

Cooperative Shielding and Localization

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Collaborations:

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MBL meeting, 24/01/2022, Mexico City, Mexico

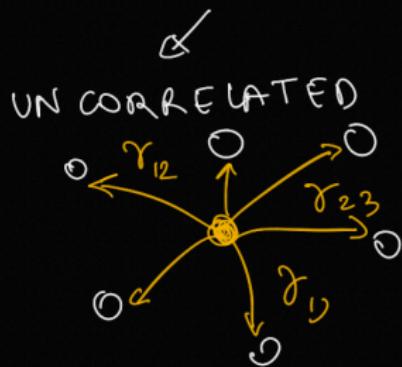
Motivations

- **Anderson localization:** a beacon to understand disordered systems. Short Range hopping, uncorrelated disorder, closed system.
- **Long-range interacting systems** cold atomic clouds, ion traps, light harvesting complexes, etc.. **Cooperative Sheilding and Correlation induced localization.** G. L. Celardo , R. Kaiser, F. Borgonovi PRB 94, 144206 (2016) ;
- **Open systems: Mobility edge in the imaginary axis.** G. L. C., M. Angeli, R. Kaiser, arXiv:1702.04506.
- **Shielding in Many body quantum Systems** L.F. Santos, F.Borgonovi and G.L. Celardo, PRL 116, 250402 (2016)
- **Experiential Verification**, Monroe Group in Maryland. NATURE PHYSICS, VOL 17, JUNe 2021, 742-747

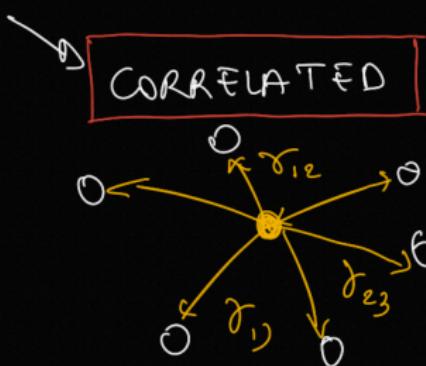
LONG RANGE INTERACTIONS

GENERAL FEATURES OF LONG RANGE INTERACTIONS.

WHICH LONG RANGE?



γ_{ij} ARE RANDOM



$$\gamma_{ij} = \gamma \equiv \text{CONSTANTS}$$

LONG RANGE INTERACTIONS

(i)

NON UNIFORM RESPONSE TO DISORDER

CO-EXISTENCE of EXTENDED and LOCALIZED EIGEN MODES.

(ii)

EXTENDED:

COOPERATIVE ROBUSTNESS

$$W_{cr} \propto N$$

(iii)

LOCALIZATION $\cancel{\Rightarrow}$

BAD TRANSPORT

DET and

DIT
Regimes

CURRENT



LOCALIZED:

HYBRID

$$|\psi(x)|^2$$

$$\text{plateau} \propto \frac{1}{N}$$

exponential
peak
= SR.
HAMILTONIAN

(iv)

COOPERATIVE SHIELDING

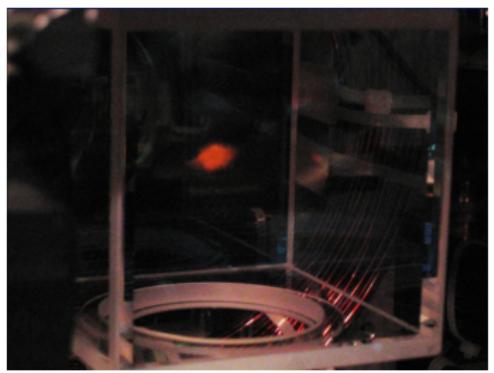
Single and many Body

Experimental Relevance of Correlated long range

Cold Atomic Clouds:

Superradiance, Mobility Edge in
the Imaginary Axis

Robin Kaiser (CNRS, France)



CAVITY PHYSICS

J. Feist and F. J. Garcia-Vidal

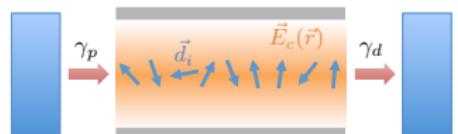
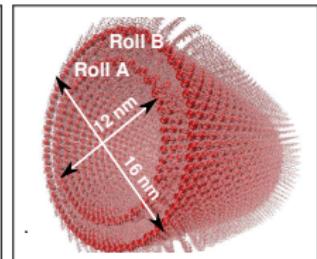
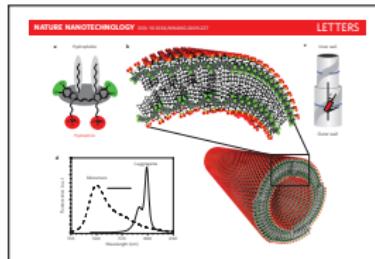
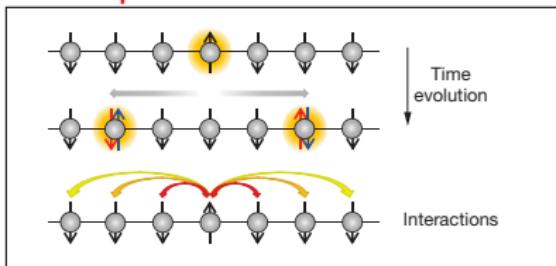


FIG. 1. Sketch of the model system. A 1D chain of (possibly disordered) quantum emitters with dipole moments \vec{d}_i inside a

Biological Systems.



