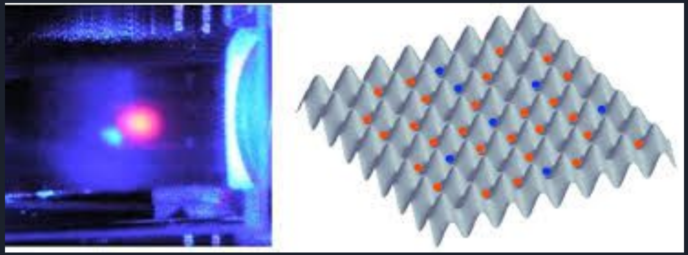
Entropía y tunelamiento dinámico en cruces evitados cerca de la ESQPT en el modelo de LMG



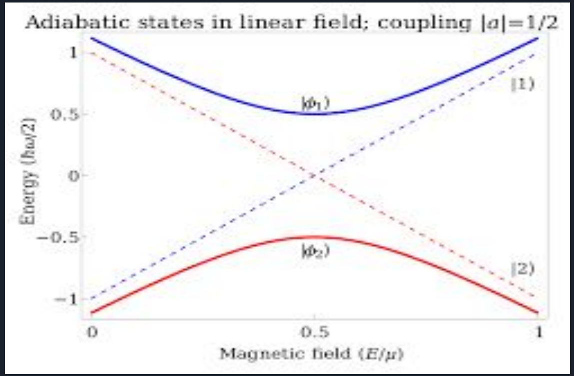
Daniel Julian Nader (UV)

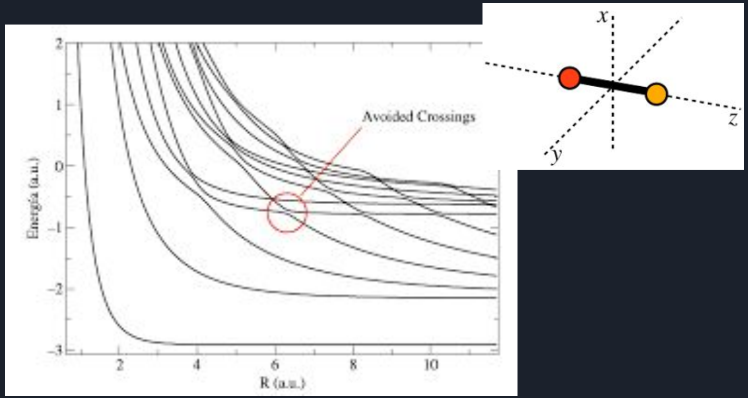
En colaboración con Sergio Lerma (UV) y
Carlos González (UV)

Septiembre 2021

Cruces evitados en niveles de energía

La energía de dos estados como función de un parámetro se acerca y se aleja sin cruzarse



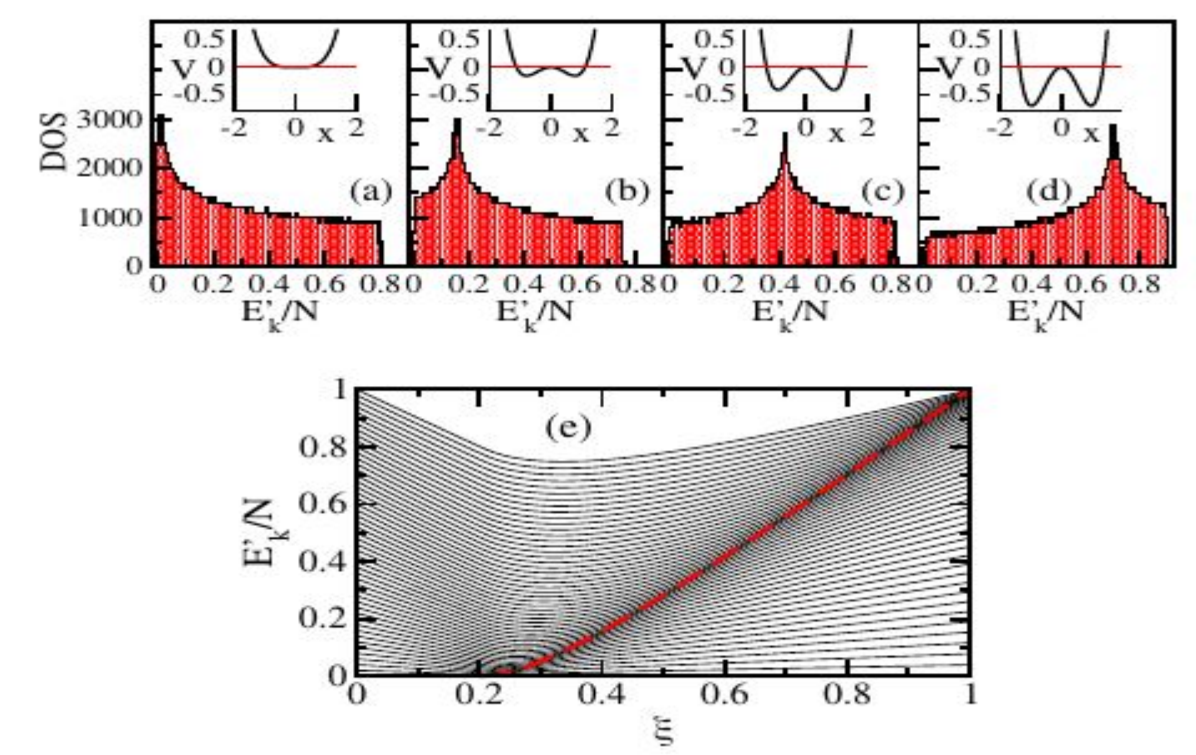


Excited-State Quantum Phase Transitions (ESQPT)

Lea F. Santos, M. Tavora and F. Pérez-Bernal,
“Excited state quantum phase transitions in many-body systems with infinite-range interaction:
localization, dynamics, and bifurcation”,
Phys. Rev. A. **94,** 012113 (2016)

$$H_{\rm b} = (1-\xi)t^\dagger t - \frac{\xi}{N}(t^\dagger s + s^\dagger t)^2$$

$$|N n_t\rangle = \frac{(t^\dagger)^{n_t}(s^\dagger)^{N-n_t}}{\sqrt{n_t!(N-n_t)!}}|0\rangle$$



