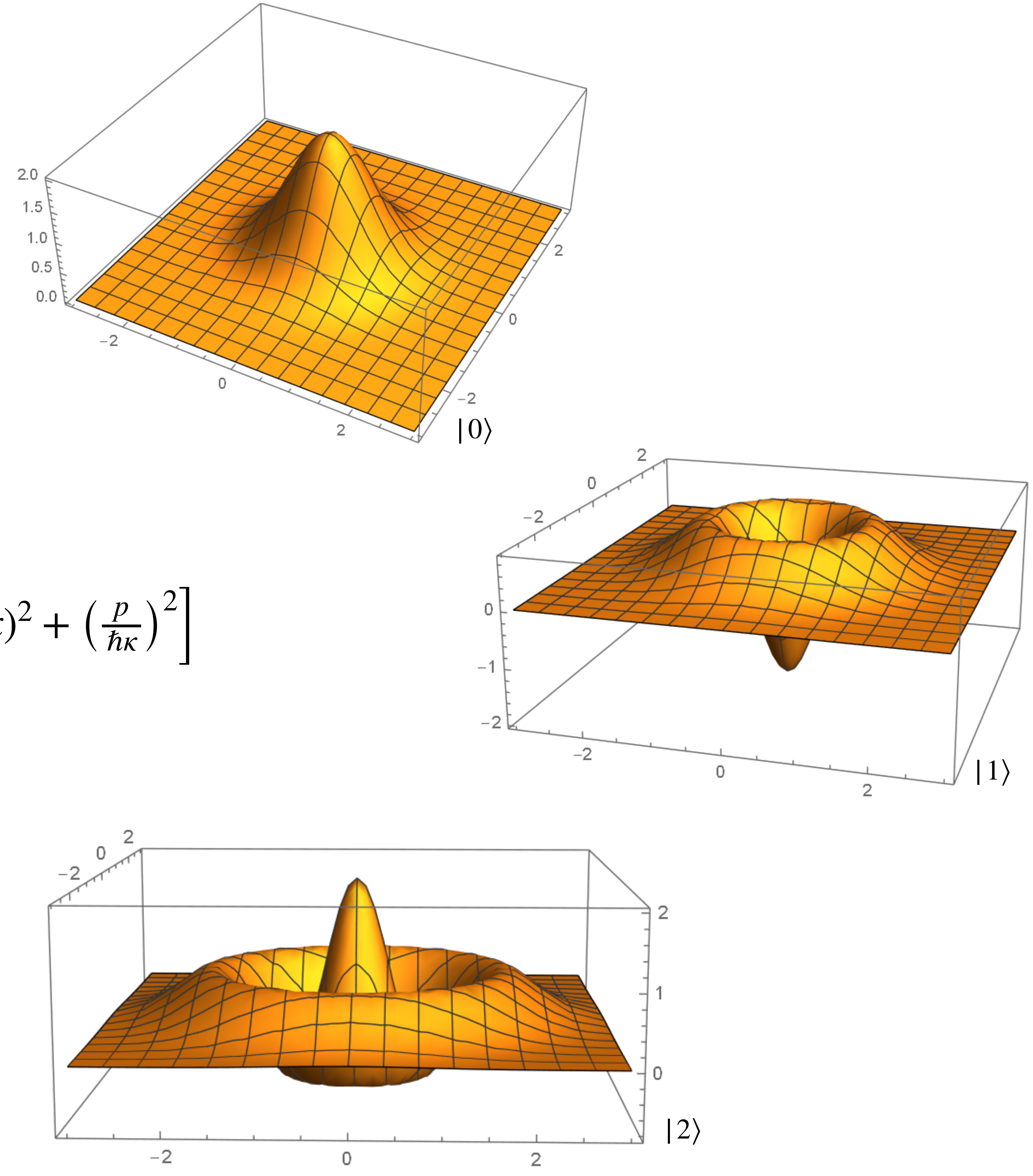
$$\langle x | n \rangle = \frac{1}{\sqrt{2^n \, n!}} \left(\frac{\kappa^2}{\pi} \right)^{\frac{1}{4}} e^{-\frac{(\kappa x)^2}{2}} H_n(\kappa x)$$

$$W(x,p) = \frac{1}{2\pi\hbar} \int dy \, \psi^*(x - \frac{y}{2}) \, e^{-\frac{iyp}{\hbar}} \, \psi(x + \frac{y}{2})$$

$$\oplus \; L_n(-2ab) = \frac{1}{\sqrt{\pi} \, 2^n \, n!} \int dx \, e^{-x^2} H_n(x+a) \, H_n(x+b)$$

Función de Wigner de los estados de Fock

$$W_{|n\rangle}(x,p) = \frac{(-1)^n}{\pi\hbar} L_n\left(2\left[(\kappa x)^2 + \left(\frac{p}{\hbar\kappa}\right)^2\right]\right) e^{-\left[(\kappa x)^2 + \left(\frac{p}{\hbar\kappa}\right)^2\right]}$$