Ion Traps.



Long-Range Interaction Contradictory features of LR

Ion Traps experiment

1d Many Body Hamiltonian:

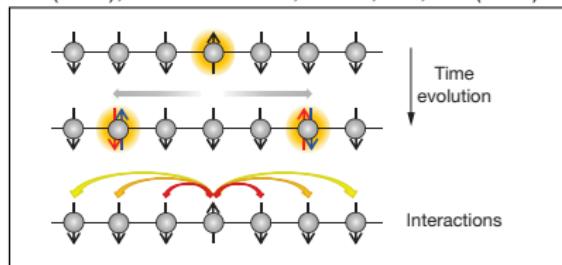
$$H = B \sum_k \sigma_k^z + J \sum_{i < j} \frac{\sigma_i^x \sigma_j^x}{|i - j|^\alpha}$$

with $0 \leq \alpha \leq 3$.

Breaking of Lieb-Robinson bounds in Ion Trap

Richerme et al., Nature Letter 511,

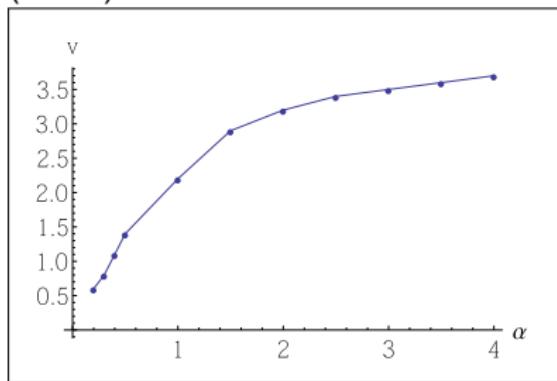
198 (2014); P. Jurcevic et. al., Nature, 511, 202 (2014).



Theoretical work:

Suppression of the velocity of spreading with the increase of the interaction range α .

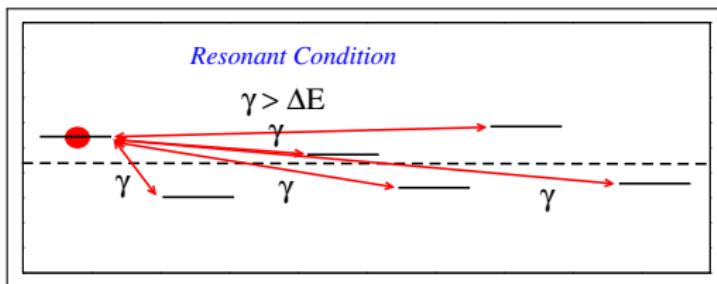
M. Kastner, New J. Phys. 17, 063021 (2015)



Cooperative Shielding can help to explain such contradictory features

Localization and long range.

- Levitov, PRL **64**, 547 1990:
"IT IS KNOWN THAT IN SYSTEMS WITH DIMENSION d WITH $r^{-\alpha}$ INTERACTION, LOCALIZATION CAN EXIST ONLY IF $\alpha > d$. FOR $\alpha \leq d$ A DIVERGING NUMBER OF RESONANCES DESTROYS LOCALIZED STATES".
- ANDERSON (1958): More distant sites are not important because the probability of finding one with the right energy increases much more slowly with distance than the interaction decreases



Number of Resonances:

$$N_{res} = \frac{V_k}{W} N_k \propto R^{d-\alpha} \rightarrow \infty \text{ for } \alpha < d$$

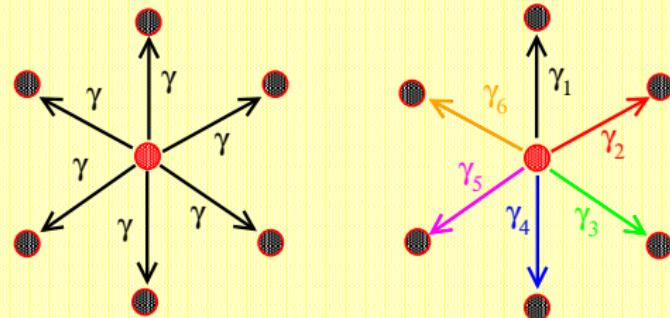
RANDOM VS NON RANDOM INTERACTIONS

- **Absence of Localization of Vibrational Modes Due to Dipole-Dipole Interaction**, L. S. Levitov, Europhys. Lett. 9, 83 (1989); Phys. Rev. Lett. 64, 547 (1990);
- **Anderson transitions**, F. Evers and A. D. Mirlin, Rev. Mod. Phys. 80, 1355 (2008).
- **Transition from localized to extended eigenstates in the ensemble of power-law random banded matrices**, A. D. Mirlin, Yan V. Fyodorov, F.-M. Dittes, J. Q., and T. H. Seligman Phys. Rev. E 54, 3221 (1996).
- Kastner, New J. Phys. 17 063021 (2015), PRX 3, 031015 (2013). Suppression of information spreading in long range systems (Lieb-Robinson Bounds).
- **Anderson localization on a simplex**, A Ossipov, Journal of Physics A: Mathematical and Theoretical, Volume 46, (2013)
$$H = \sum E_i^0 |i\rangle\langle i| - \gamma \sum |i\rangle\langle i|$$
PR and all its moments independent of N .

