SPONTANEOUS STABILIZATION OF PRODUCT STATES IN THE RABI MODEL

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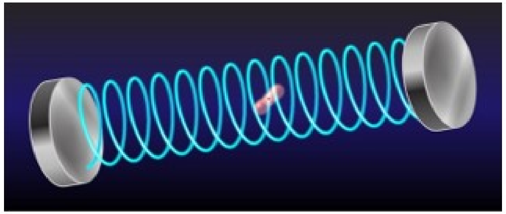
Outline

- Extended Rabi model and its classical limit
- QPTs and ESQPTs
- Stability of the noninteracting ground state
- Parity violation

Rabi model



Experiments:

- F. Yoshihara et al., *Nat. Phys.* **13**, 44 (2017)
- N.K. Langford et al., *Nat. Commun.* **8**, 1715 (2017)
- M.-L. Cai et al., *Nat. Commun.* **12**, 1126 (2021)

Two-level system

(atom, qubit, spin $j = \frac{1}{2}$, ...)

$$\hat{H}_{\mathrm{spin}} = \omega_0 \hat{J}_z$$

$$\hat{\boldsymbol{J}} = \frac{1}{2}(\sigma_x, \sigma_y, \sigma_z)$$

Strong detuning

$$R \equiv \frac{\omega_0}{\omega} \gg 1$$

Harmonic oscillator

(EM field,...)

$$H_{\mathrm{osc}} = \omega \hat{b}^+ \hat{b}$$

$$\hat{b} = \sqrt{\frac{R}{2}}(\hat{q} + i\hat{p})$$

Interaction

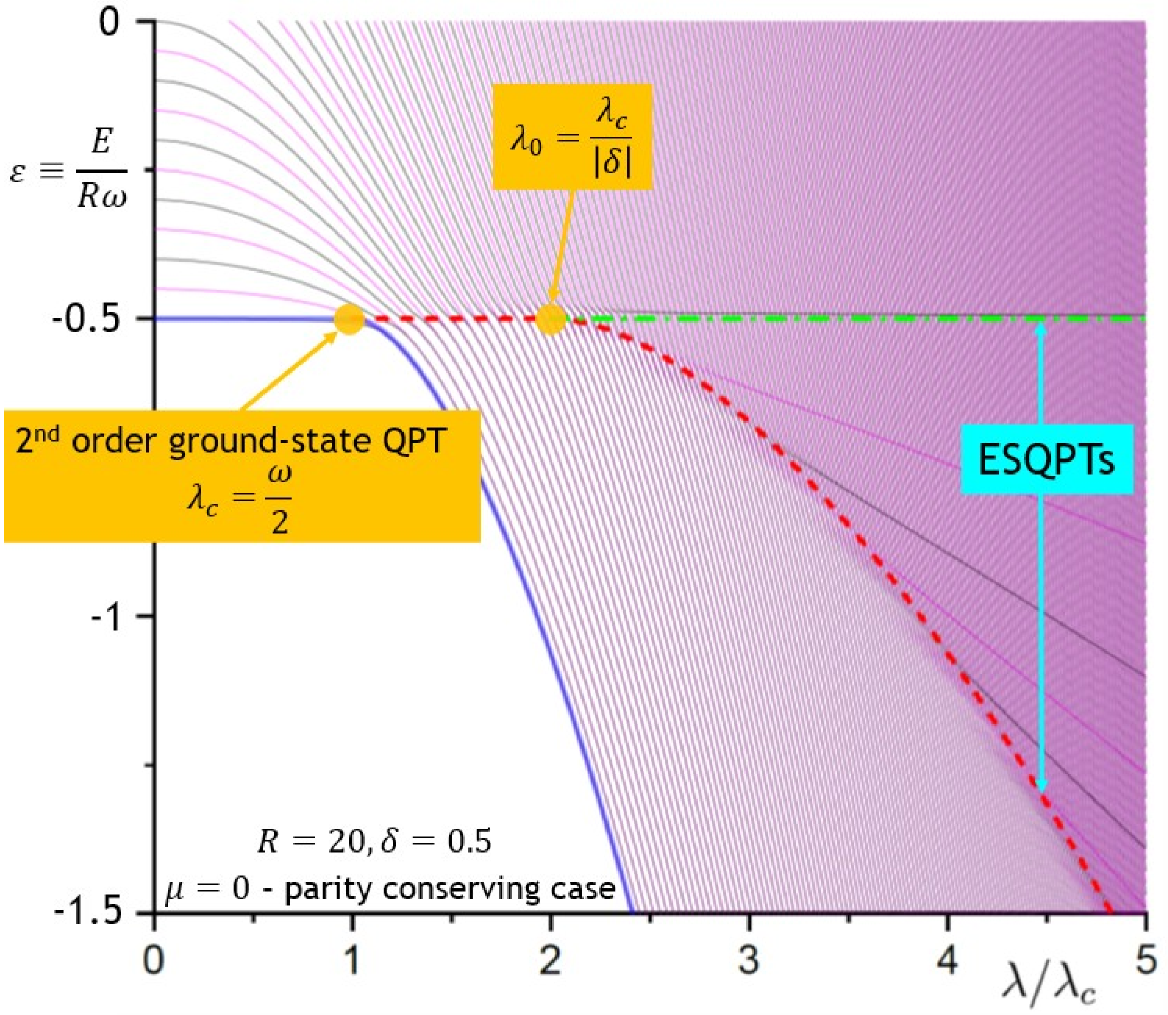
Conserving parity: $$H_{\mathrm{int}} = \lambda\sqrt{R}\left[(1+\delta)\left(\hat{b}^+\hat{J}_- + \hat{b}\hat{J}_+\right) + (1-\delta)\left(\hat{b}^+\hat{J}_+ + \hat{b}\hat{J}_-\right)\right]$$ $\lambda \in [0, \infty), \delta \in [-1,1]$