El modelo de Lipkin-Meshkov-Glick (LGM)

Originalmente [H.. J. Lipkin, N. Meshkov, and A. J. Glick, Nucl. Phys. **62**, 188 (1965)] **sistema fermiónico de dos niveles.**

1.C

Nuclear Physics **62** (1965) 188—198; © *North-Holland Publishing Co., Amsterdam*

VALIDITY OF MANY-BODY APPROXIMATION METHODS FOR A SOLVABLE MODEL

(I). Exact Solutions and Perturbation Theory

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Received 18 February 1964

Abstract: In order to test the validity of various techniques and formalisms developed for treating many-particle systems, a model is constructed which is simple enough to be solved exactly in some cases, but yet is non-trivial. The construction of such models is based on the observation that bilinear products of creation and annihilation operators can be considered as generators of Lie groups. Thus the problem of finding eigenvalues can be greatly simplified by the additio-nal integrals of the motion which are present if the Hamiltonian is constructed so as to commute with invariants of the group. In the present case, the model consists of N fermions distributed in two N-fold degenerate levels and interacting via a monopole-monopole force. It is shown that the model Hamiltonian is easily expressed in terms of quasi-spin operators and exact eigenvalues are obtained. In addition, eigenvalues are calculated with ordinary perturbation theory using values for the number of particles and interaction strength which are appropriate to the more realistic problems of finite nuclei. In subsequent papers we consider the results obtained by various other approximation methods for comparison with the exact results presented here.

Alcanzable experimentalmente

epl A Letters Journal Exploring the Frontiers of Physics

EPL, **90** (2010) 54001
doi: 10.1209/0295-5075/90/54001

Circuit QED scheme for the realization of the Lipkin-Meshkov-Glick model

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received 3 May 2010; accepted in final form 1 June 2010
published online 15 June 2010

PRL **100**, 040403 (2008) PHYSICAL REVIEW LETTERS week ending 1 FEBRUARY 2008

Dynamical Quantum Phase Transitions in the Dissipative Lipkin-Meshkov-Glick Model with Proposed Realization in Optical Cavity QED

S. Morrison[1,2] and A. S. Parkins[3]

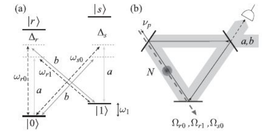
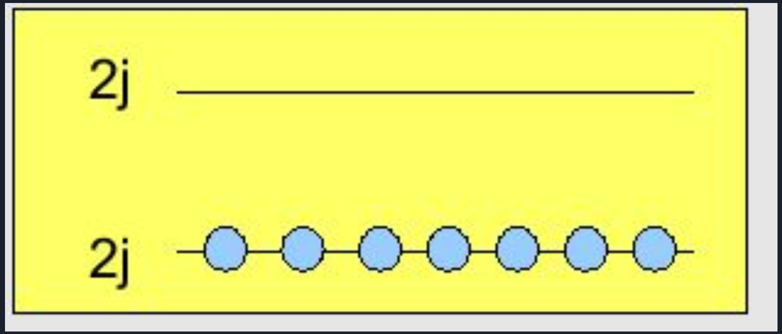


FIG. 1. (a) Atomic level and excitation scheme. (b) Potent ring-cavity setup. The laser fields (dashed lines) are at freque cies that are not supported by the resonator, but can be inject through one of the resonator mirrors so as to be copropagati with the cavity fields through the ensemble.

El modelo de Lipkin-Meshkov-Glick (LGM)



$$\hat{H} = \frac{\epsilon}{2}\sum_{p\nu}\sigma a^{\dagger}_{p\nu}a_{p\nu} + \frac{V}{2}\sum_{pp'\sigma}a^{\dagger}_{p\sigma}a^{\dagger}_{p'\sigma}a_{p'-\sigma}a_{p-\sigma} + \frac{W}{2}\sum_{pp'\sigma}a^{\dagger}_{p\sigma}a^{\dagger}_{p'-\sigma}a_{p'\sigma}a_{p'-\sigma}$$

Representacion en algebra SU(2)

$$\hat{J}_z = \frac{1}{2}\sum_{p,\sigma} \sigma a^{\dagger}_{p,\sigma} a_{p,\sigma}$$

$$\hat{J}_\pm = \sum_{p,\sigma} a^{\dagger}_{p\mp} a_{p\pm}$$

→

$$[\hat{J}_z, \hat{J}_\pm] = \pm\hat{J}_\pm$$
$$[\hat{J}_+, \hat{J}_-] = \hat{J}_z$$

Hamiltoniano

$$\hat{H} = \epsilon_0 \hat{J}_z + \frac{V}{2}\left(\hat{J}_+^2 + \hat{J}_-^2\right) + \frac{W}{2}\left(\hat{J}_+\hat{J}_- + \hat{J}_-\hat{J}_+\right)$$

reparametrización

$$\gamma_x = \left(\frac{2J-1}{\epsilon_0}\right)(W+V), \quad \gamma_y = \left(\frac{2J-1}{\epsilon_0}\right)(W-V)$$