Estados coherentes y estados comprimidos

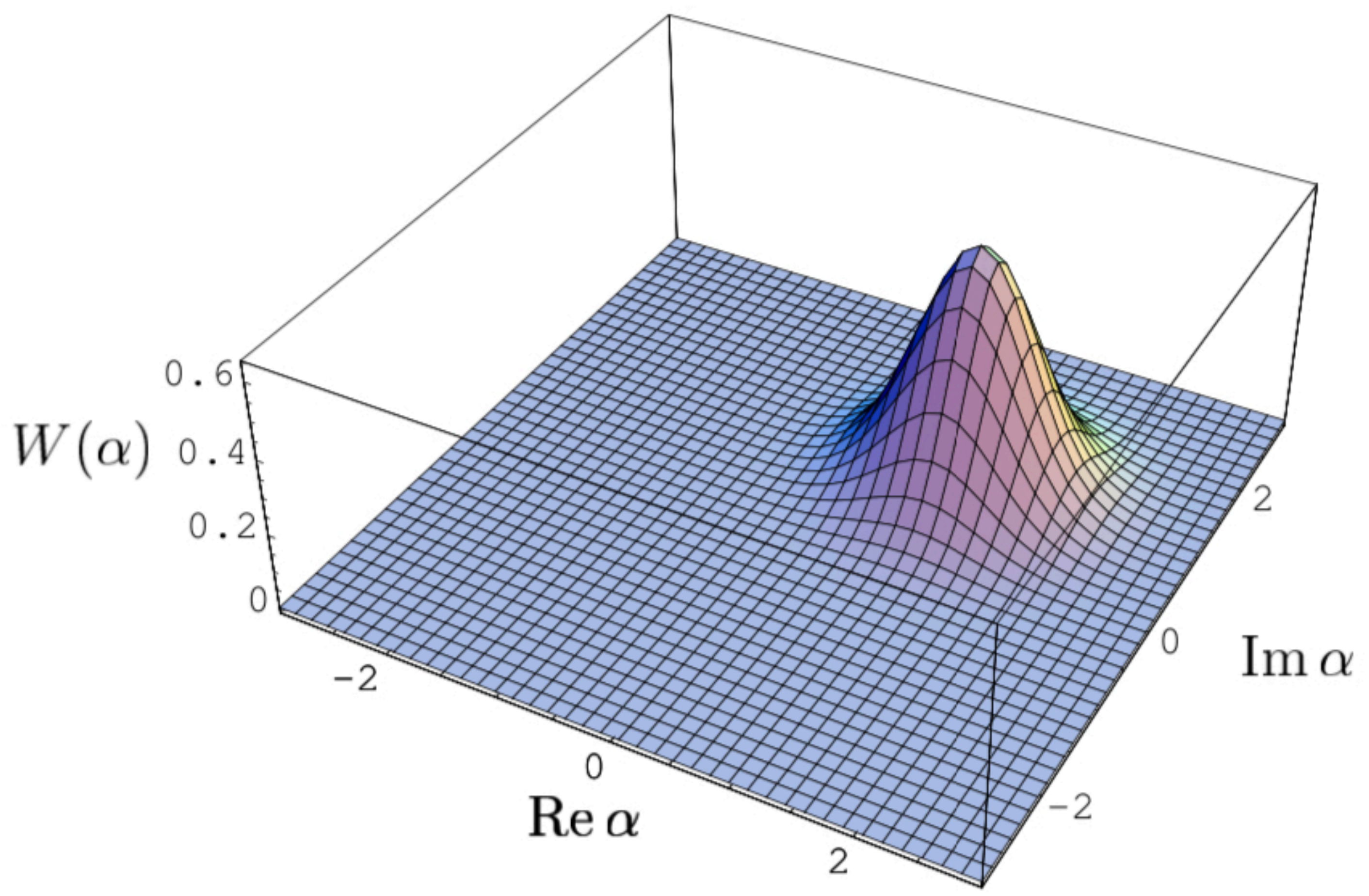


FIG. 7: Wigner function of a coherent state with $\alpha_0 = 1 + i$.

