Cooperative Shielding and Localization

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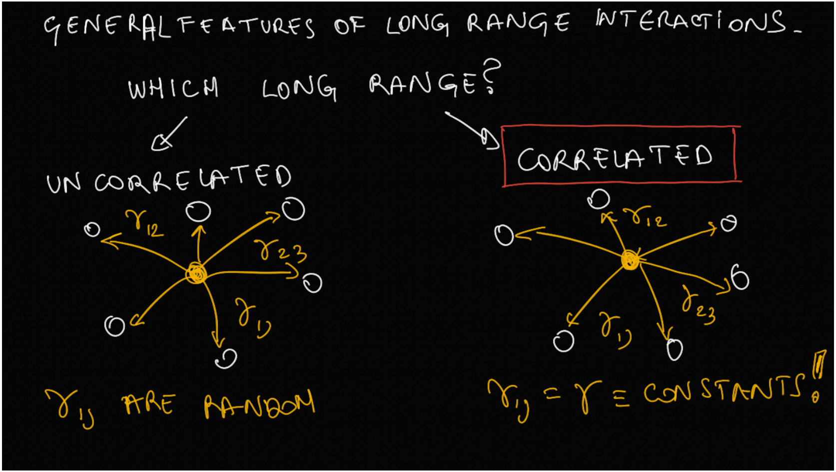
MBL meeting, 24/01/2022, Mexico City, Mexico

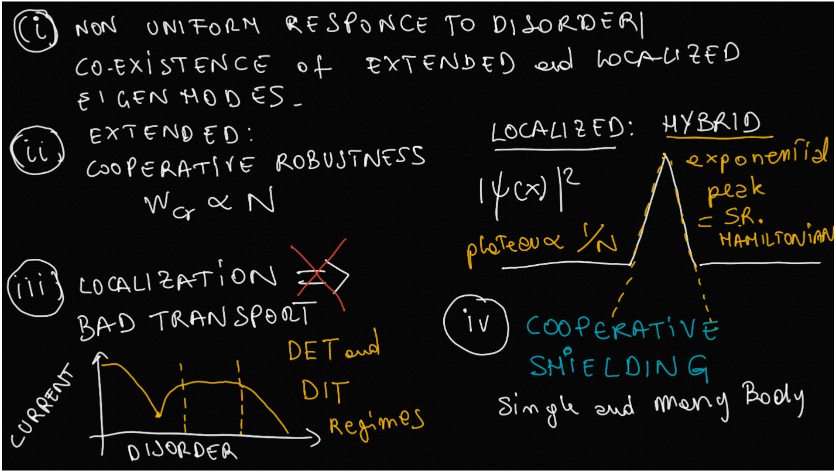
Motivations

- Anderson localization: a beacon to understand disordered systems. Short Range hopping, uncorrelated disorder, closed system.
- Long-range interacting systems cold atomic clouds, ion traps, light harvesting complexes, etc.. **Cooperive Sheilding and Correlation induced localization**. G. L. Celardo , R. Kaiser, F. Borgonovi PRB 94, 144206 (2016) ;
- Open systems: **Mobility edge in the imaginary axis**. G. L. C., M. Angeli, R. Kaiser, arXiv:1702.04506.
- Shielding in Many body quantum Systems L.F. Santos, F.Borgonovi and G.L. Celardo, PRL 116, 250402 (2016)
- Experiental Verification, Monroe Group in Maryland. NATURE PHYSICS, VOL 17, JUNe 2021, 742-747

LONG RANGE INTERACTIONS



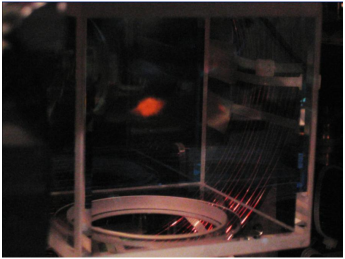
LONG RANGE INTERACTIONS



Experimental Relevance of Correlated long range

Cold Atomic Clouds: Superradiance, Mobility Edge in the Imaginary Axis

Robin Kaiser (CNRS, France)