$$\hat{\Pi} \equiv (-1)^{\hat{n}+\hat{n}_*} \qquad \hat{n} \equiv \hat{b}^+\hat{b} \qquad \hat{n}_* \equiv \hat{J}_z + j$$

- $\delta = 1$: **Jaynes-Cummings**
- $\delta = 0$: **Rabi**
- $\delta = -1$: **Anti-Jaynes-Cummings**

Violating parity: $$+\mu\sqrt{R}\left(\hat{b}^+ + \hat{b}\right)\left(\hat{J}_z + j\right)$$

Quantum critical effects



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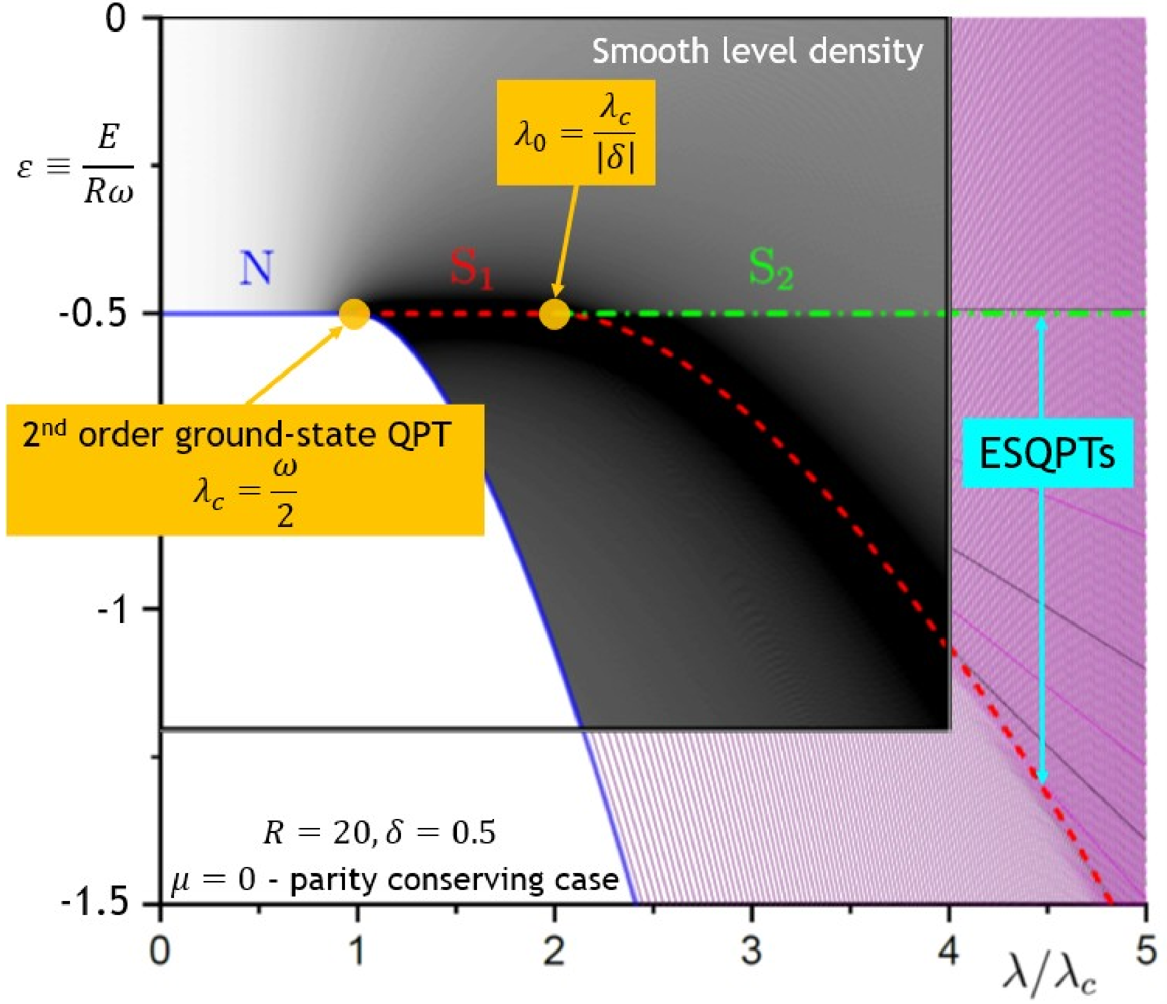