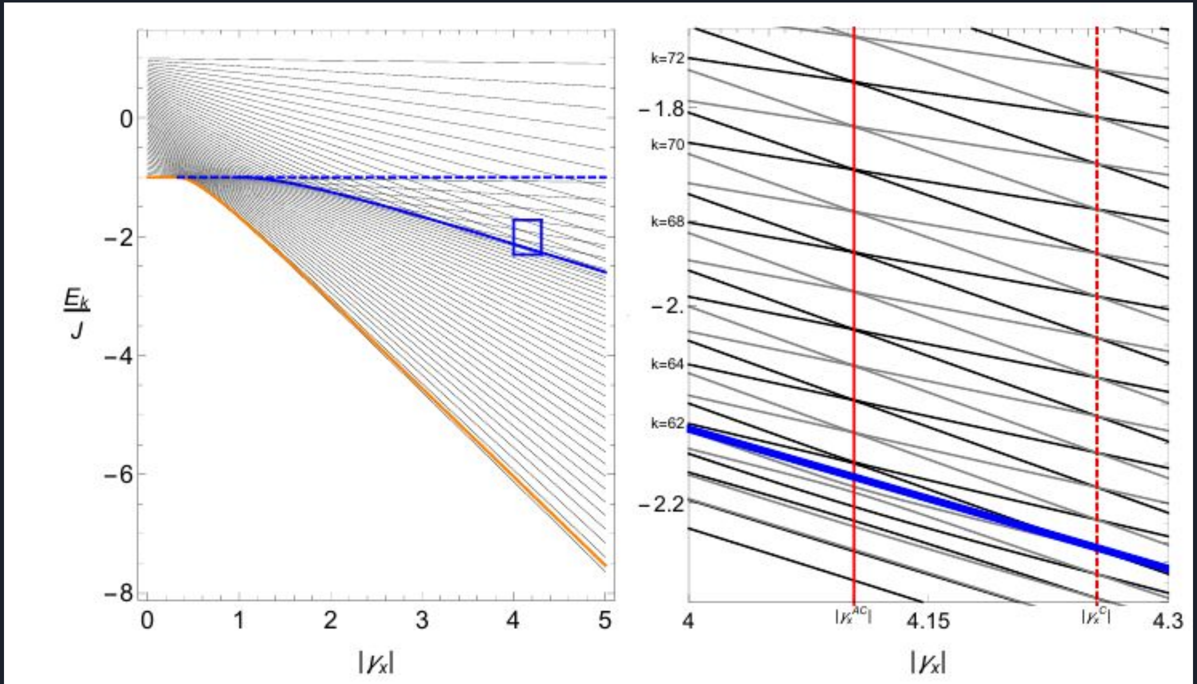
Podemos llevar a cabo diagonalización exacta para momento angular definido

S. Lerma H. and J. Dukelsky, “The lipkin–meshkov–glick model as a particular limit of the su(1,1) richardson–gaudin integrable models,” Nuclear Physics B 870, 421–443 (2013).

Espectro cuántico y cruces evitados

Octavio Castaños, Ramón López-Peña, Jorge G. Hirsch,
and Enrique López-Moreno, “Phase transitions and accidental degeneracy in nonlinear spin systems,” Phys. Rev.
B 72, 012406 (2005).

$$\gamma_x^{C(AC)}\gamma_y^{C(AC)} = \left(\frac{2J-1}{2J-N_{\mathrm{o(e)}}}\right)^2$$



Estado coherente de Bloch

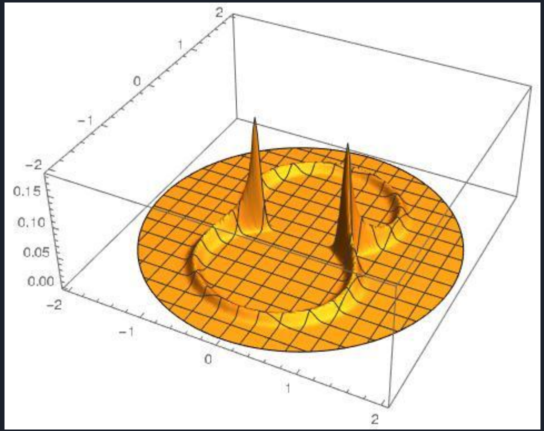
$$|\alpha\rangle = \frac{1}{(1+|\alpha|^2)^J} e^{\alpha \hat{J}_+} |J - J\rangle$$

$$\alpha = \tan(\theta/2) e^{i\phi}$$

$$Q = \sqrt{2(1-\cos\theta)}\cos\phi, \quad P = -\sqrt{2(1-\cos\theta)}\sin\phi,$$

Representación de Husimi

$$\mathcal{Q}_k(\alpha) = |\langle\alpha|E_k\rangle|^2$$



Un enfoque clásico al modelo LMG

Hamiltoniano clásico

$$H \equiv \langle\alpha|\hat{H}|\alpha\rangle = \epsilon_0 J \left[z + \frac{\gamma_x}{2}x^2 + \frac{\gamma_y}{2}y^2\right]$$

$$z = -\cos\theta, \quad x = \sin\theta\cos\phi, \quad y = \sin\theta\sin\phi,$$

Variables canónicas

$$Q = \sqrt{2(1-\cos\theta)}\cos\phi, \quad P = -\sqrt{2(1-\cos\theta)}\sin\phi,$$