CAVITY PHYSICS

J. Feist and F. J. Garcia-Vidal

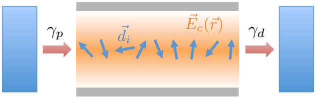
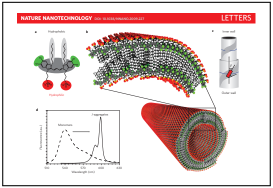
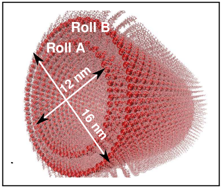


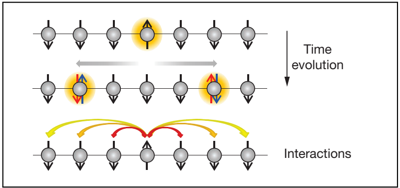
FIG. 1. Sketch of the model system. A 1D chain of (possibly disordered) quantum emitters with dipole moments $\vec{d}_i$ inside a

Biological Systems.





Ion Traps.



Ion Traps experiment

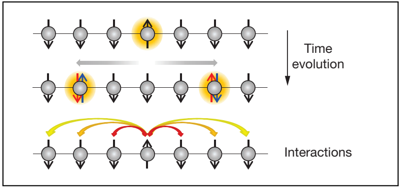
1d Many Body Hamiltonian:

$$H = B\sum_k \sigma_k^z + J\sum_{i<j} \frac{\sigma_i^x \sigma_j^x}{|i-j|^\alpha}$$

with $0 \leq \alpha \leq 3$.

Breaking of Lieb-Robinson bounds in Ion Trap

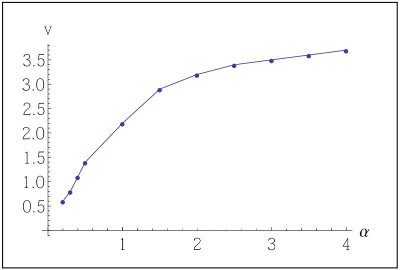
Richerme et al., Nature Letter **511**, 198 (2014); P. Jurcevic et. al., Nature, **511**, 202 (2014).



Theoretical work:

Suppression of the velocity of spreading with the increase of the interaction range α.

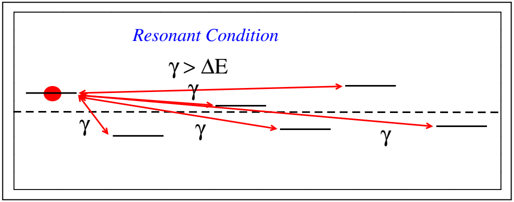
M. Kastner, New J. Phys. **17**, 063021 (2015)



Cooperative Shielding can help to explain such contradictory features

Localization and long range.

- Levitov, PRL **64**, 547 1990: "IT IS KNOWN THAT IN SYSTEMS WITH DIMENSION d WITH $r^{-\alpha}$ INTERACTION, LOCALIZATION CAN EXIST ONLY IF $\alpha > d$. FOR $\alpha \leq d$ A DIVERGING NUMBER OF RESONANCES DESTROYS LOCALIZED STATES".
- ANDERSON (1958): More distant sites are not important because the probability of finding one with the right energy increases much more slowly with distance than the interaction decreases



Number of Resonances:

$$N_{res} = \frac{V_k}{W} N_k \propto R^{d-\alpha} \to \infty \;\; \text{for} \;\; \alpha < d$$