$R \to \infty$ classical limit

$$[\hat{q}, \hat{p}] = \frac{i}{R}$$

$$\frac{\hat{H}}{R} = -\frac{1}{2R} + \frac{\hat{p}^2 + \hat{q}^2}{2} + \sqrt{2}\frac{\mu}{\omega}\hat{q} + \underbrace{\left(\sqrt{8}\frac{\lambda}{\omega}\hat{q}, -\sqrt{8}\frac{\lambda\delta}{\omega}\hat{p}, 1 + \sqrt{8}\frac{\mu}{\omega}\hat{q}\right)}_{\hat{B}(\hat{q},\hat{p})} \cdot \hat{\boldsymbol{J}}$$

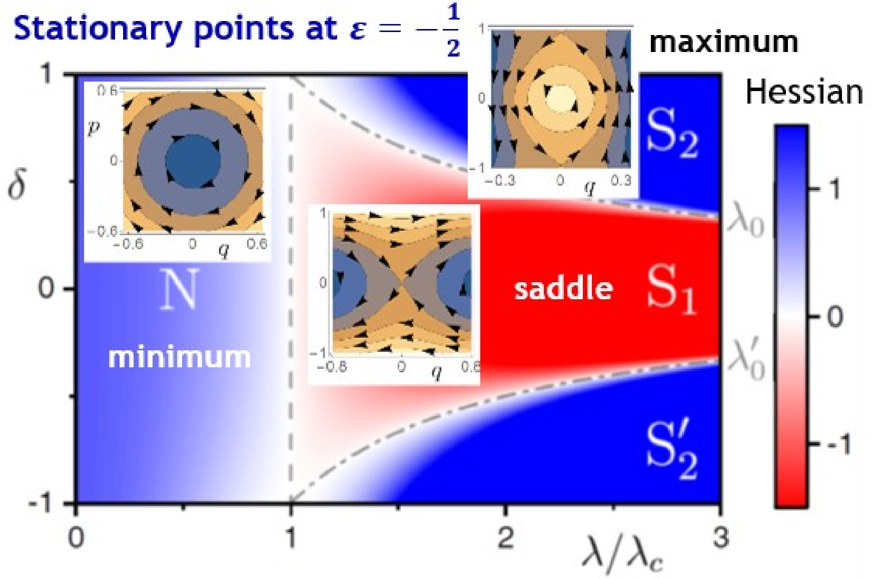
Magnetic dipole in an external field

In the $R \to \infty$ limit the field becomes classical

(adiabatic separation of the field and qubit subsystems)

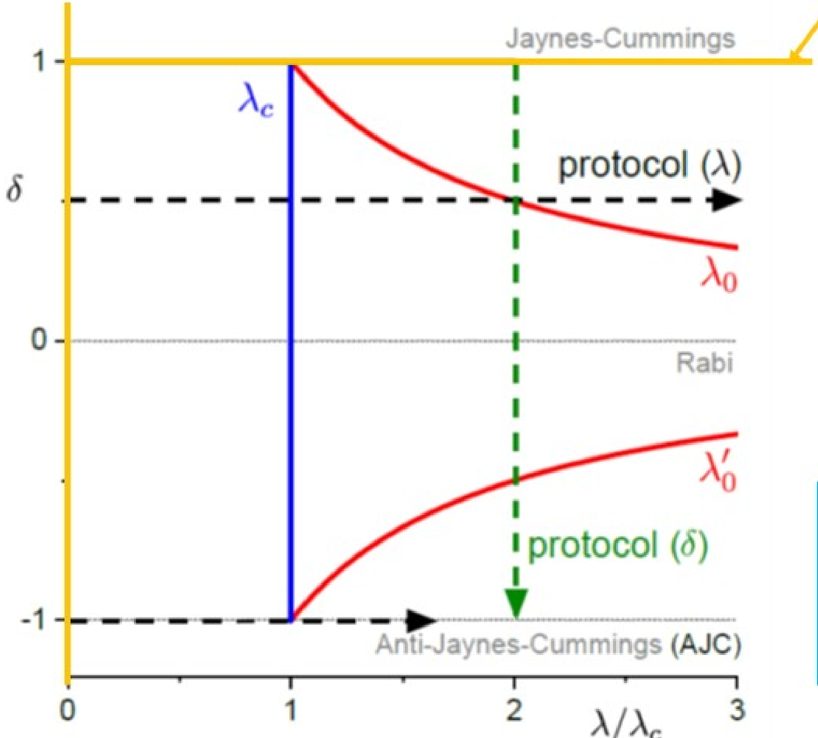
$$H_{cl} = \frac{q^2 + p^2}{2} + \sqrt{2}\frac{\mu}{\omega}q \pm \frac{1}{2}\sqrt{\frac{8\lambda^2}{\omega^2}q^2 + \frac{8\lambda^2\delta^2}{\omega^2}p^2 + \left(1 + \sqrt{8}\frac{\mu}{\omega}q\right)^2}$$