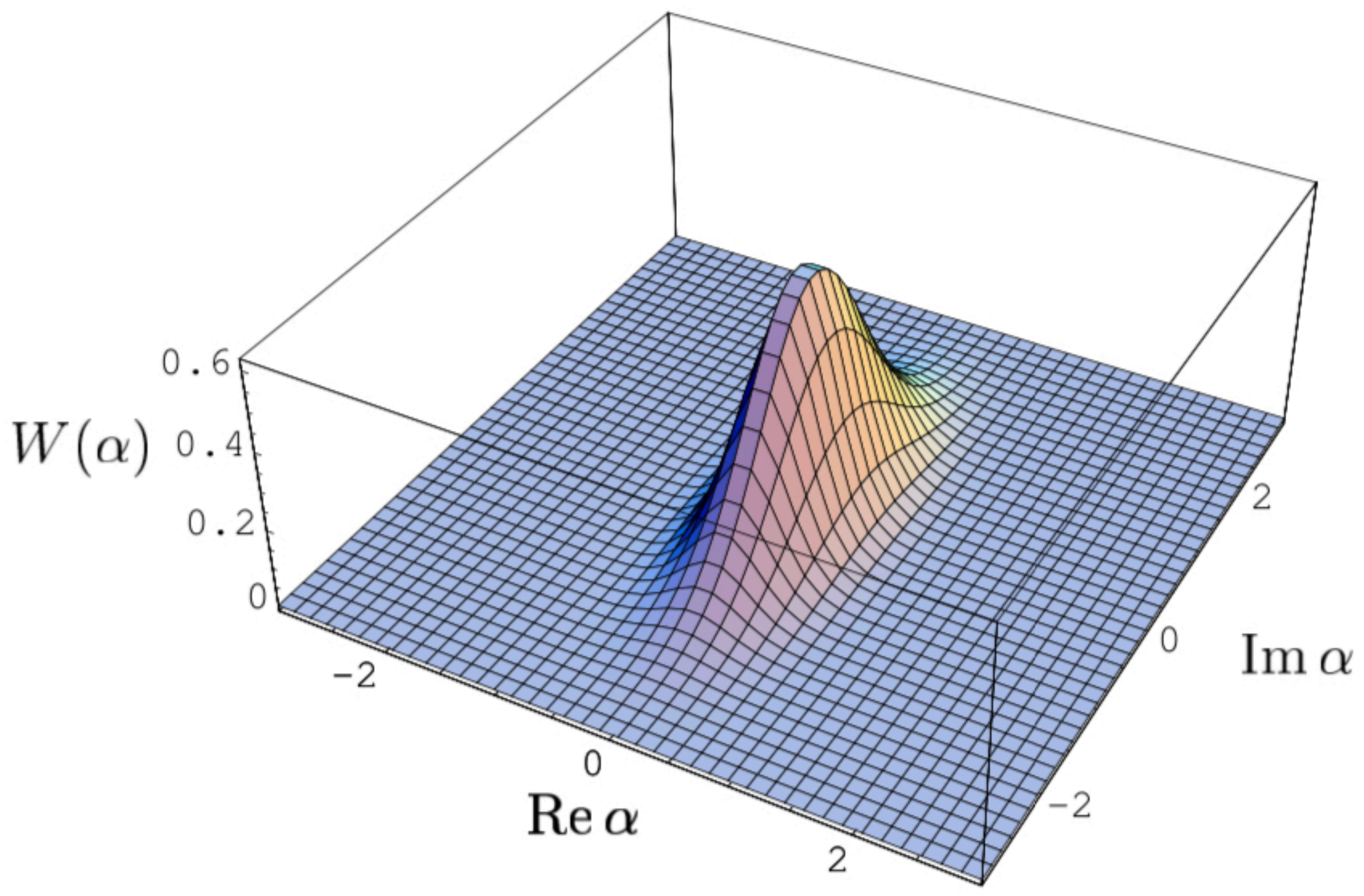


FIG. 8: Wigner function of a squeezed vacuum state with $\xi = 1$.

https://www.lkv.uni-rostock.de/storages/uni-rostock/Alle_MNF/Physik_Qms/Lehre_Scheel/quantenoptik/Quantenoptik-Vorlesung4.pdf

Estados Coherentes

y su evolución temporal

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$\langle x|\alpha(t)\rangle = \left(\frac{\kappa^2}{\pi}\right)^{\frac{1}{4}} e^{-\frac{1}{2}\left[|\alpha|^2 + (\kappa x)^2 + i\omega t + \alpha^2 e^{-i2\omega t} - 2\sqrt{2}\kappa x \alpha e^{-i\omega t}\right]}$$

Estados Coherentes

función de Wigner y su evolución temporal

$$W_{|\alpha\rangle}(x,p,t) = \frac{1}{\pi\hbar}\, e^{-\left[|\alpha|^2 + \Re\left(\alpha^2 e^{-i2\omega t}\right) - 2\sqrt{2}\kappa x\, \Re\left(\alpha e^{-i\omega t}\right) + (\kappa x)^2 + \left(\frac{p}{\hbar\kappa}\right)^2 - 2\sqrt{2}\left(\frac{p}{\hbar\kappa}\right)\Im\left(\alpha e^{-i\omega t}\right) + 2\left(\Im\left(\alpha e^{-i\omega t}\right)\right)^2\right]}$$