Formalismo Hamiltoniano →

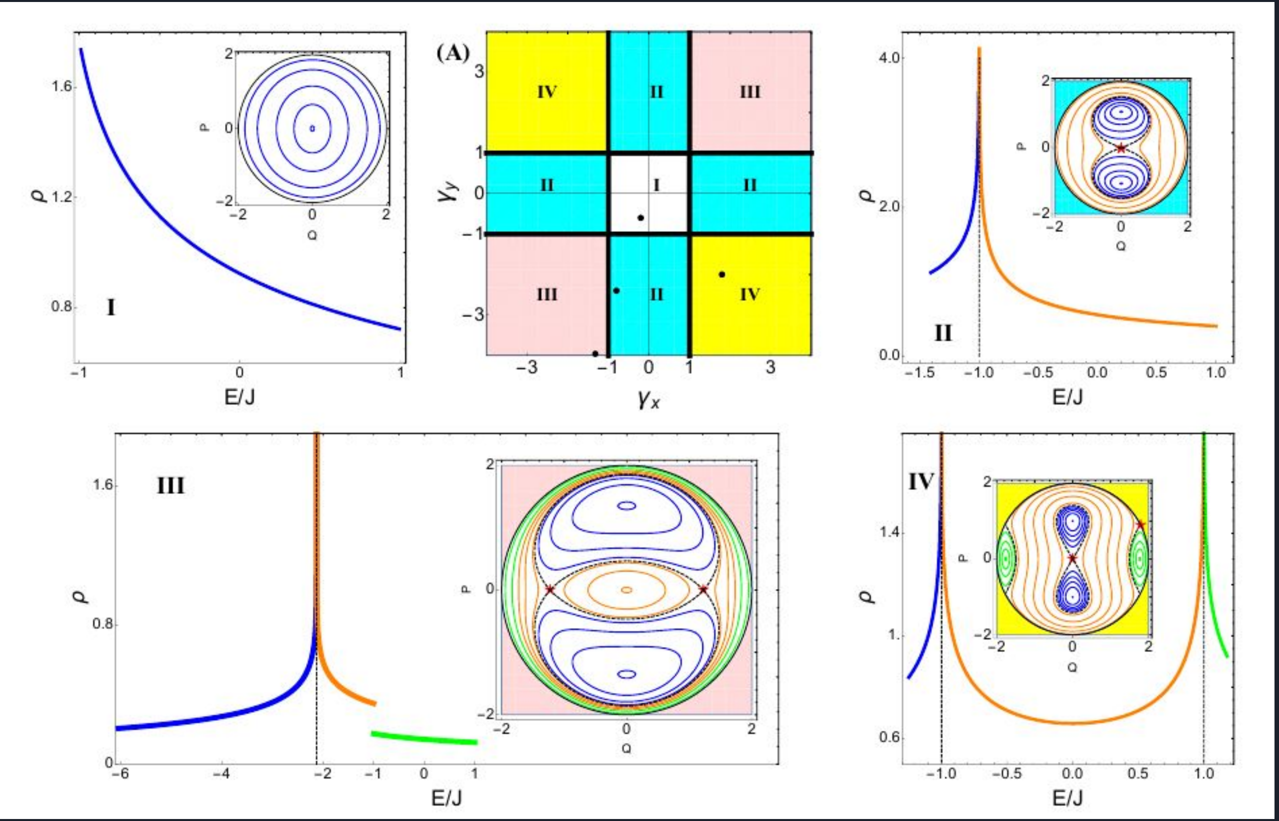
$$\dot{Q} = -\frac{\partial H}{\partial P}$$

$$\dot{P} = \frac{\partial H}{\partial Q}$$

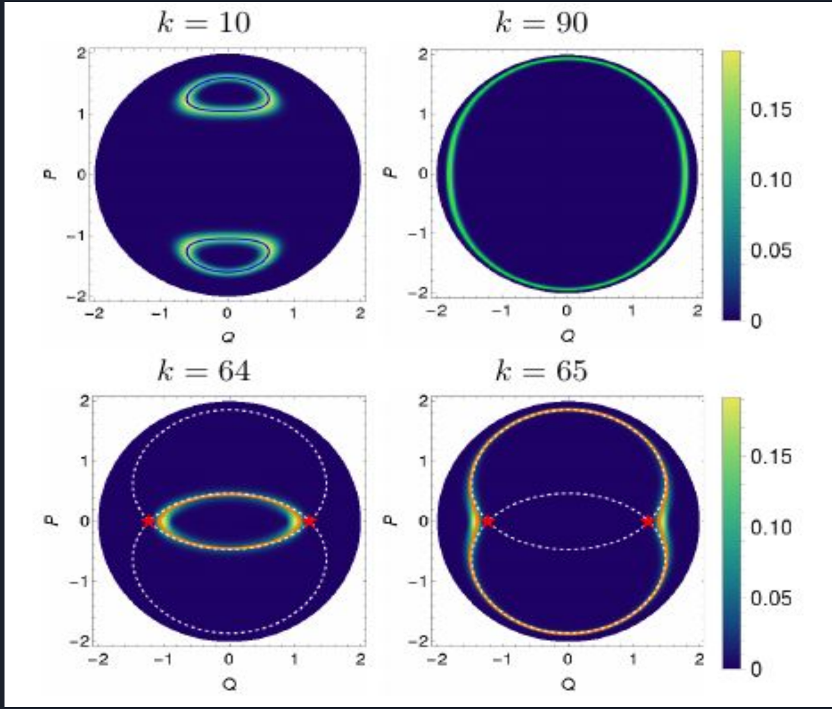
Regiones de parámetros en el Modelo de LMG

Octavio Castaños, Ramón López-Peña, Jorge G. Hirsch, and Enrique López-Moreno, “Classical and quantum phase transitions in the lipkin-meshkov-glick model,” Phys. Rev. B 74, 104118 (2006).

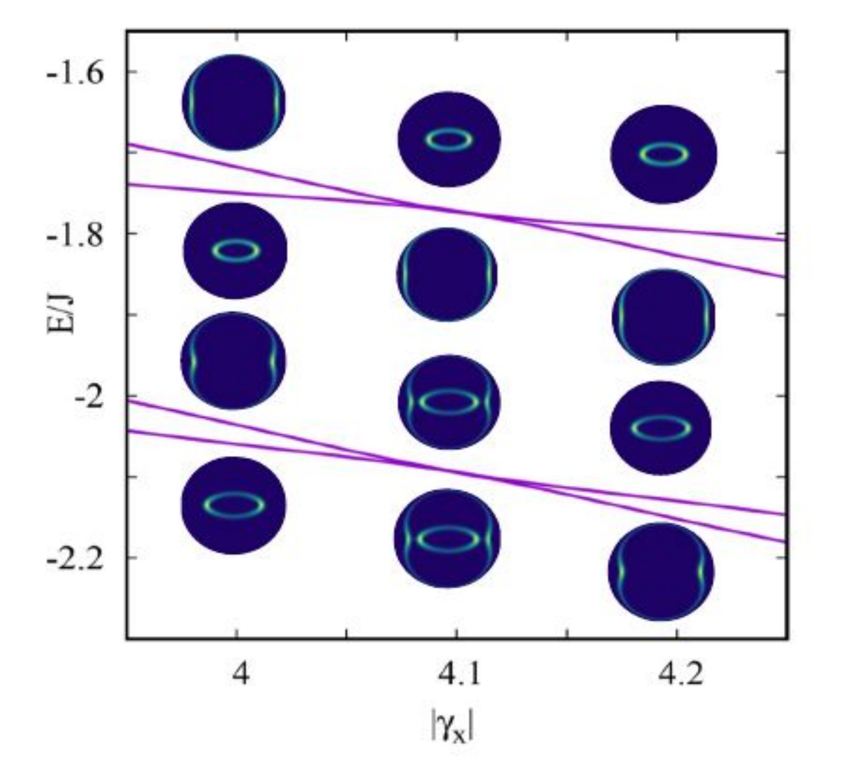
S. Lerma H. and J. Dukelsky, “The lipkin–meshkov–glick model as a particular limit of the su(1,1) richardson–gaudin integrable models,” Nuclear Physics B 870, 421–443 (2013).