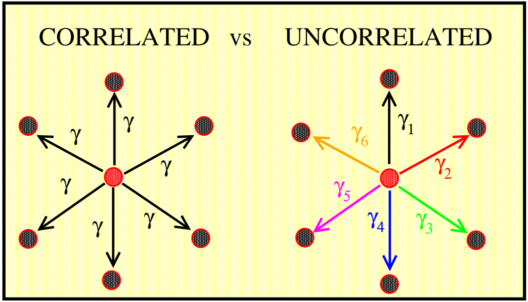
RANDOM VS NON RANDOM INTERACTIONS

- **Absence of Localization of Vibrational Modes Due to Dipole-Dipole Interaction**, L. S. Levitov, Europhys. Lett. 9, 83 (1989); Phys. Rev. Lett. 64, 547 (1990);
- **Anderson transitions**, F. Evers and A. D. Mirlin, Rev. Mod. Phys. 80, 1355 (2008).
- **Transition from localized to extended eigenstates in the ensemble of power-law random banded matrices**, A. D. Mirlin, Yan V. Fyodorov, F.-M. Dittes, J. Q., and T. H. Seligman Phys. Rev. E 54, 3221 (1996).
- Kastner, New J. Phys. 17 063021 (2015), PRX 3, 031015 (2013). Suppression of information spreading in long range systems (Lieb-Robinson Bounds).
- **Anderson localization on a simplex**, A Ossipov, Journal of Physics A: Mathematical and Theoretical, Volume 46, (2013) $H = \sum E_i^0 |i\rangle\langle i| - \gamma \sum |i\rangle\langle i|$

PR and all its moments independent of N.

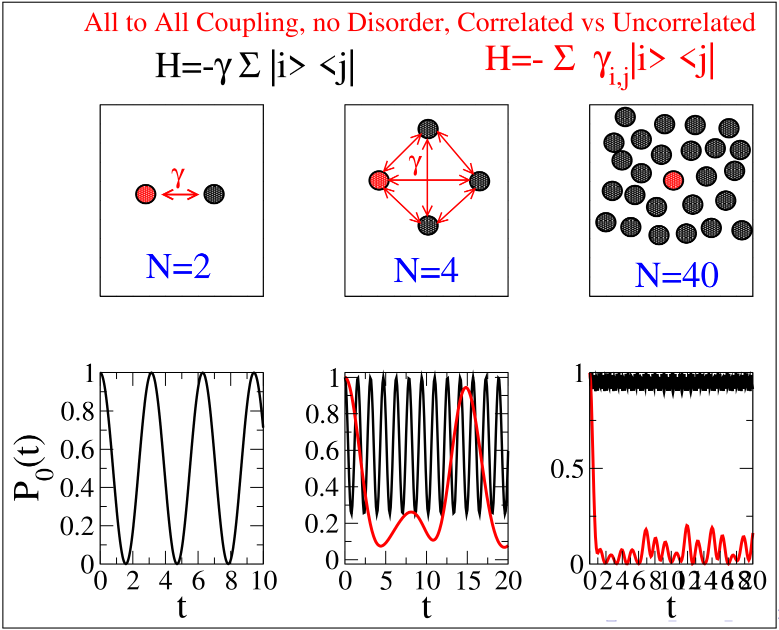
How do we explain such contradiction?

Correlation Induced Localization



- X. Deng, V.E. Kravtsov, G.V. Shlyapnikov, and L. Santos Phys.Rev. Lett. 120, 110602 (2018).
- Rahul M. Nandkishore and S.L. Sondhi Phys. Rev. X 7, 041021 (2017).
- J. T. Cantin, T. Xu, and R. V. Krems Phys. Rev. B 98, 014204 (2018).
-P. A. Nosov, I. M. Khaymovich, and V. E. Kravtsov Phys. Rev. B 99, 104203 (2019)
-A. Lerose, B. Zunkovic, A. Silva and A. Gambassi, Phys. Rev. B 99, 121112(R) (2019)
-F. Liu, R. Lundgren, P. Titum, G. Pagano, J. Zhang, C. Monroe, and A. V. Gorshkov, Phys. Rev. Lett. 122, 150601 (2019).

Suppression of Long Range for non-random case



The Shielding effect

- Let us consider a system:

$$H = H_0 + V, \qquad \text{with} \qquad [H_0, V] = 0$$

with V highly degenerate $V|v_k\rangle = v|v_k\rangle$

- $|\psi_0\rangle = \sum_{k=1}^{g} c_k|v_k\rangle$
- V contributes only with global phase

$$|\psi(t)\rangle = e^{iHt}|\psi_0\rangle = e^{ivt}e^{iH_0t}|\psi_0\rangle$$

We have shielding from V!!. H_0: emerging Hamiltonian.