How do we explain such contradiction?

Correlation Induced Localization

CORRELATED vs UNCORRELATED



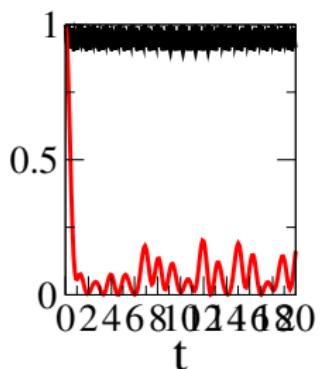
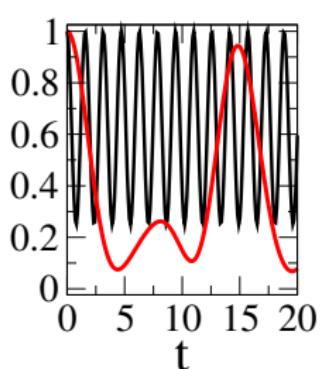
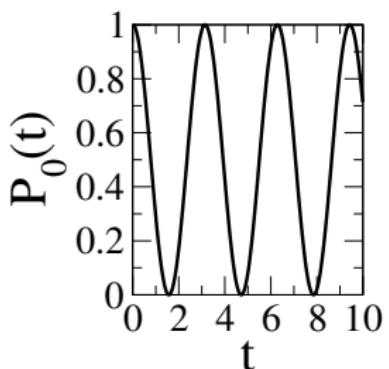
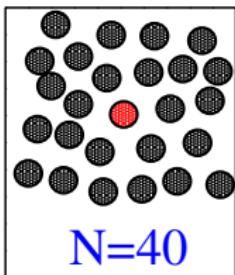
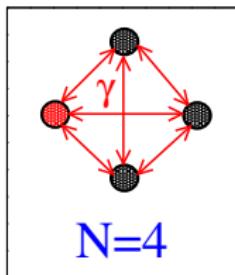
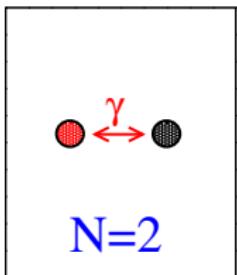
- X. Deng, V.E. Kravtsov, G.V. Shlyapnikov, and L. Santos Phys. Rev. Lett. 120, 110602 (2018).
- Rahul M. Nandkishore and S.L. Sondhi Phys. Rev. X 7, 041021 (2017).
- J. T. Cantin, T. Xu, and R. V. Krems Phys. Rev. B 98, 014204 (2018).
- P. A. Nosov, I. M. Khaymovich, and V. E. Kravtsov Phys. Rev. B 99, 104203 (2019)
- A. Lerose, B. Zunkovic, A. Silva and A. Gambassi, Phys. Rev. B 99, 121112(R) (2019)
- F. Liu, R. Lundgren, P. Titum, G. Pagano, J. Zhang, C. Monroe, and A. V. Gorshkov, Phys. Rev. Lett. 122, 150601 (2019).

Suppression of Long Range for non-random case

All to All Coupling, no Disorder, Correlated vs Uncorrelated

$$H = -\gamma \sum |i\rangle \langle j|$$

$$H = -\sum \gamma_{i,j} |i\rangle \langle j|$$



The Shielding effect

- Let us consider a system:

$$H = H_0 + V, \quad \text{with} \quad [H_0, V] = 0$$

with V highly degenerate $V|v_k\rangle = v|v_k\rangle$

- $|\psi_0\rangle = \sum_{k=1}^g c_k |v_k\rangle$
- V contributes only with global phase

$$|\psi(t)\rangle = e^{iHt} |\psi_0\rangle = e^{ivt} e^{iH_0 t} |\psi_0\rangle$$

We have shielding from V !!.

H_0 : emerging Hamiltonian.

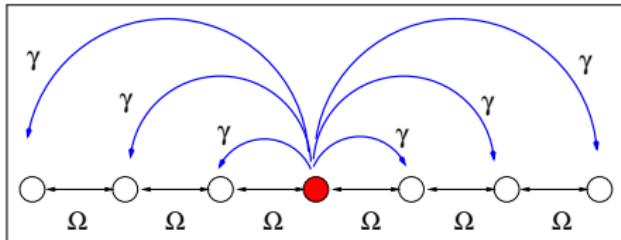
- What if $[H_0, V] \neq 0$?
- What if spectrum of V is not degenerate? What is the connection with long range? Is this a cooperative effects? What is the emergent Hamiltonian?

Cooperative Shielding. Single excitation transport.