Stationary points at $\boldsymbol{\varepsilon = -\frac{1}{2}}$

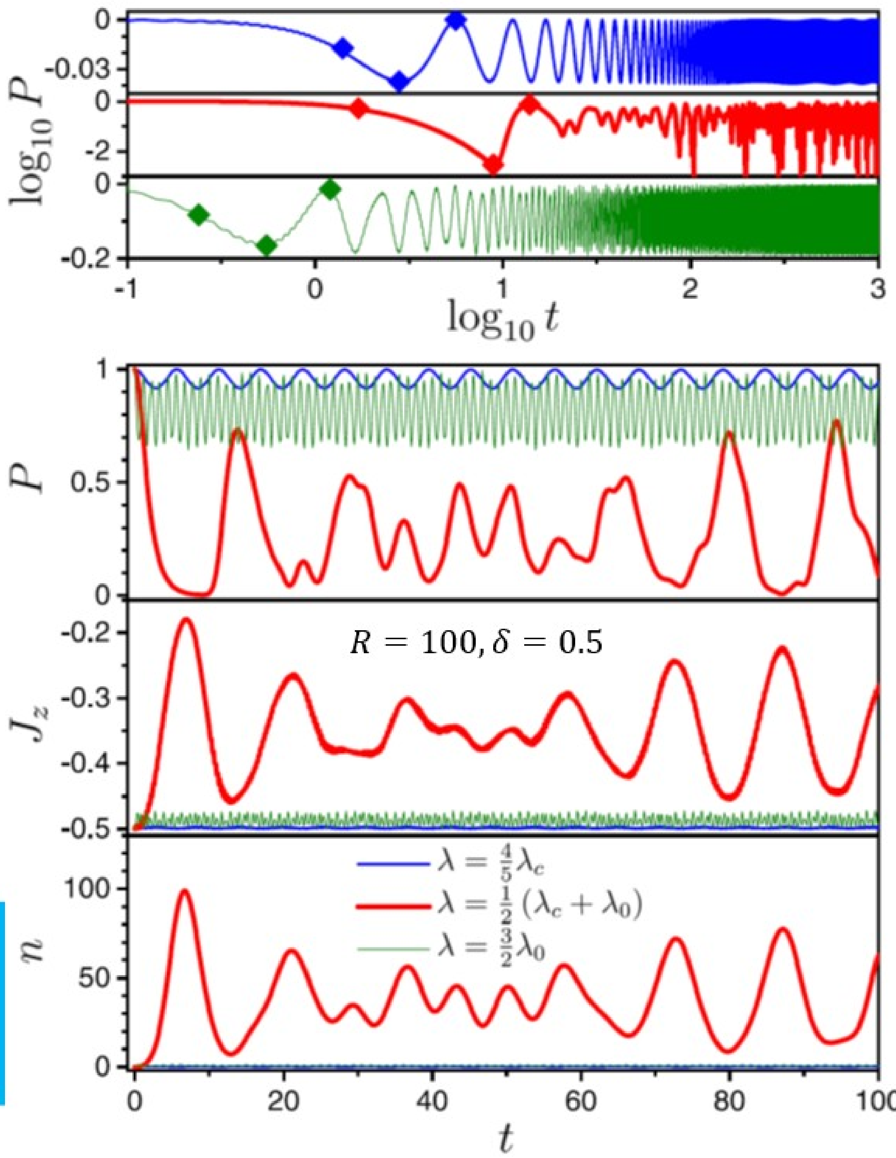


Quench of the noninteracting state

- Noninteracting ground state $|\psi_0\rangle = |n=0\rangle \otimes |m=-\frac{1}{2}\rangle$ at $q=p=0$ stationary point (remains g.s. in the whole Jaynes-Cummings regime)
- Expectation value $A(t) = \langle\psi_0|e^{i\hat{H}t}\hat{A}e^{-i\hat{H}t}|\psi_0\rangle$
- Survival probability $\hat{P} = |\psi_0\rangle\langle\psi_0|$