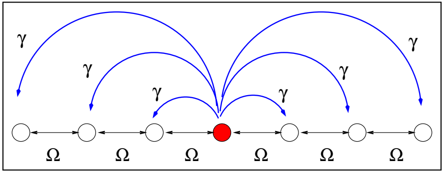
- What if $[H_0, V] \neq 0$?
- What if spectrum of V is not degenerate? What is the connection with long range? Is this a cooperative effects? What is the emergent Hamiltonian?

Cooperative Shielding. Single excitation transport.

- 1d Anderson model with long range hopping:

$$H = D + H_{\rm NN} + V_{\rm LR} = \sum_i \epsilon_i^0 |i\rangle\langle i| - \Omega \sum_{\langle i,j\rangle} (|j\rangle\langle i| + |i\rangle\langle j|) - \gamma \sum_{i\neq j} \frac{|i\rangle\langle j|}{r_{\rm i,j}^{\alpha}}$$

- ϵ_j^0: are random energies $[-W/2, +W/2]$; $r_{i,j} = |i-j|$; long range for $\alpha < 1$. $\alpha = 0$: all to all.
- $\Omega > 0, \gamma > 0$: the tunnelling transition amplitude.

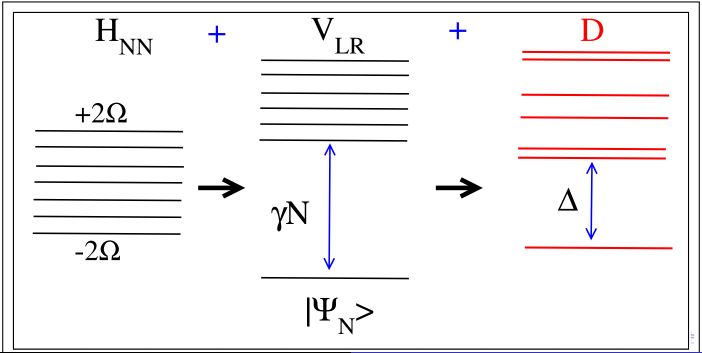


G.L.C., R. Kaiser, and F. Borgonovi, PRB **94**, 144206 (2016).

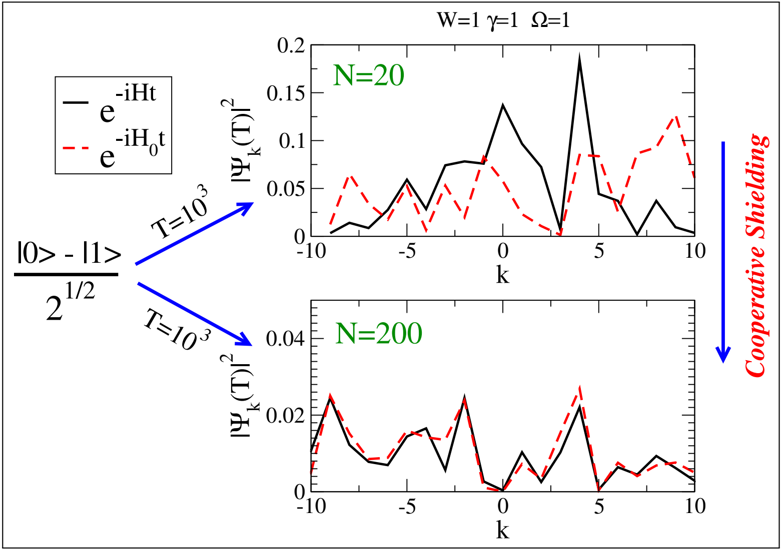
Spectrum and Energy Gap: Does shielding survive disorder?

$$H = H_{NN} + V_{LR} + D$$

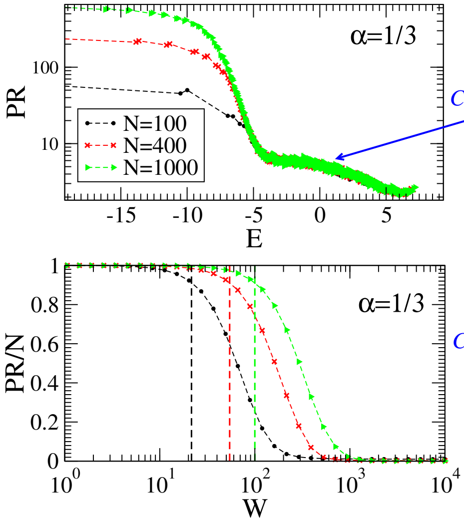
$$H = -\Omega \sum_{i} (|i\rangle\langle i+1| + h.c.) - \gamma \sum_{i \neq j} |i\rangle\langle j| + \sum_{i} \epsilon_i^0 |i\rangle\langle i|$$