- 1d Anderson model with long range hopping:

$$H = D + H_{\text{NN}} + V_{\text{LR}} = \sum_i \epsilon_i^0 |i\rangle\langle i| - \Omega \sum_{\langle i,j \rangle} (|j\rangle\langle i| + |i\rangle\langle j|) - \gamma \sum_{i \neq j} \frac{|i\rangle\langle j|}{r_{i,j}^\alpha}$$

- ϵ_j^0 : are random energies $[-W/2, +W/2]$; $r_{i,j} = |i - j|$; long range for $\alpha < 1$. $\alpha = 0$: all to all.
- $\Omega > 0, \gamma > 0$: the tunnelling transition amplitude.

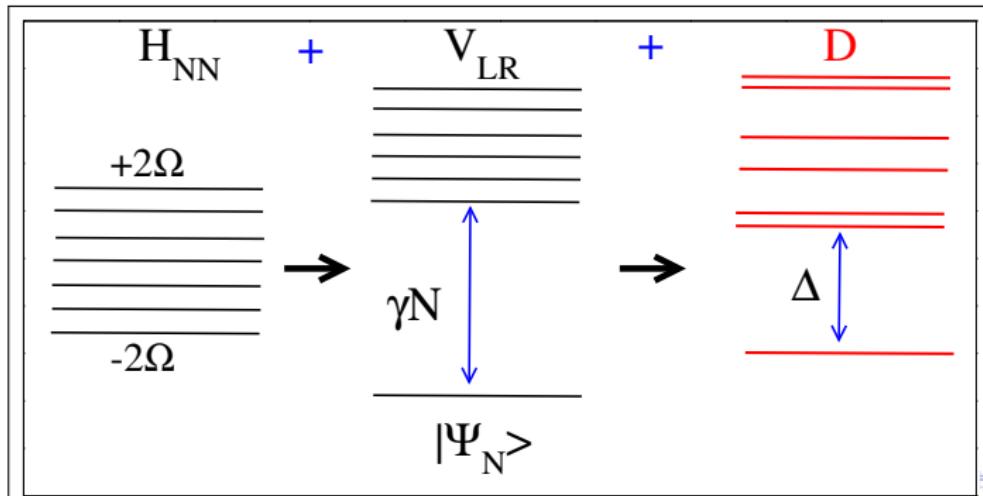


G.L.C., R. Kaiser, and F. Borgonovi, PRB **94**, 144206 (2016).

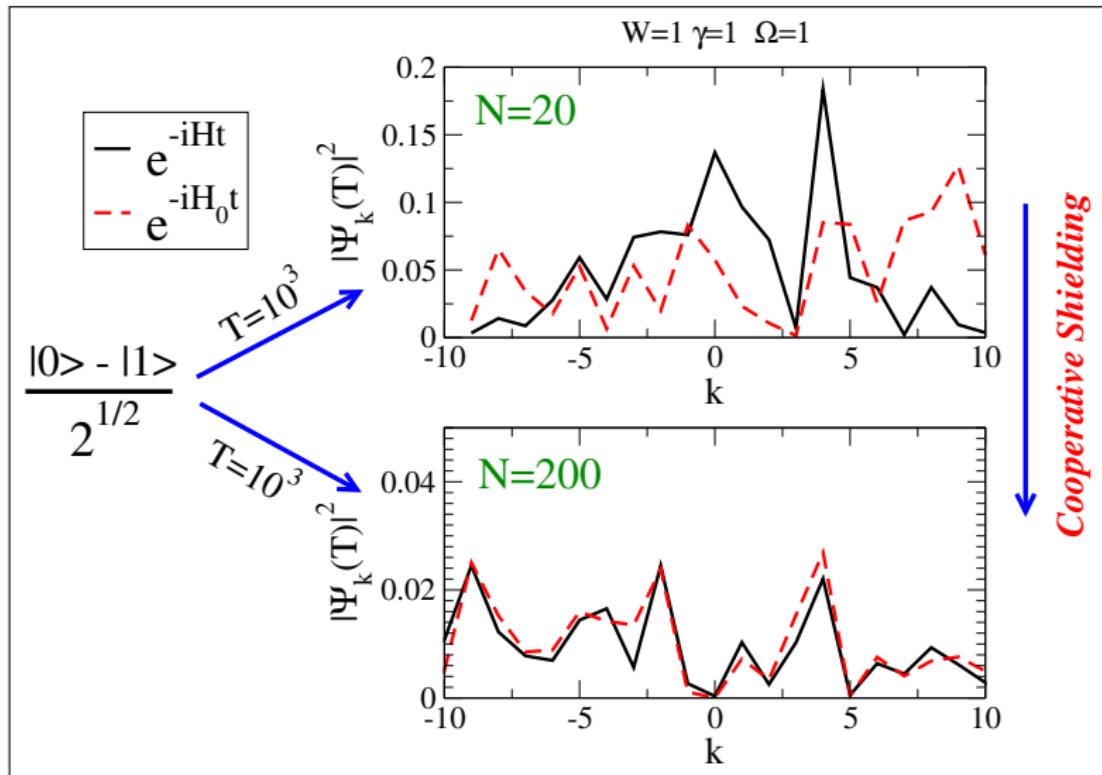
Spectrum and Energy Gap: Does shielding survive disorder?