Estados Coherentes

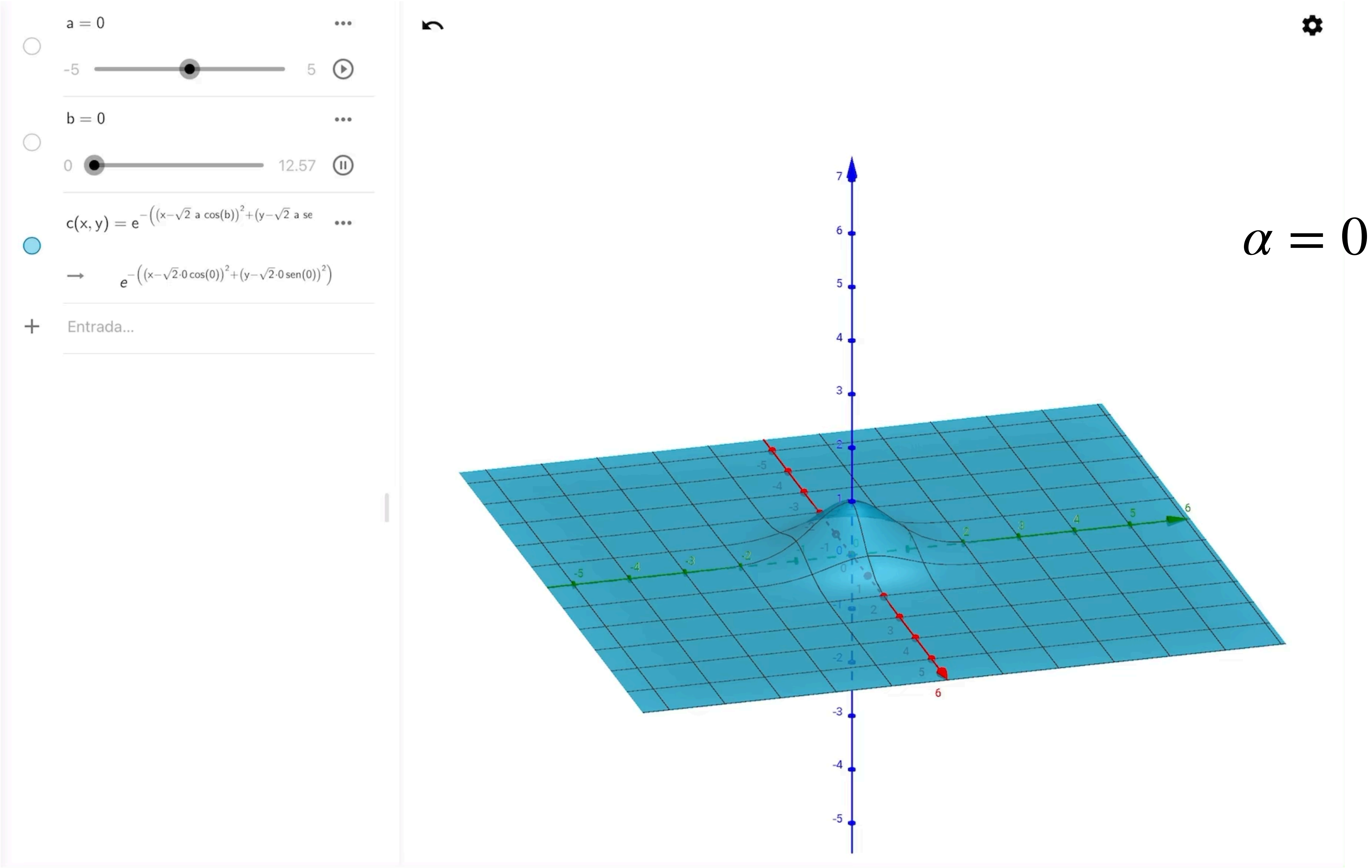
función de Wigner y su evolución temporal

$$W_{|\alpha\rangle}(x,p,t) = \frac{1}{\pi\hbar}\, e^{-\left(\left[\kappa x - \sqrt{2}\alpha\cos(\omega t)\right]^2 + \left[\frac{p}{\hbar\kappa} - \sqrt{2}\alpha\sin(\omega t)\right]^2\right)}$$