La función de Husimi se distribuye alrededor de las trayectorias clásicas



Intercambio de la función de Husimi en el cruce evitado

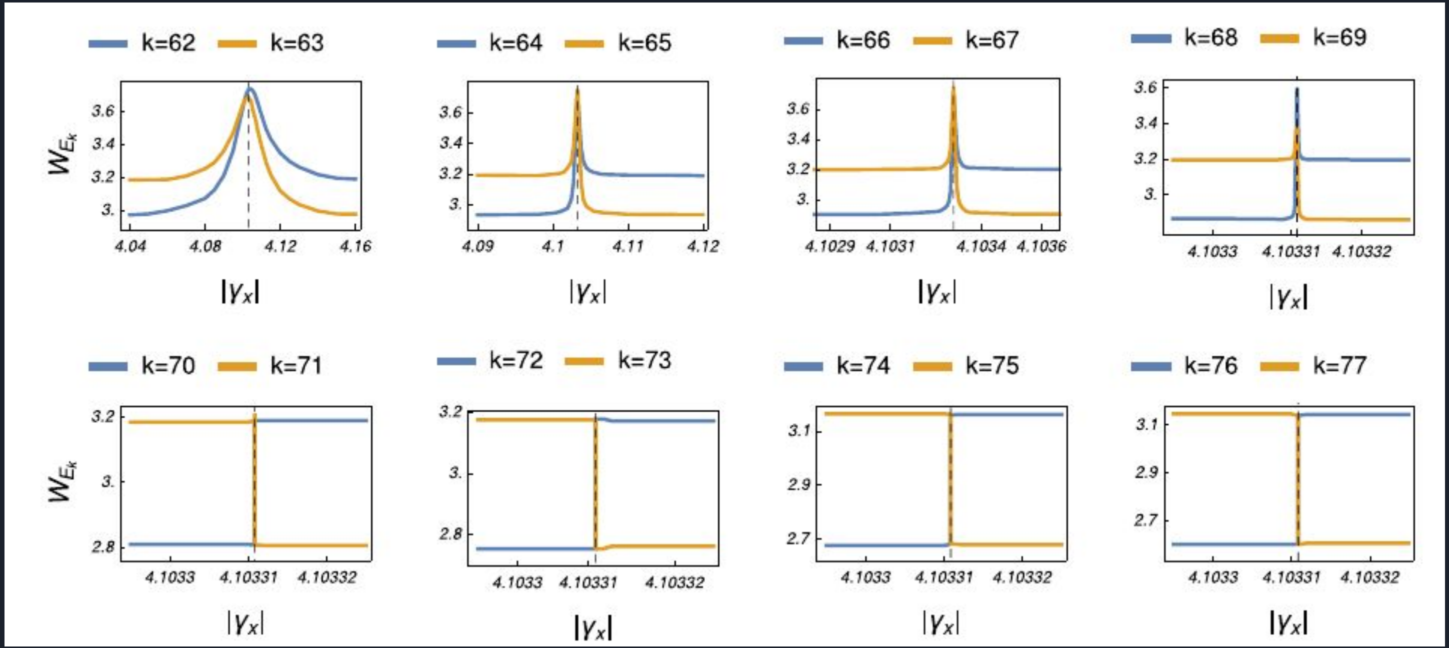


Entropía de Wehrl cerca del cruce evitado

$$W_{E_k} = -\int \mathcal{Q}_k(\alpha) \ln \mathcal{Q}_k(\alpha) d\Omega$$
$$= -\int \mathcal{Q}_k(Q,P) \ln \mathcal{Q}_k(Q,P) dQdP$$

Elvira Romera, Octavio Castaños, Manuel Calixto, and Francisco Pérez-Bernal, “Delocalization properties at isolated avoided crossings in lipkin–meshkov–glick type hamiltonian models,” Journal of Statistical Mechanics: Theory and Experiment 2017, 013101 (2017).

F. J. Arranz, R. M. Benito, and F. Borondo, “Shannon entropy at avoided crossings in the quantum transition from order to chaos,” Phys. Rev. E 99, 062209 (2019).



Dinámica cuántica del estado coherente

- Perfil del estado coherente inicial

$$|\alpha_0\rangle = \sum_k c_k |E_k\rangle$$

- Evolución de la función de Husimi

$$\mathcal{Q}_{\alpha_o}(\alpha, t) = |\langle\alpha|\hat{U}(t)|\alpha_o\rangle|^2$$

- Probabilidad de supervivencia

$$SP(t) = |\langle\alpha_0|\hat{U}(t)|\alpha_0\rangle|^2$$

- Valor esperado del operador de ocupación

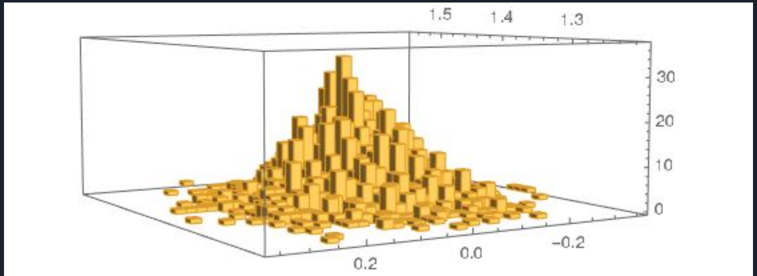
$$j_z(t) \equiv \frac{\langle\hat{J}_z\rangle(t)}{J} = \frac{1}{J}\langle\alpha_0|\hat{U}^\dagger(t)\hat{J}_z\hat{U}(t)|\alpha_0\rangle$$