Cooperative Shielding



Generalization to $\alpha > 0$: Shielding and Localization



Cooperative Shielding

$PR = (\Sigma_k |\Psi_k|^4)^{-1}$

Cooperative Robustness

Cooperative Shielding in many-body.

Experimentally accessible 1d spin $1/2$ Hamiltonian:

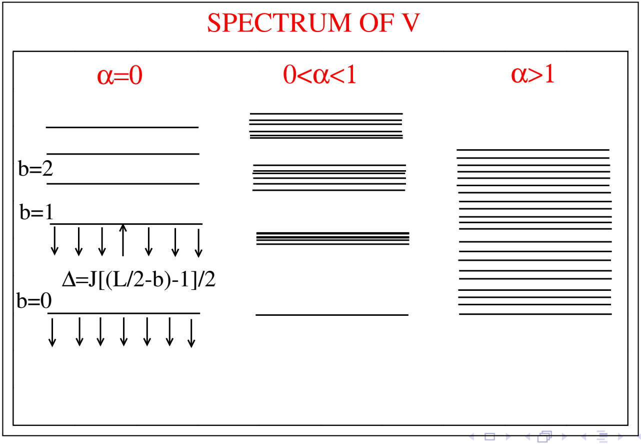
$$
\begin{aligned}
H &= H_0 + V, \\
H_0 &= B \sum_{n=1}^{L} \sigma_n^z \\
V &= \sum_{n<m} \frac{J}{|n-m|^\alpha} \sigma_n^x \sigma_m^x .
\end{aligned}
\tag{1}
$$

- $\alpha < 1$: long range. $\alpha > 1$: short range.

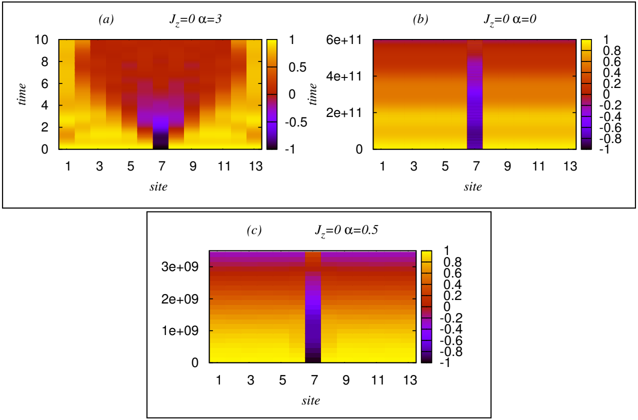
The case $\alpha = 0$:

$$V = J\sum_{n<m}\sigma_n^x\sigma_m^x = \frac{JM_x^2}{2} - \frac{JL}{2} \quad \text{where} \quad M_x = \sum_n \sigma_n^x$$

$$V_b = J(L/2-b)^2/2 - JL/2, \quad \text{where} \quad b = 0, 1, \ldots L/2$$



Light-cones

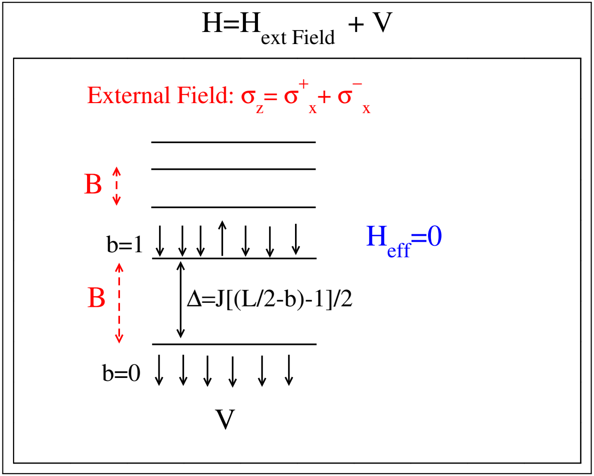


Initial State:

$$|\psi_0\rangle = |\uparrow,\uparrow,..,\downarrow,..,\uparrow,\uparrow\rangle x$$