Parity conservation:
$0 = J_x(t) = J_y(t) = p(t) = q(t) = 0$



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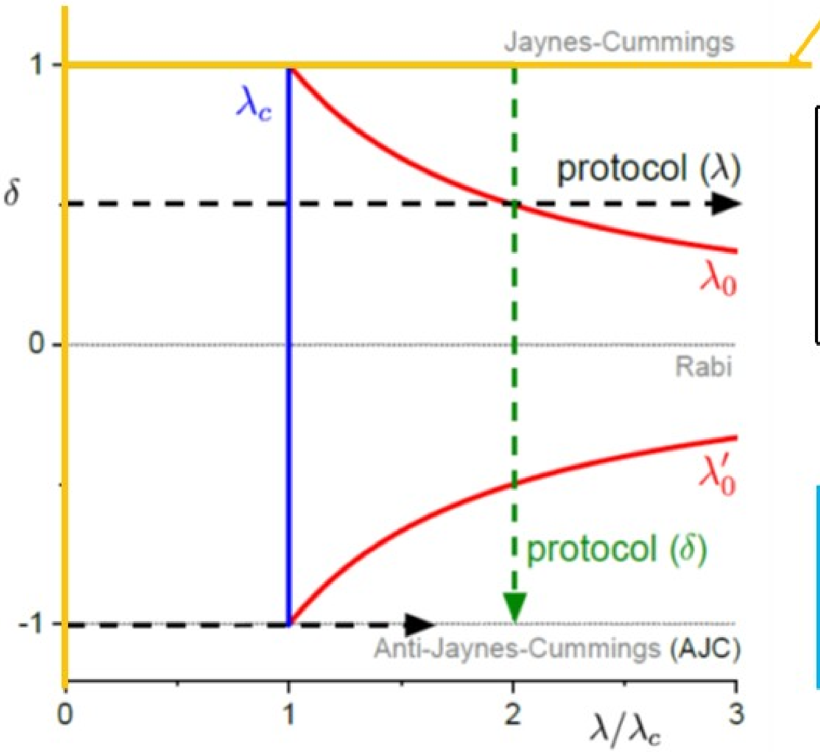

Infinite-time average