Dinámica semiclásica del estado coherente

- Función de Wigner (aprox)

$$w_{\alpha_0}(\theta,\phi) \approx \frac{J}{\pi} e^{-J\Theta^2}$$

$$\cos\Theta = \cos\theta\cos\theta_0 + \sin\theta\sin\theta_0\cos(\phi-\phi_0)$$



- Promedios clásicos

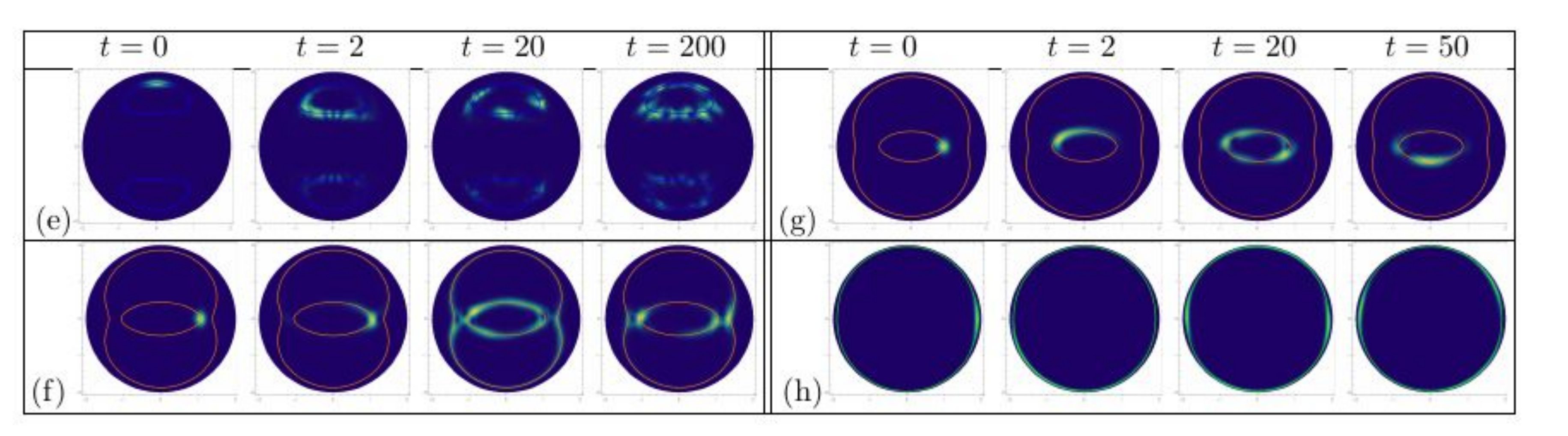
$$SP(t) \approx \frac{2\pi}{J}\int d\mathbf{u}\; w(\mathbf{u},0)w(\varphi^{-t}(\mathbf{u}),0)$$

$$j_z(t) \approx \int d\mathbf{u}\; w_{\alpha_o}(\varphi^{-t}(\mathbf{u}))z(\mathbf{u})$$

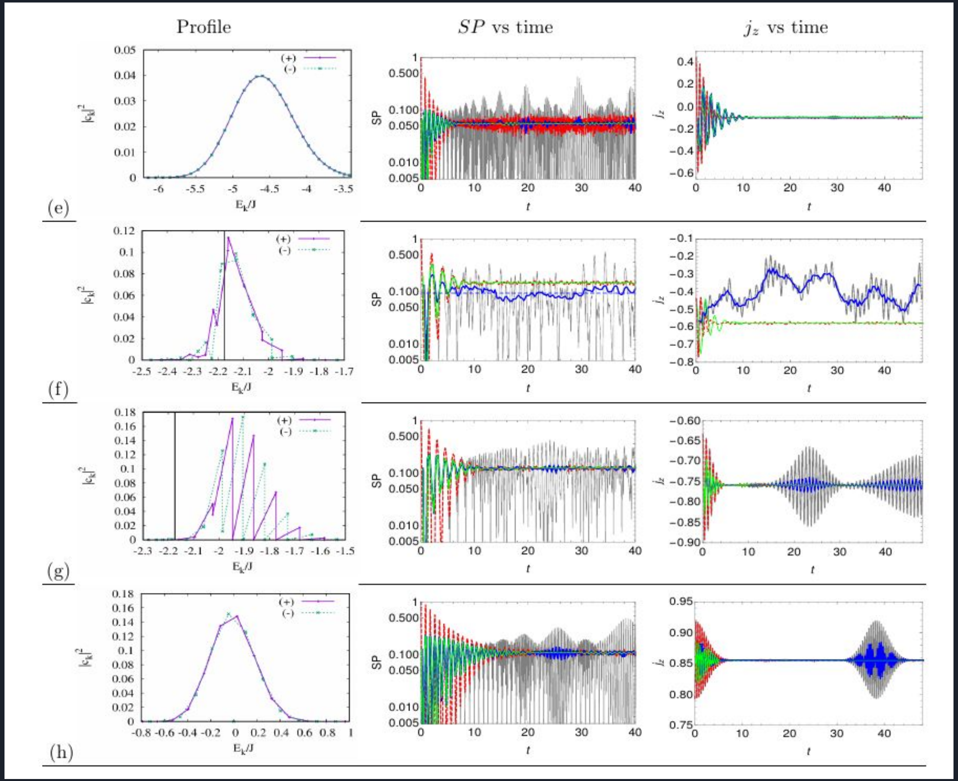
$$\mathbf{u}(t) = \varphi^t(\mathbf{u}_0)$$

D Villaseñor, S Pilatowsky-Cameo, M A Bastarrachea-Magnani, S Lerma-Hernández, L F Santos, and J G Hirsch, “Quantum vs classical dynamics in a spin-boson system: manifestations of spectral correlations and scarring,” New Journal of Physics 22, 063036 (2020).