*siempre y cuando α sea número real

Estados Coherentes

animación de la evolución temporal de la función de Wigner

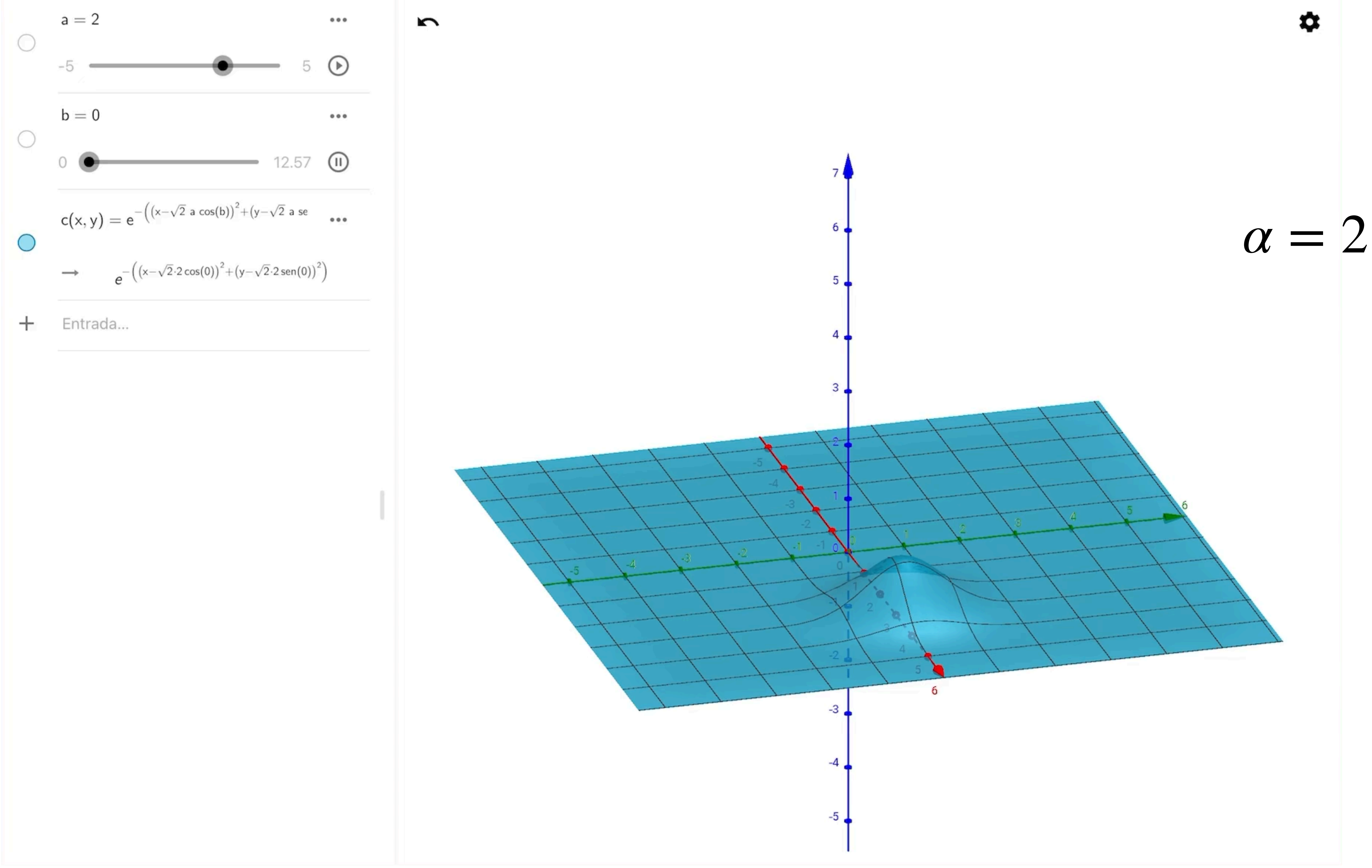


$\alpha = 0$

*siempre y cuando α sea número real

Estados Coherentes

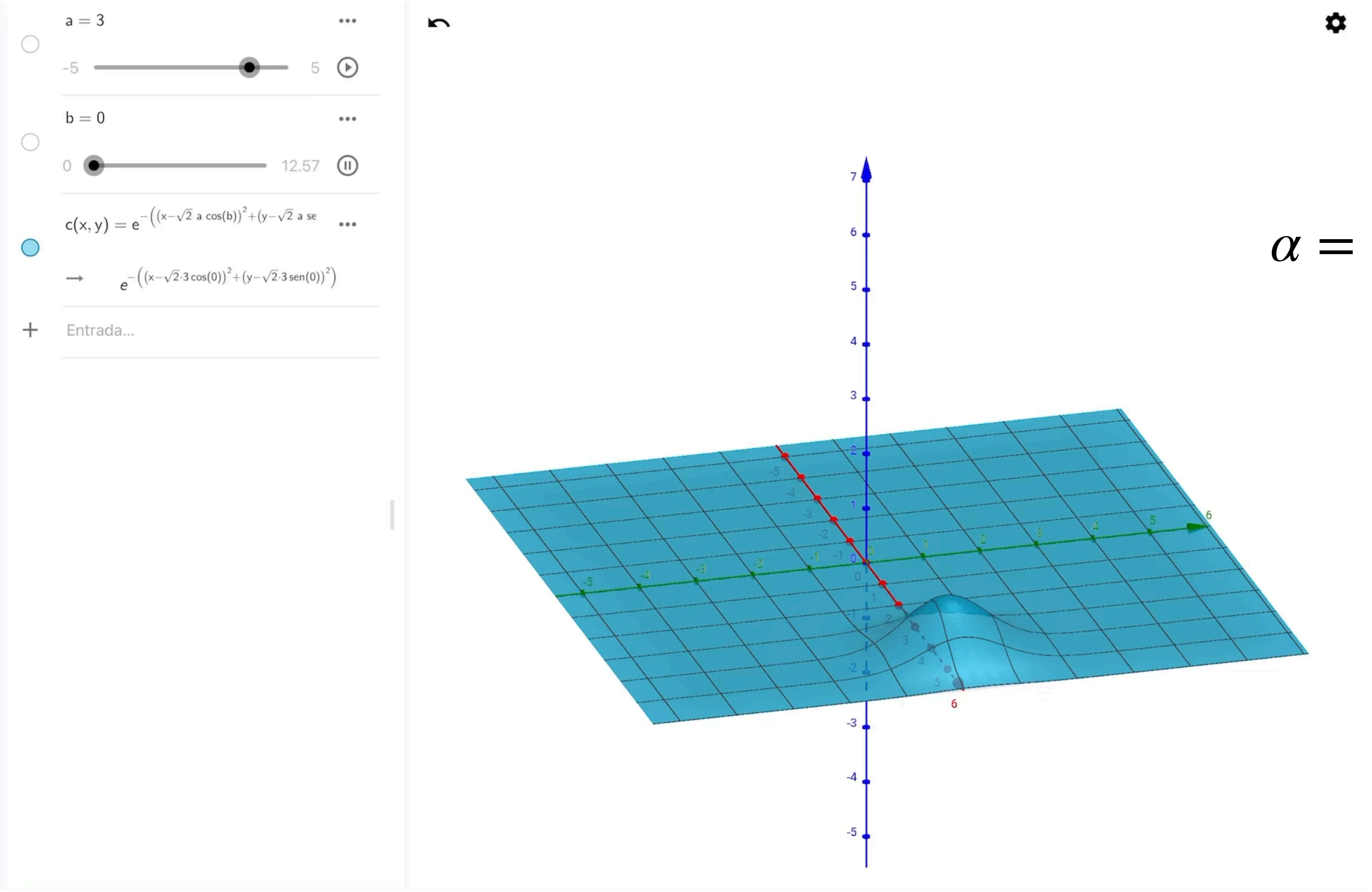
animación de la evolución temporal de la función de Wigner