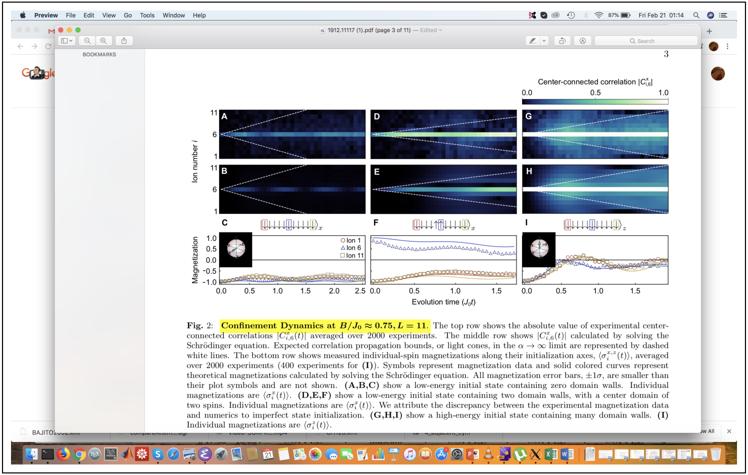
a) $B = 0.5, \alpha = 3$ light-cone;
b) $B = 0.5, \alpha = 0$ localization without disorder;
c) $B = 0.5, \alpha = 0.5$

Invariant Subspaces



$P_{leak} \propto (W/J)^2/L$ for random field and no NN interaction

Experimental Verification



Observation of Domain Wall Confinement and Dynamics in a Quantum Simulator, Monroe Group, Maryland, USA. CONNECTION WITH QUARK CONFINAMENT.

Cooperative Shielding in many-body.

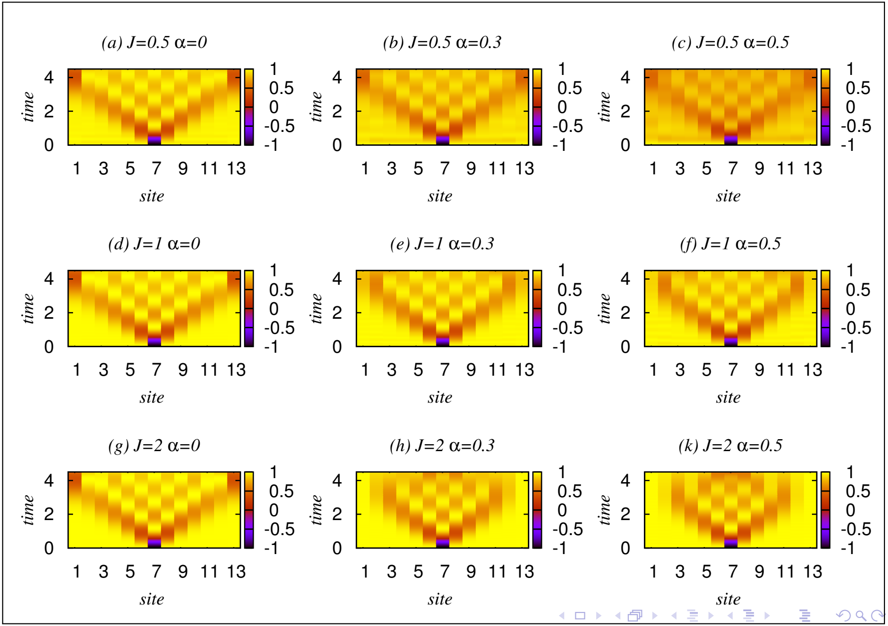
Experimentally accessible spin $1/2$ Hamiltonian:

$$H = H_0 + V, \tag{2}$$
$$H_0 = \sum_{n=1}^{L-1} J_z \sigma_n^z \sigma_{n+1}^z,$$
$$V = \sum_{n<m} \frac{J}{|n-m|^\alpha} \sigma_n^x \sigma_m^x.$$

- $\alpha < 1$: long range. $\alpha > 1$: short range.