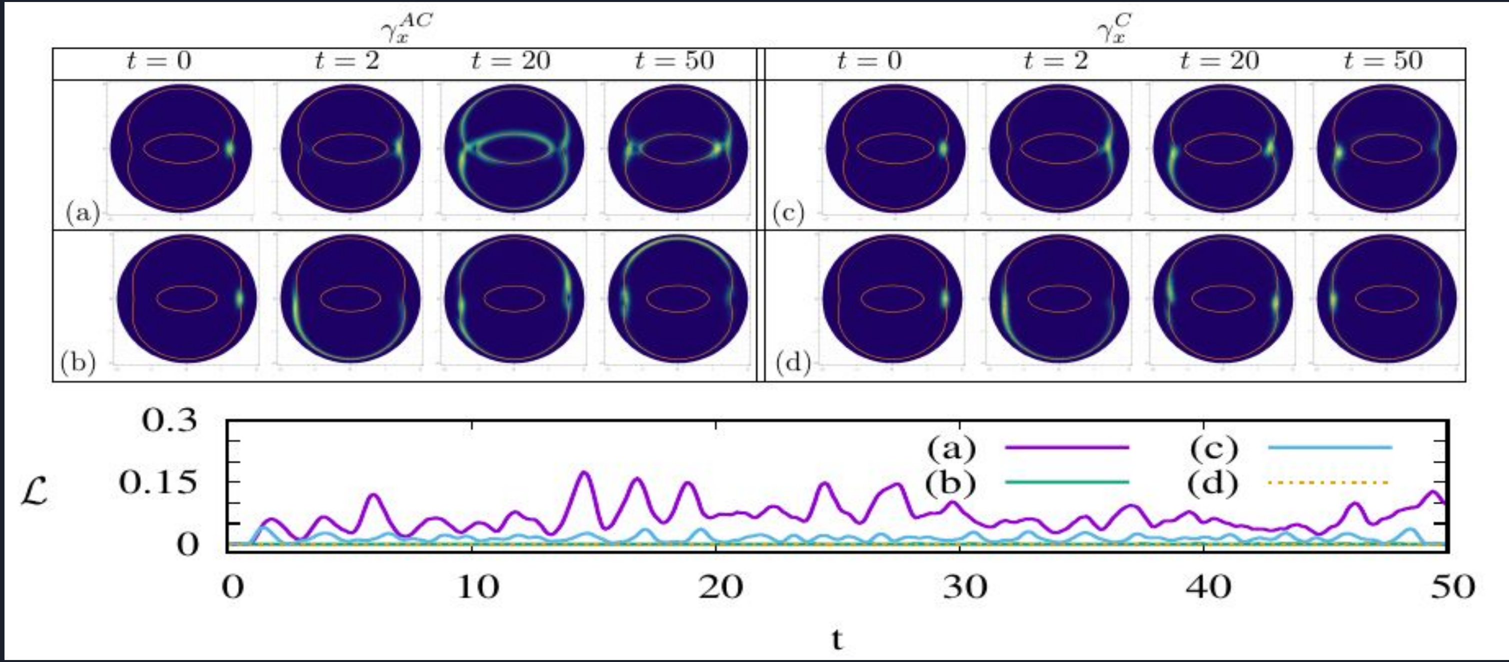
Evolución de la función de Husimi de un estado coherente



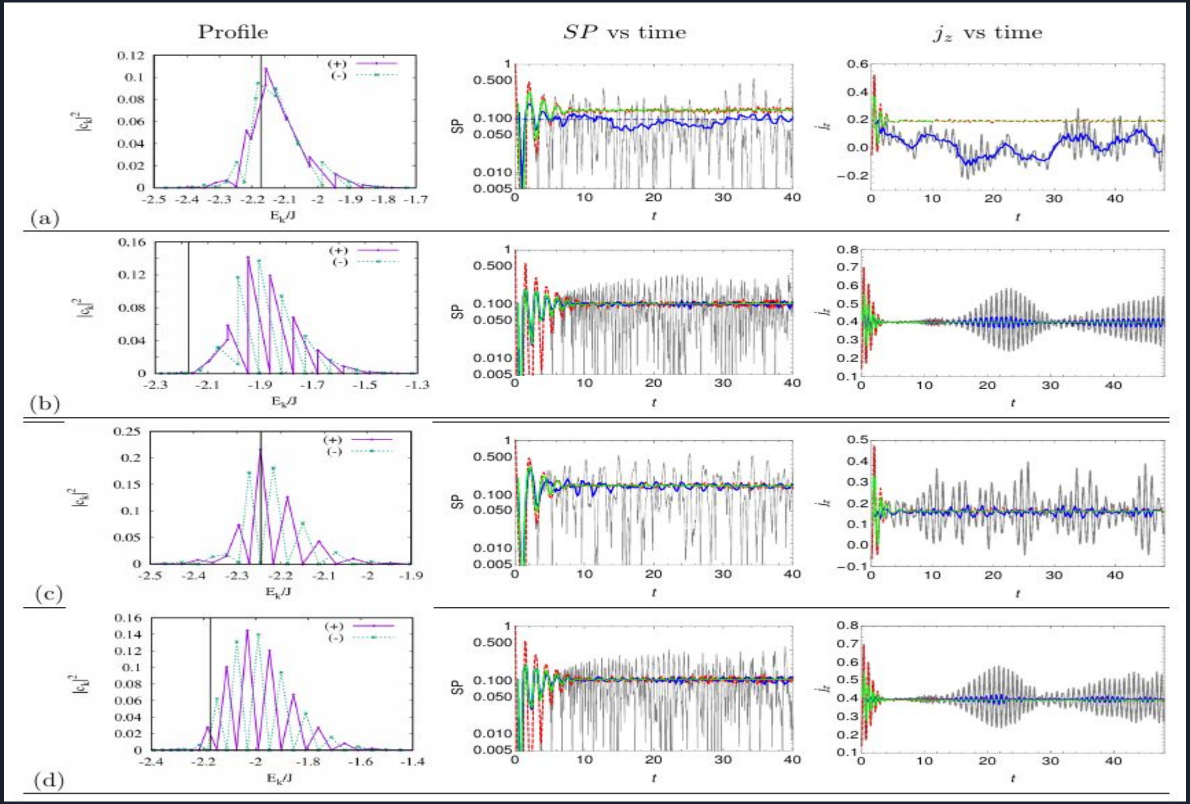
Correspondencia en la probabilidad de supervivencia y observables