*siempre y cuando α sea número real

Estados Coherentes

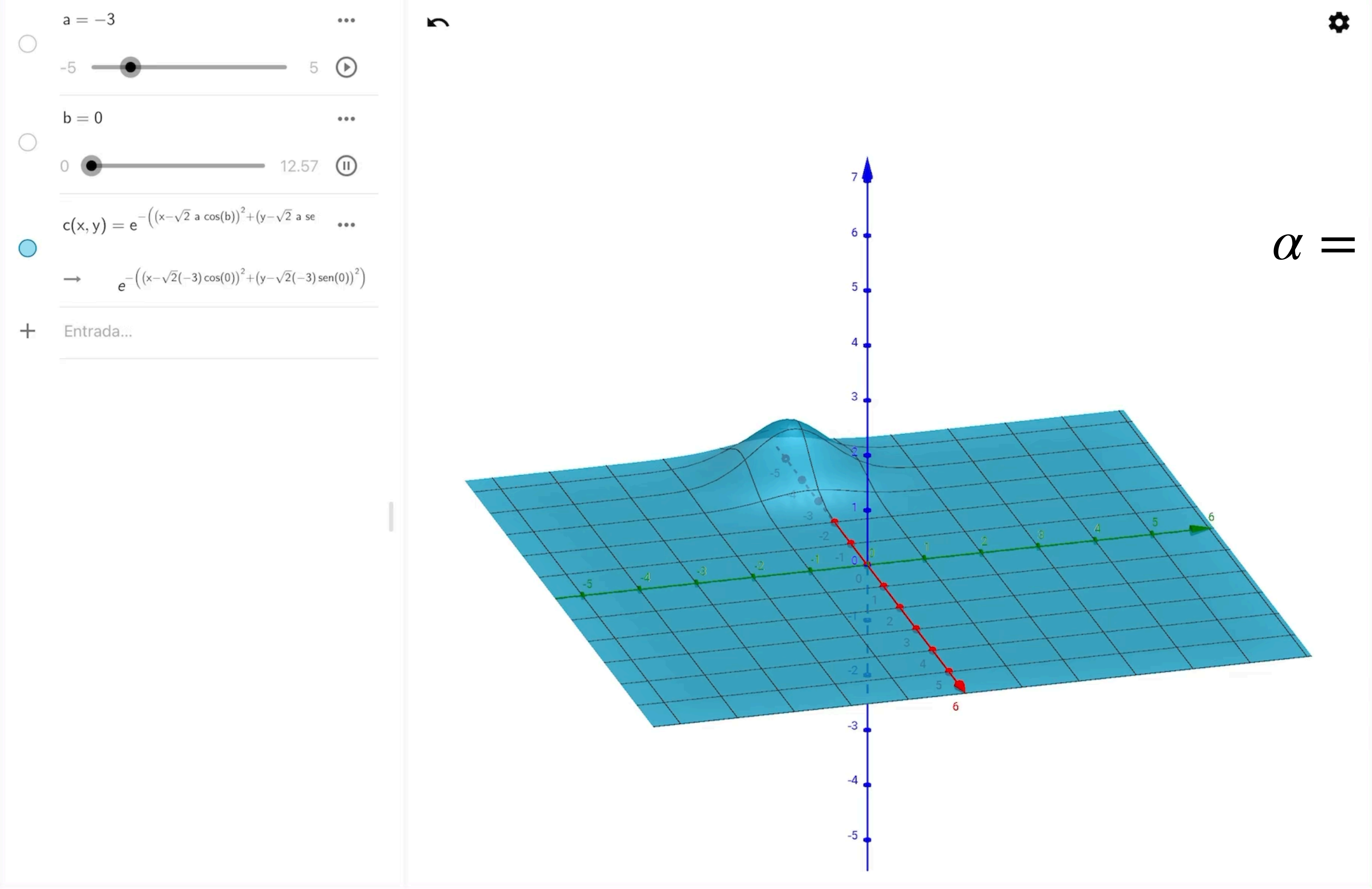
animación de la evolución temporal de la función de Wigner