$$\bar{A} = \lim_{T\to\infty}\frac{1}{T}\int_0^T A(t)dt$$

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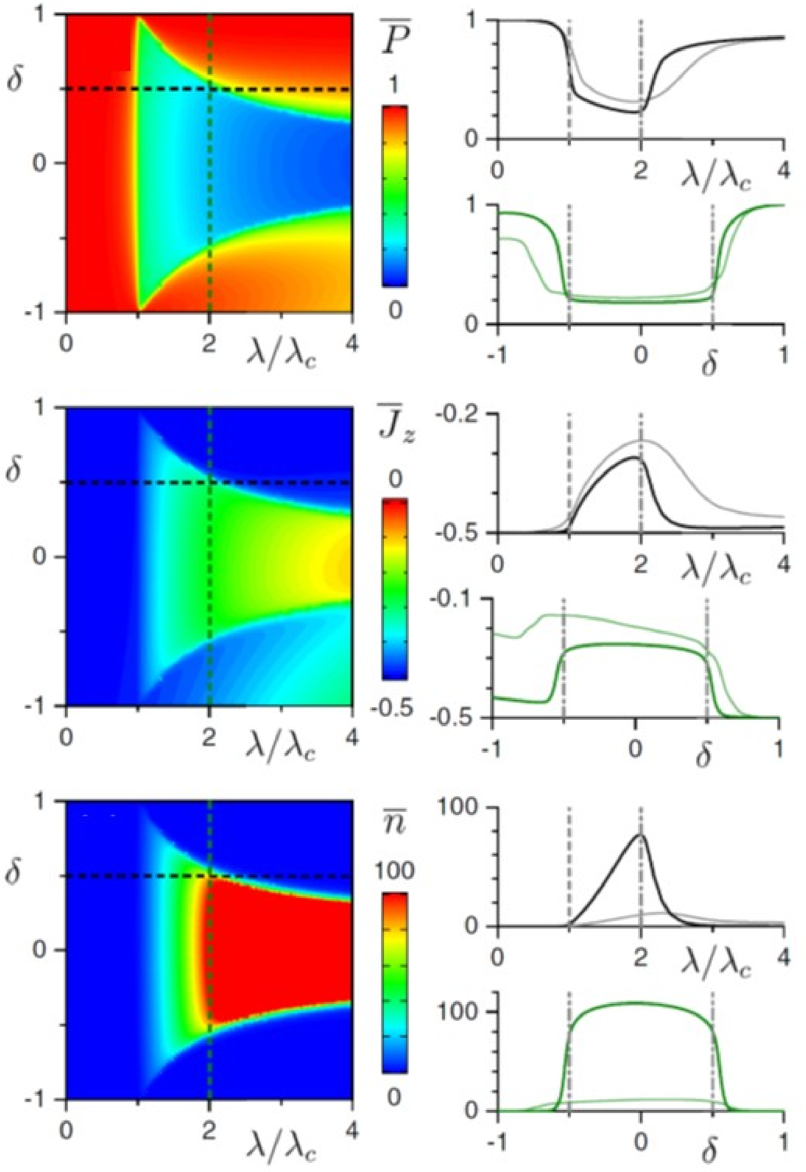
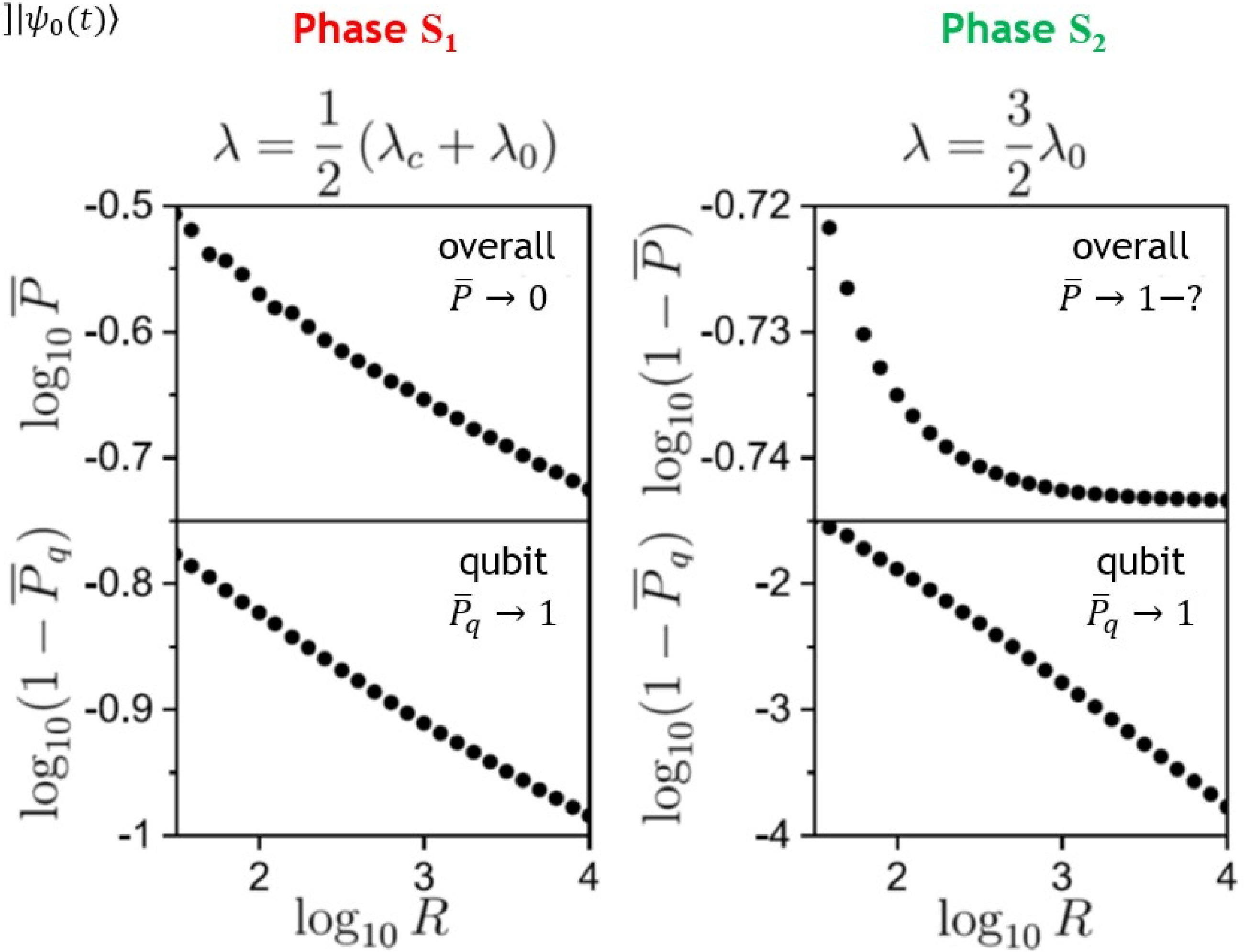