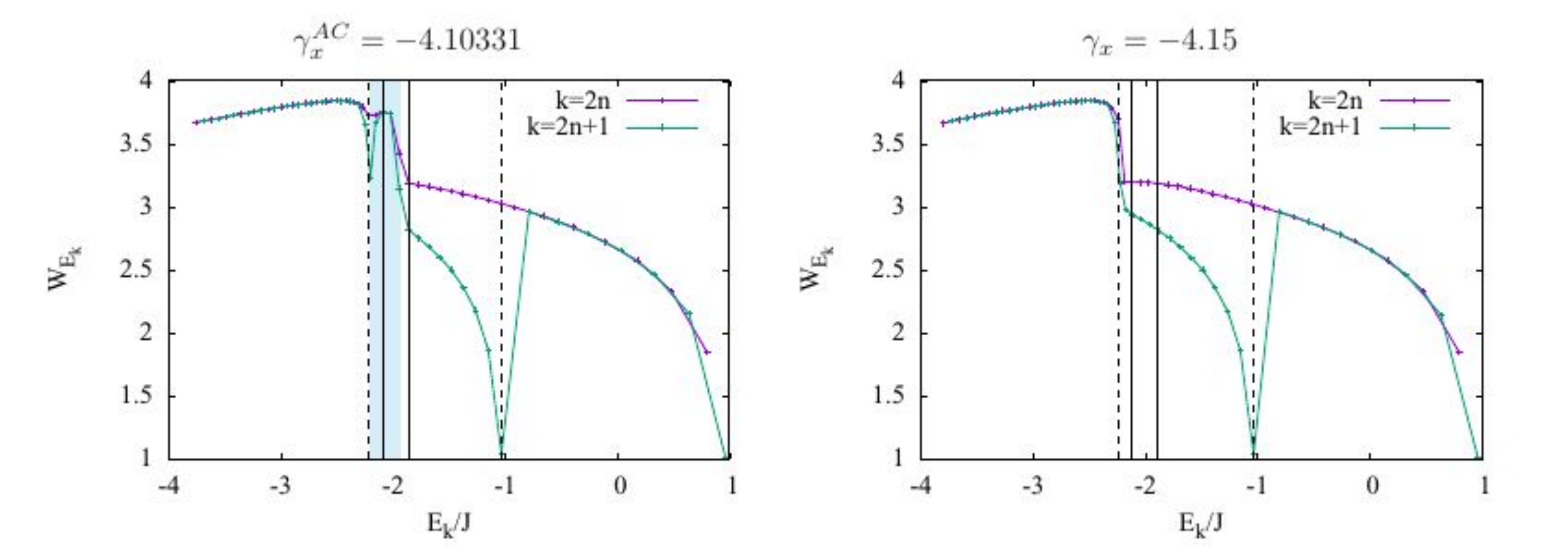
Incremento del tunelamiento dinámico



Rompimiento de la correspondencia clásico-cuántica



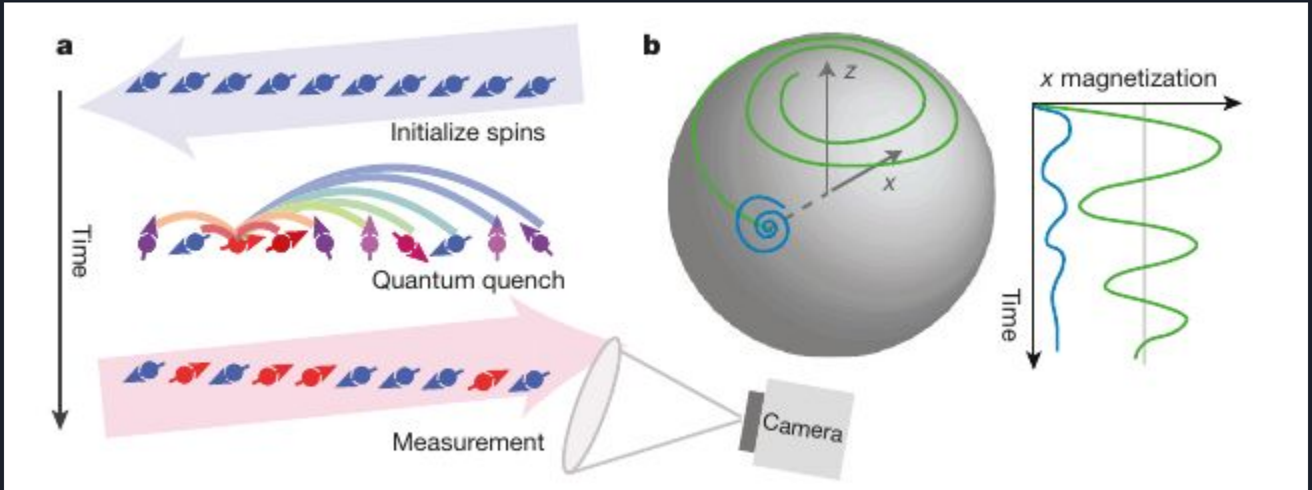
Conexión entre incremento en la entropía y tunelamiento



¿Como se comporta el intervalo de incremento si nos desplazamos en la constante de acoplamiento o aumentamos el tamaño del sistema?

Estudios de quantum quench

Un estado estacionario inicial a t=0 evoluciona de acuerdo a un Hamiltoniano diferente a t>0



J. Zhang et al, “Observation of a many-body dynamical phase transition with a 53-qubit quantum simulator” Nature **551** (2017)

Conclusiones

- La entropía aumenta súbitamente en el cruce evitado de algunos estados (cerca de la ESQPT)
- Existe una correspondencia clásico-cuántica en la probabilidad de supervivencia de un estado coherente inicial y en observables
- Se rompe la correspondencia clásico-cuántica si se cumple lo siguiente:
  1) El acoplamiento satisface la condición de cruce evitado

  2) El estado coherente tiene energía cercana a la ESQPT
- La entropía brinda información sobre la dinámica del estado coherente distinguiendo los casos en donde aparece tunelamiento dinámico

Gracias!