*siempre y cuando α sea número real

Estados Coherentes

animación de la evolución temporal de la función de Wigner

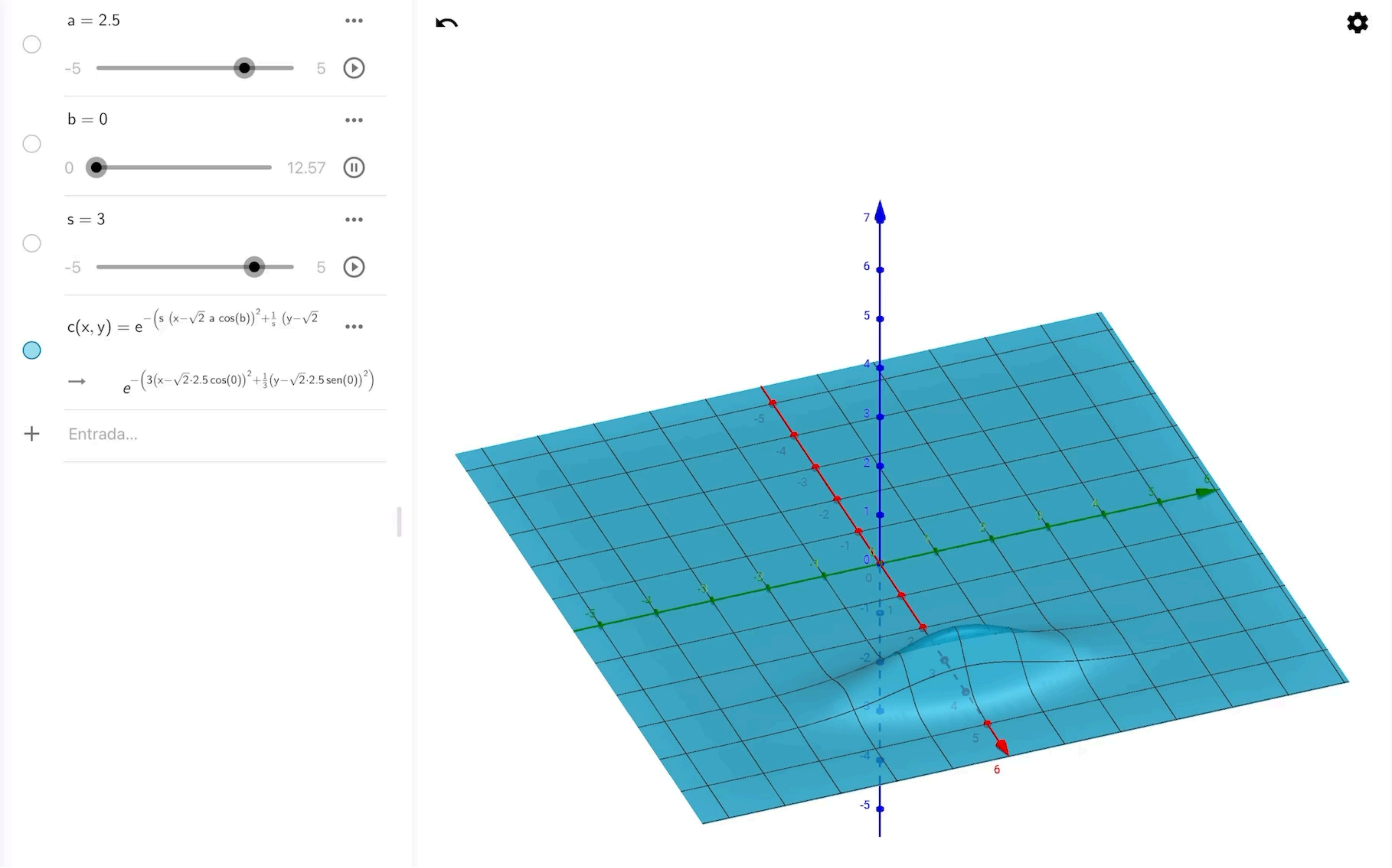


$\alpha = -3$

*siempre y cuando α sea número real

Estados Comprimidos

animación de la evolución temporal de la función de Wigner



$\alpha = 2.5, s = 3$

Schleich, W. (2001) Quantum Optics in Phase Space. Wiley.