$$H = H_{NN} + V_{LR} + D$$

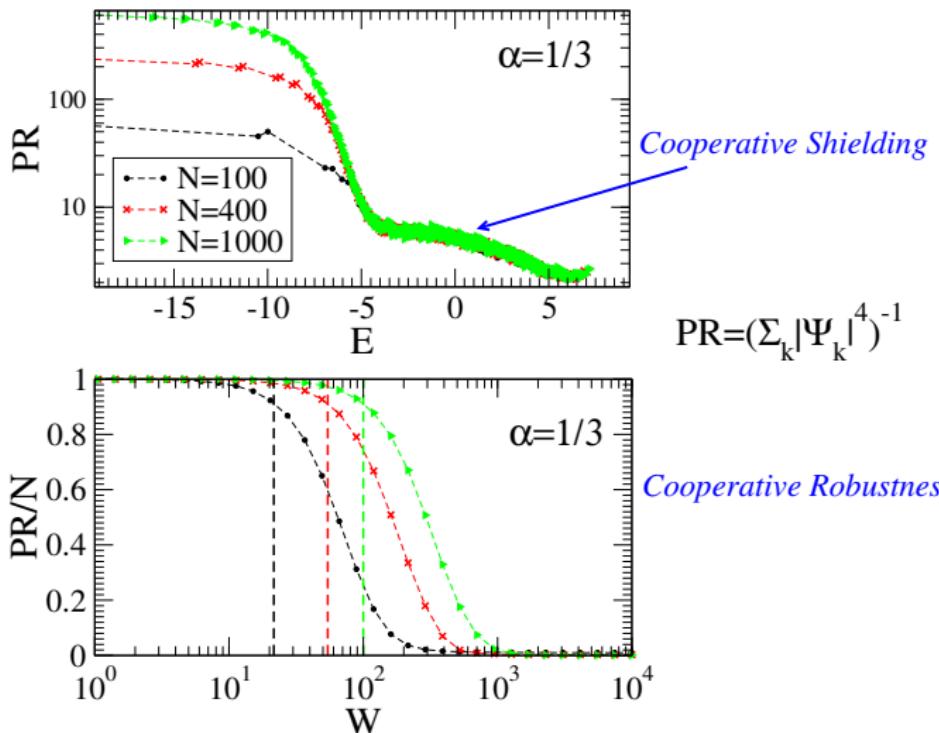
$$H = -\Omega \sum_i (|i\rangle\langle i+1| + h.c.) - \gamma \sum_{i \neq j} |i\rangle\langle j| + \sum_i \epsilon_i^0 |i\rangle\langle i|$$



Cooperative Shielding



Generalization to $\alpha > 0$: Shielding and Localization



$$PR = (\sum_k |\Psi_k|^4)^{-1}$$

Cooperative Robustness

Cooperative Shielding in many-body.

Experimentally accessible 1d spin 1/2 Hamiltonian:

$$H = H_0 + V, \quad (1)$$

$$H_0 = B \sum_{n=1}^L \sigma_n^z$$

$$V = \sum_{n < m} \frac{J}{|n - m|^\alpha} \sigma_n^x \sigma_m^x.$$

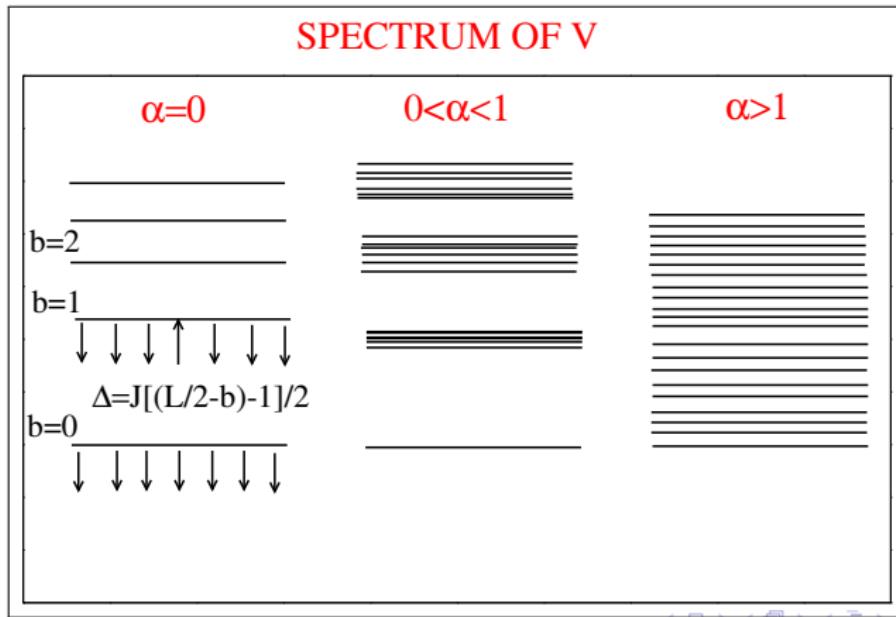
- $\alpha < 1$: long range. $\alpha > 1$: short range.