Subsystem survival probabilities

$$P(t) = \langle\psi_0(t)|[|n_* = 0\rangle\langle n_* = 0| \otimes |n = 0\rangle\langle n = 0|]|\psi_0(t)\rangle$$

$$P_q(t) = \langle\psi_0(t)|[n_* = 0\rangle\langle n_* = 0| \otimes \hat{1}]|\psi_0(t)\rangle$$
$$= \frac{1}{2} - J_z(t)$$



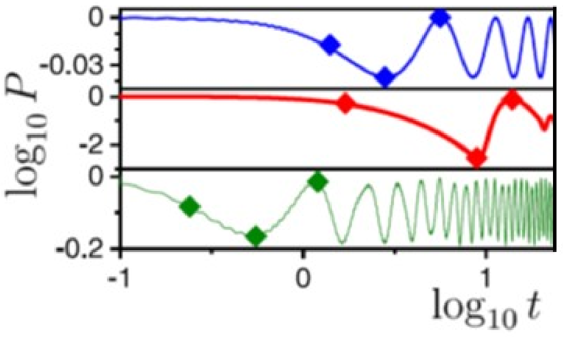
Effective separation of the qubit & field states for $R \to \infty$:
$P(t) = P_q(t)P_b(t)$

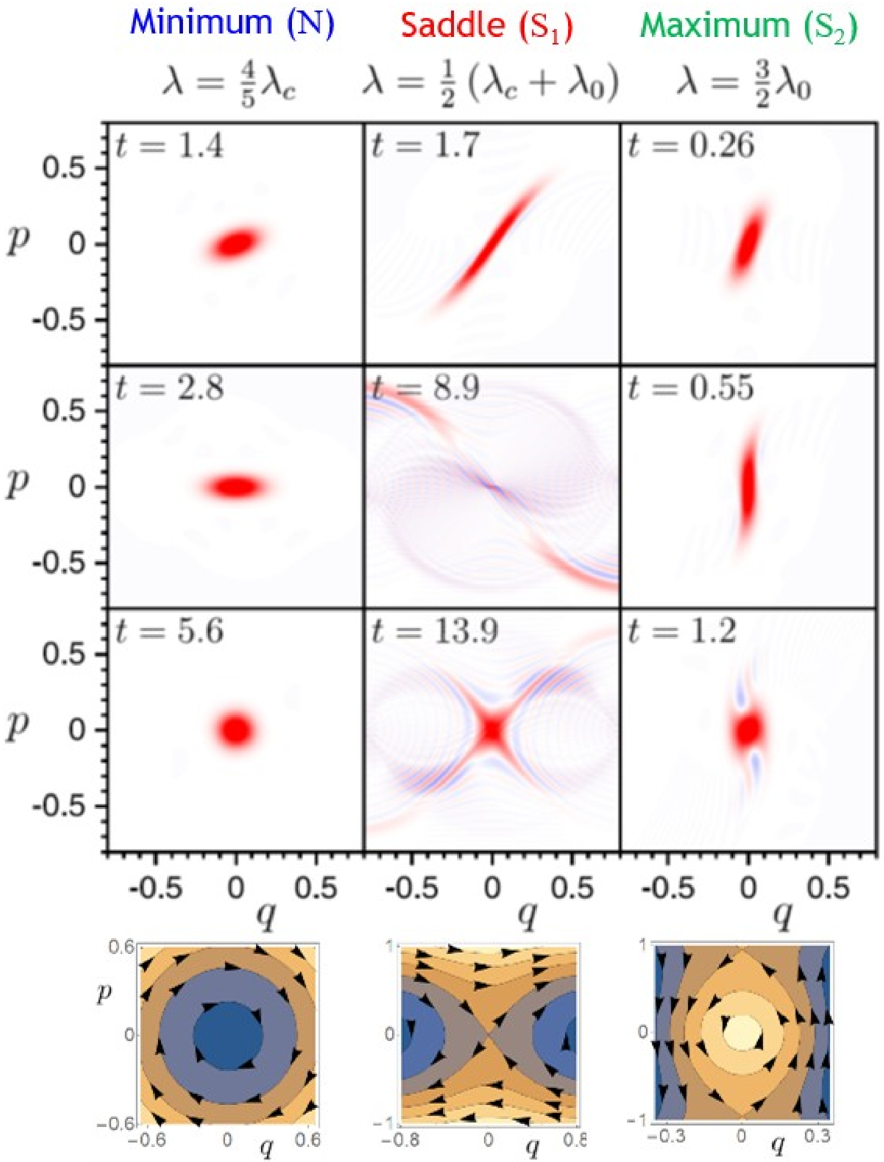
Wigner function

- Phase-space evolution of the **bosonic** degree of freedom

$$W_b(q,p,t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \langle q+q'|\hat{\rho}_b(t)|q-q'\rangle e^{-2ipq'}dq'$$

$$P(t) = P_b(t) = 2\pi \iint_{-\infty}^{\infty} W_b(q,p,t) W_b(q,p,0)\, dq\, dp$$