*siempre y cuando α sea número real

Otras funciones de cuasi-probabilidad

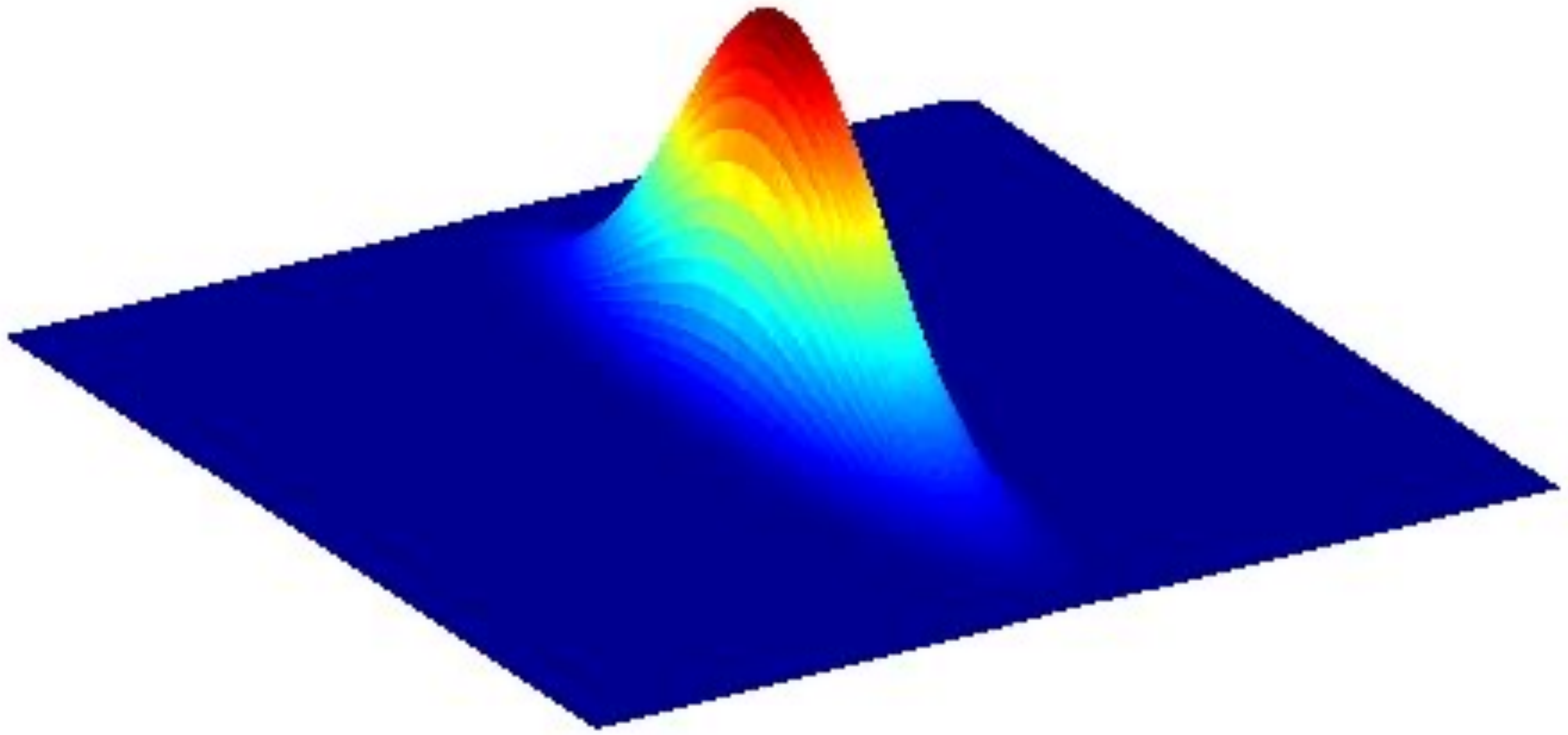
Representación de Husimi Q

$$Q(\alpha) = \frac{1}{\pi}\langle \alpha \,|\, \hat{\rho} \,|\, \alpha \rangle$$

O alternativamente

$$Q(q,p) = \frac{1}{\pi\hbar}\int W(q',p')\, e^{-(q'-q)^2/2s^2} e^{-(p'-p)^2(2s^2/\hbar^2)}\, dq dp$$

Ballentine, L. E.: Quantum mechanics: A modern development, capítulo 15



Representación de Husimi de un estado comprimido

Conclusiones

- La función de Wigner es una función de cuasiprobabilidad que permite visualizar el espacio fase de un estado cuántico (puro o mixto).
- Recupera probabilidades marginales.
- Existen expresiones cerradas para los estados de Fock, coherentes y comprimidos.

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