Spectrum of V

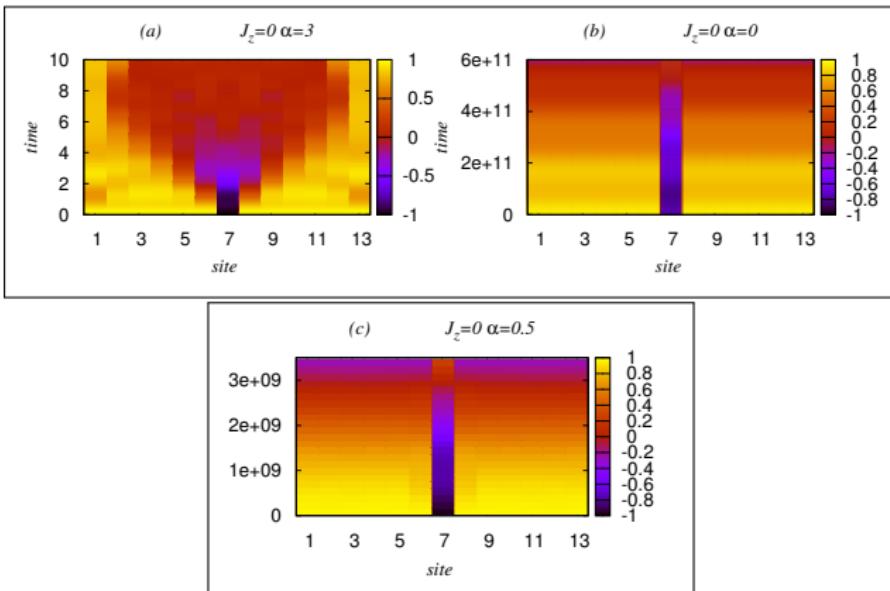
The case $\alpha = 0$:

$$V = J \sum_{n \leq m} \sigma_n^x \sigma_m^x = \frac{JM_x^2}{2} - \frac{JL}{2} \quad \text{where} \quad M_x = \sum_n \sigma_n^x$$

$$V_b = J(L/2 - b)^2/2 - JL/2, \quad \text{where} \quad b = 0, 1, \dots, L/2$$



Light-cones



Initial State:

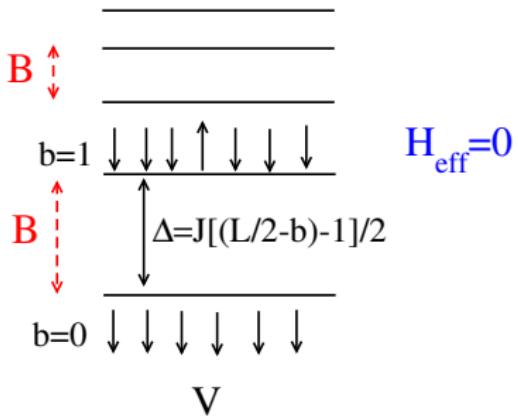
$$|\psi_0\rangle = |\uparrow, \uparrow, \dots, \downarrow, \dots, \uparrow, \uparrow\rangle_x$$

- a) $B = 0.5, \alpha = 3$ light-cone;
- b) $B = 0.5, \alpha = 0$ localization without disorder;
- c) $B = 0.5, \alpha = 0.5$