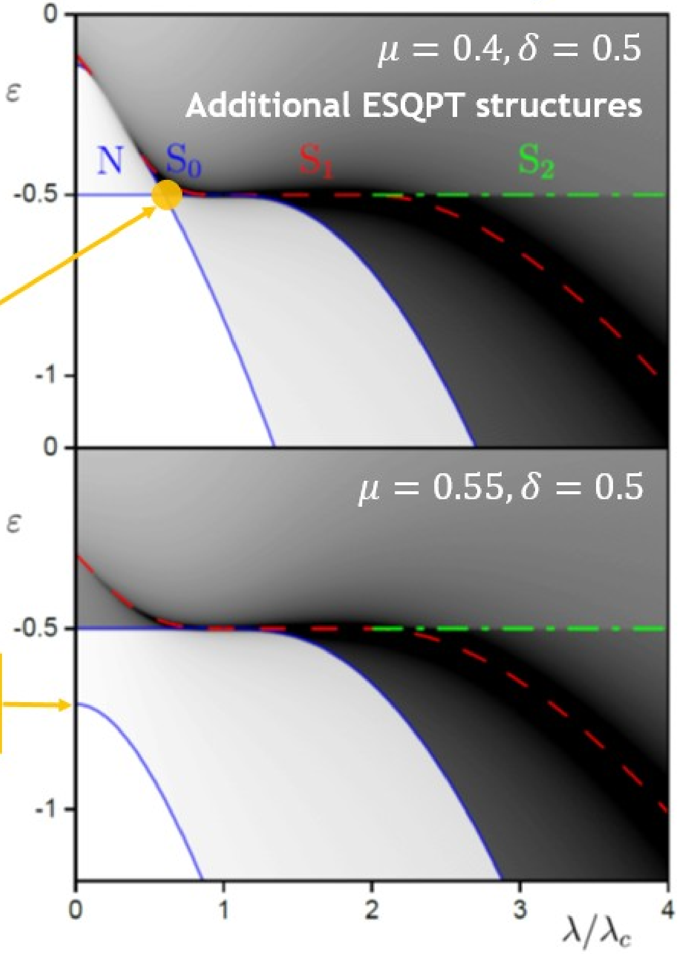


Animations available at

http://pavelstransky.cz/wigner_rabi.php

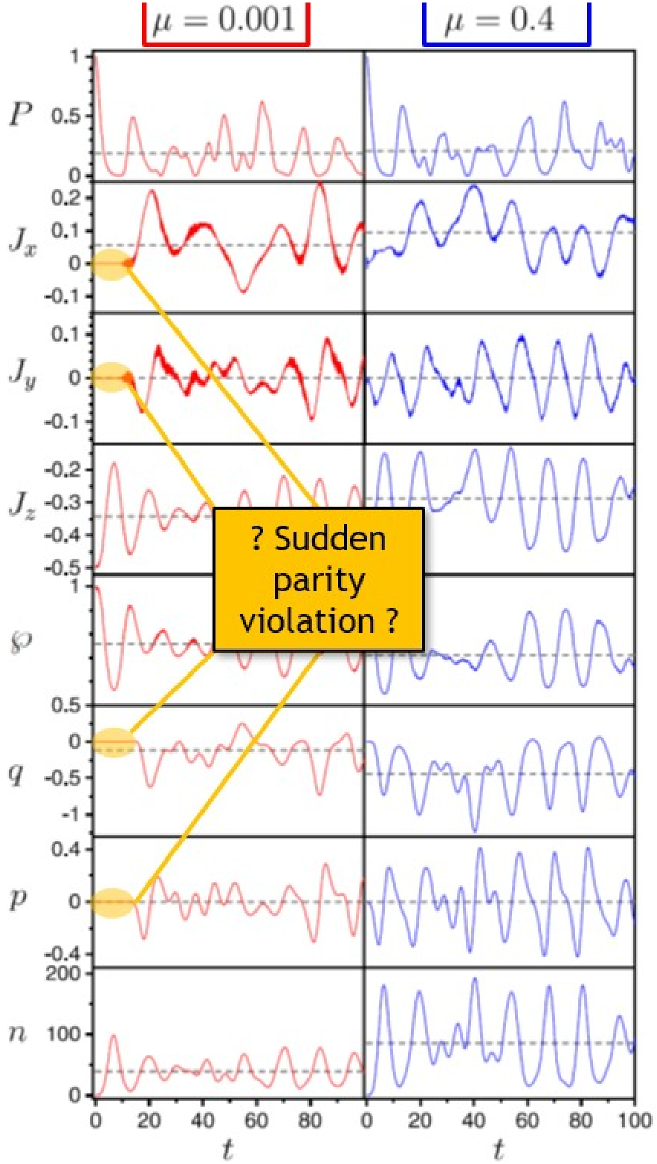
Parity violation $\mu > 0$

Smooth level density

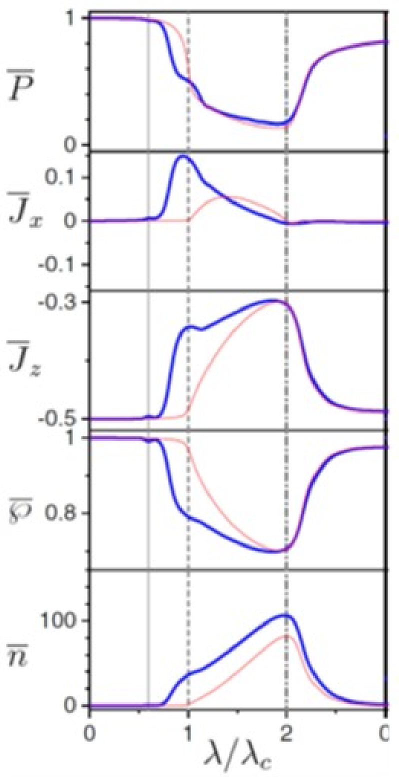


1st order ground-state QPT

NO ground-state QPT



Infinite-time average



$\bar{J}_y = \bar{p} = 0$

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L.F. Santos, M. Távora, F. Pérez-Bernal, Phys. Rev. A 94, 012113 (2016)
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M. Kloc, P. Stránský, P. Cejnar, Phys. Rev. A 98, 013836 (2018)

- … and destabilization (depending od the quench protocol)

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M. Kloc et al., Phys Rev. A 103, 032213 (2021)

- Integrability and solvability of the Rabi model

D. Braak, Phys. Rev. Lett. 107, 100401 (2011)
R. Puebla, *Equilibrium and Nonequilibrium Aspects of Phase Transitions in Quantum Physics* (Springer 2018)

Conclusions

1. The **Rabi model** is a handy tool for analysing various dynamical effects of the qubit-environment coupling
2. Although only a single qubit is present, the system shows various types of **quantum critical effects** (the role of size parameter is played by $R = \omega_0/\omega$)

- The system in a strong coupling regime exhibits an efficient (increasing with R) **spontaneous stabilisation** of a decoupled qubit state, which is connected to the semiclassical stability of the field vacuum.

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