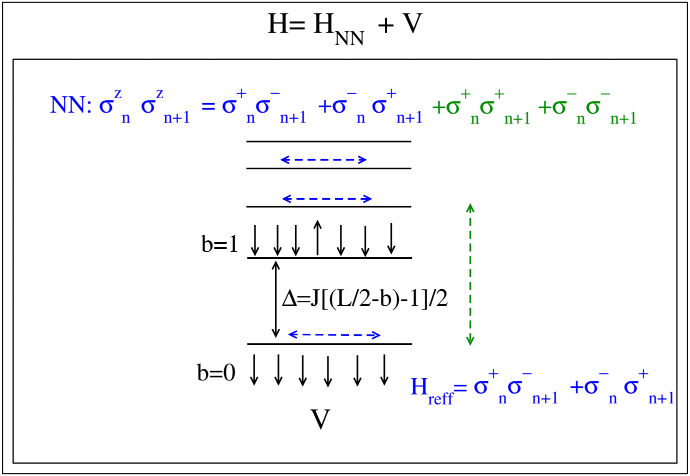
NN+ LONG RANGE

Shielding



(a) J=0.5 α=0
(b) J=0.5 α=0.3
(c) J=0.5 α=0.5
(d) J=1 α=0
(e) J=1 α=0.3
(f) J=1 α=0.5
(g) J=2 α=0
(h) J=2 α=0.3
(k) J=2 α=0.5
time
0
2
4
site
1 3 5 7 9 11 13
1
0.5
0
-0.5
-1

Invariant Subspaces II



$P_{leak} \propto (J_z/J)^2/L$ for NN interaction only.

(Cooperative) Zeno Dynamics

- QZE: Observation freeze dynamics in invariant subspaces.

$$H = H_0 + K H_{meas}$$

As K increases, eigensubspace of H_{meas} becomes invariant.

- Zeno Hamiltonian: in our case: $H = H_0 + V_{LR}$, $V_{LR} \leftarrow H_{meas}$.

$$H_Z = \sum_b \left[P_b H_0 P_b + V_b P_b \right] =$$

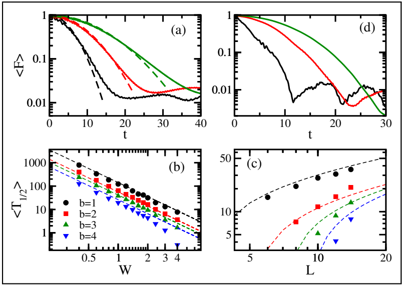
$$= diag(H_0) + \sum_b V_b P_p$$

where P_b are the projectors on the eigensubspace of V corresponding to the eigenvalues V_b.

For $\alpha = 0$ $H_{eff} = H_Z$!

Zeno Fidelity:

$$F(t) = |\langle \Psi(0) | e^{iH_Z t} e^{-iHt} | \Psi(0) \rangle|^2$$



Fidelity decay slows down with N!

Classical *vs* Quantum Shielding

Questions:

- Is it a classical or quantum effect?
- Is the energy gap essential?
- What if we rescale the long range term $(J/N^{1-\alpha})$?
- Classical case...continuum spin of modulus one.

The classical model:

$$H = \sum_{j=1}^{N} h_j S_j^z + J_z \sum_{j=1}^{N-1} S_j^z S_{j+1}^z +$$

$$+\frac{J}{2N^{1-\alpha}} \sum_{j,m\neq j} \frac{S_j^x S_m^x}{|\mathbf{r}_j - \mathbf{r}_m|^\alpha},$$

Conclusions and Perspectives

1. Cooperative Shielding and Correlation Induced localization
2. Mobility Edge in the Imaginary axis.
3. Cooperative shielding in Many Body Systems.

Cooperative Shielding: A Guiding principle in closed and open quantum systems with long range interactions