Invariant Subspaces

$$H = H_{\text{ext Field}} + V$$

External Field: $\sigma_z = \sigma_x^+ + \sigma_x^-$



$P_{\text{leak}} \propto (W/J)^2/L$ for random field and no NN interaction

Experimental Verification

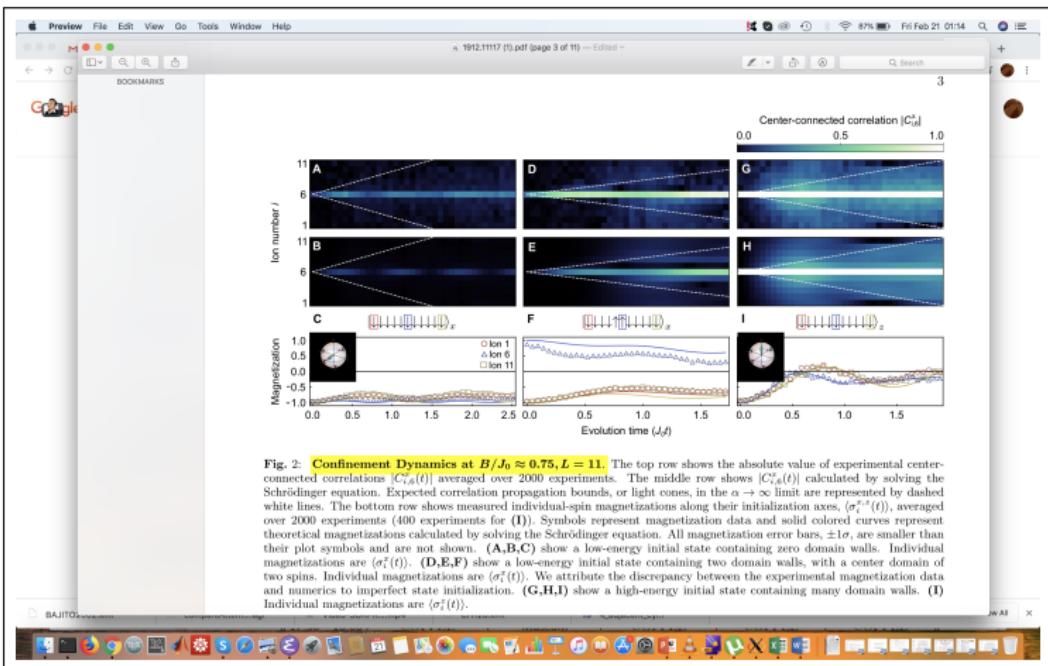


Fig. 2: Confinement Dynamics at $B/J_0 \approx 0.75, L = 11$. The top row shows the absolute value of experimental center-connected correlations $|C_{i,j}(t)|$ averaged over 2000 experiments. The middle row shows $|C_{i,j}^*(t)|$ calculated by solving the Schrödinger equation. Expected correlation propagation bounds, or light cones, in the $\alpha \rightarrow \infty$ limit are represented by dashed white lines. The bottom row shows measured individual-spin magnetizations along their initialization axes, $\langle \sigma_i^{x,y}(t) \rangle$, averaged over 2000 experiments (400 experiments for (I)). Symbols represent magnetization data and solid colored curves represent theoretical magnetizations calculated by solving the Schrödinger equation. All magnetization error bars, $\pm 1\sigma$, are smaller than their plot symbols and are not shown. (A,B,C) show a low-energy initial state containing zero domain walls. Individual magnetizations are $\langle \sigma_i^z(t) \rangle$. (D,E,F) show a low-energy initial state containing two domain walls, with a center domain of two spins. Individual magnetizations are $\langle \sigma_i^z(t) \rangle$. We attribute the discrepancy between the experimental magnetization data and numerics to imperfect state initialization. (G,H,I) show a high-energy initial state containing many domain walls. (I) Individual magnetizations are $\langle \sigma_i^z(t) \rangle$.

**Observation of Domain Wall Confinement and Dynamics in a Quantum Simulator, Monroe Group, Maryland, USA.
CONNECTION WITH QUARK CONFINEMENT.**

Cooperative Shielding in many-body.

Experimentally accessible spin 1/2 Hamiltonian:

$$H = H_0 + V, \quad (2)$$

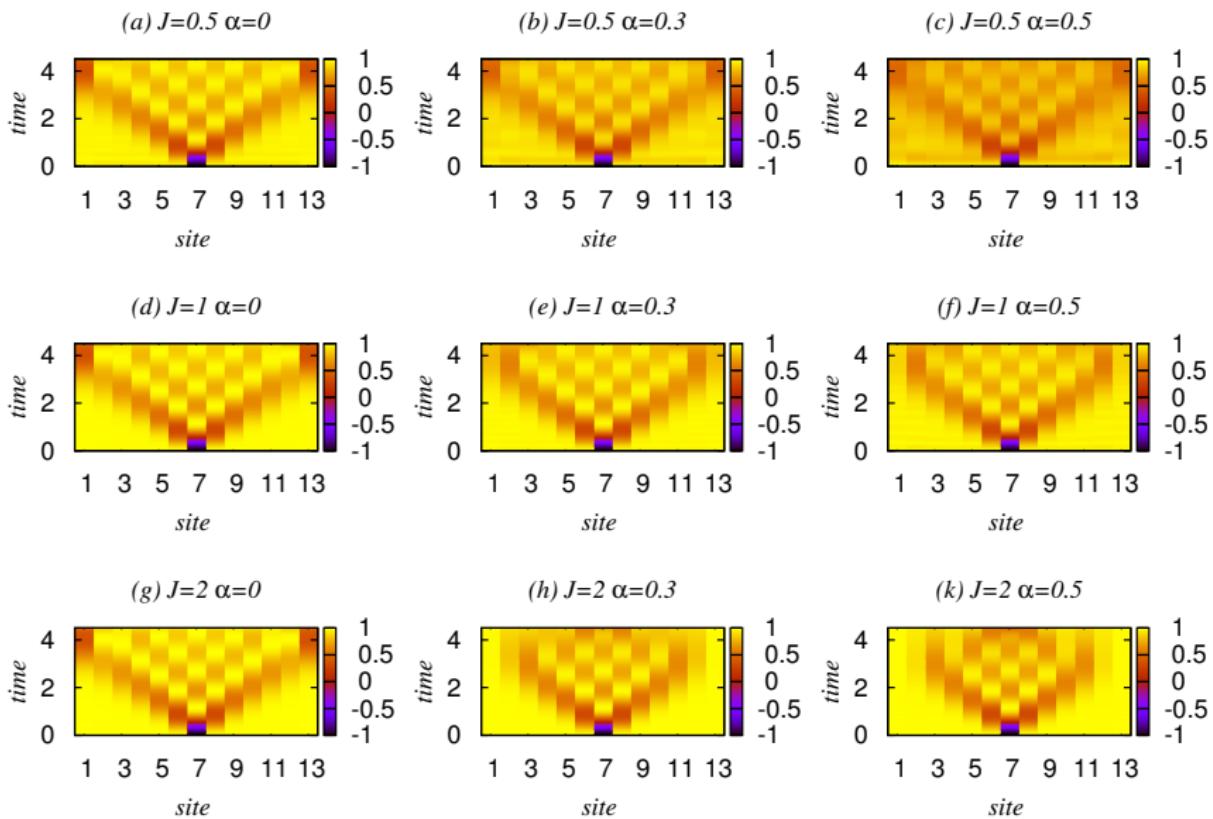
$$H_0 = \sum_{n=1}^{L-1} J_z \sigma_n^z \sigma_{n+1}^z,$$

$$V = \sum_{n < m} \frac{J}{|n - m|^\alpha} \sigma_n^x \sigma_m^x.$$

- $\alpha < 1$: long range. $\alpha > 1$: short range.

NN+ LONG RANGE

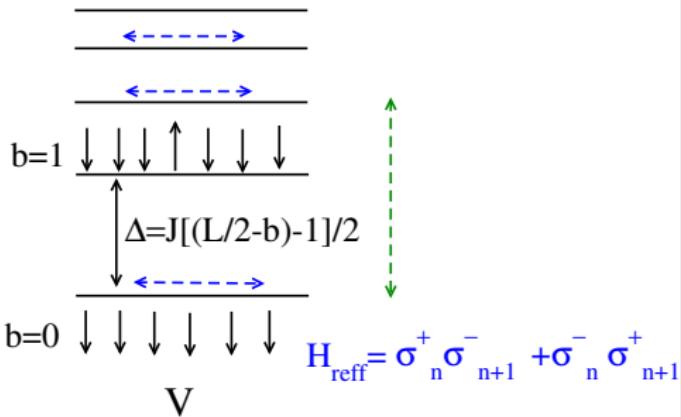
Shielding



Invariant Subspaces II

$$H = H_{NN} + V$$

$$\text{NN: } \sigma_n^z \sigma_{n+1}^z = \sigma_n^+ \sigma_{n+1}^- + \sigma_n^- \sigma_{n+1}^+ + \sigma_n^+ \sigma_{n+1}^+ + \sigma_n^- \sigma_{n+1}^-$$



$$H_{\text{eff}} = \sigma_n^+ \sigma_{n+1}^- + \sigma_n^- \sigma_{n+1}^+$$

$P_{\text{leak}} \propto (J_z/J)^2/L$ for NN interaction only.

(Cooperative) Zeno Dynamics

- **QZE:** Observation freeze dynamics in invariant subspaces.

$$H = H_0 + KH_{\text{meas}}$$

As K increases, eigensubspace of H_{meas} becomes invariant.

- **Zeno Hamiltonian:** in our case:
 $H = H_0 + V_{LR}$, $V_{LR} \leftarrow H_{\text{meas}}$.

$$H_Z = \sum_b [P_b H_0 P_b + V_b P_b] =$$

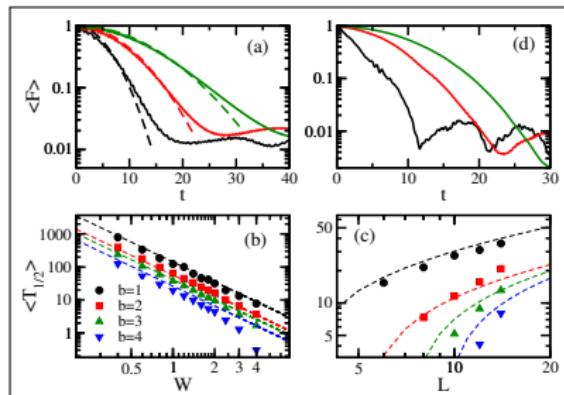
$$= \text{diag}(H_0) + \sum_b V_b P_p$$

where P_b are the projectors on the eigensubspace of V corresponding to the eigenvalues V_b .

For $\alpha = 0$ $H_{\text{eff}} = H_Z$!

Zeno Fidelity:

$$F(t) = |\langle \Psi(0) | e^{iH_Z t} e^{-iHt} | \Psi(0) \rangle|^2$$



Fidelity decay slows down with
N!

Classical vs Quantum Shielding

Questions:

- Is it a classical or quantum effect?
- Is the energy gap essential?
- What if we rescale the long range term ($J/N^{1-\alpha}$)?
- Classical case...continuum spin of modulus one.

The classical model:

$$H = \sum_{j=1}^N h_j S_j^z + J_z \sum_{j=1}^{N-1} S_j^z S_{j+1}^z + \frac{J}{2N^{1-\alpha}} \sum_{j,m \neq j} \frac{S_j^x S_m^x}{|\mathbf{r}_j - \mathbf{r}_m|^\alpha},$$

Conclusions and Perspectives

1. Cooperative Shielding and Correlation Induced localization
2. Mobility Edge in the Imaginary axis.
3. Cooperative shielding in Many Body Systems.

Cooperative Shielding: A Guiding principle in closed and open quantum systems with long range interactions