Timescales and OTOCs in quantum many-body systems

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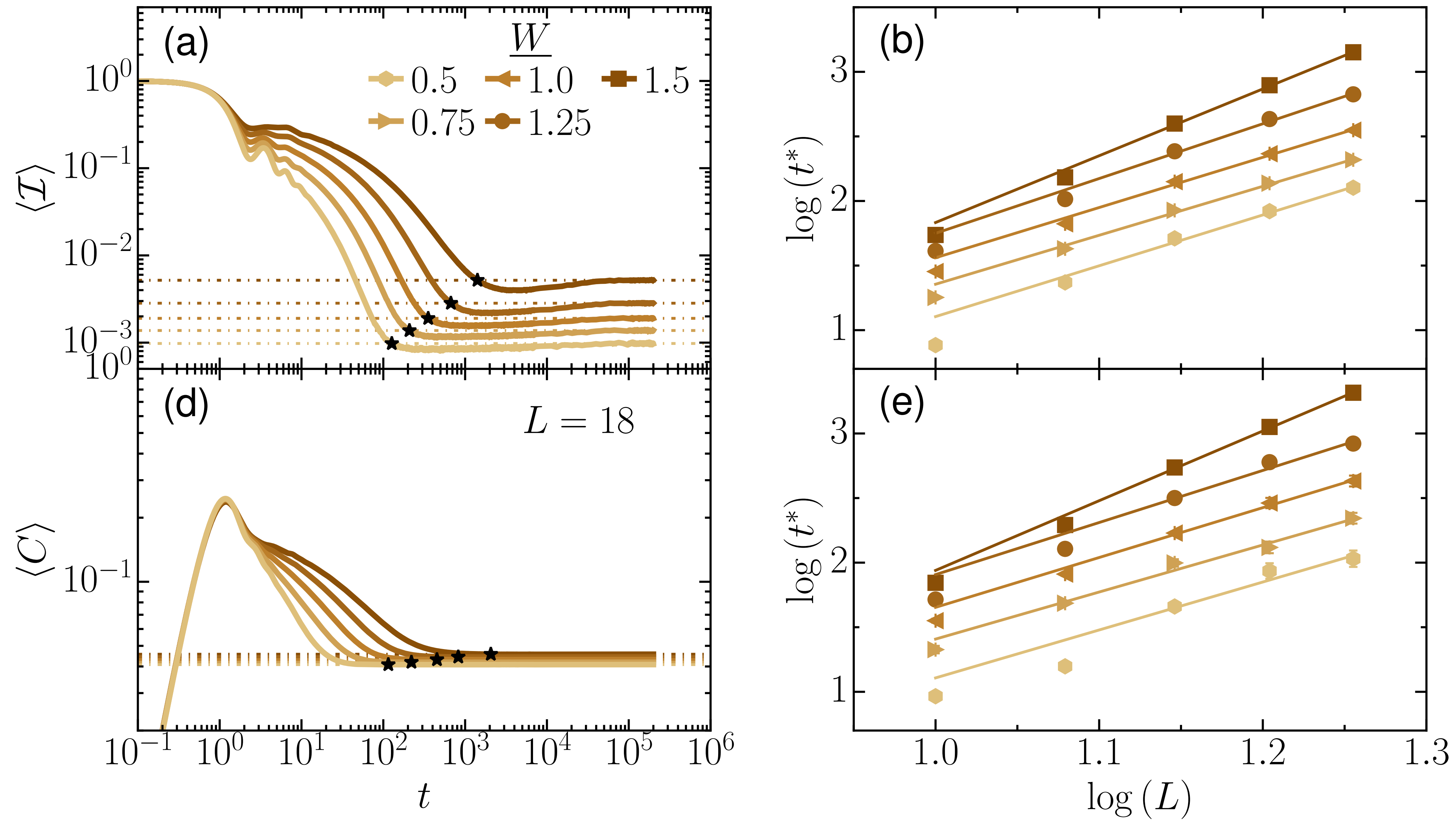
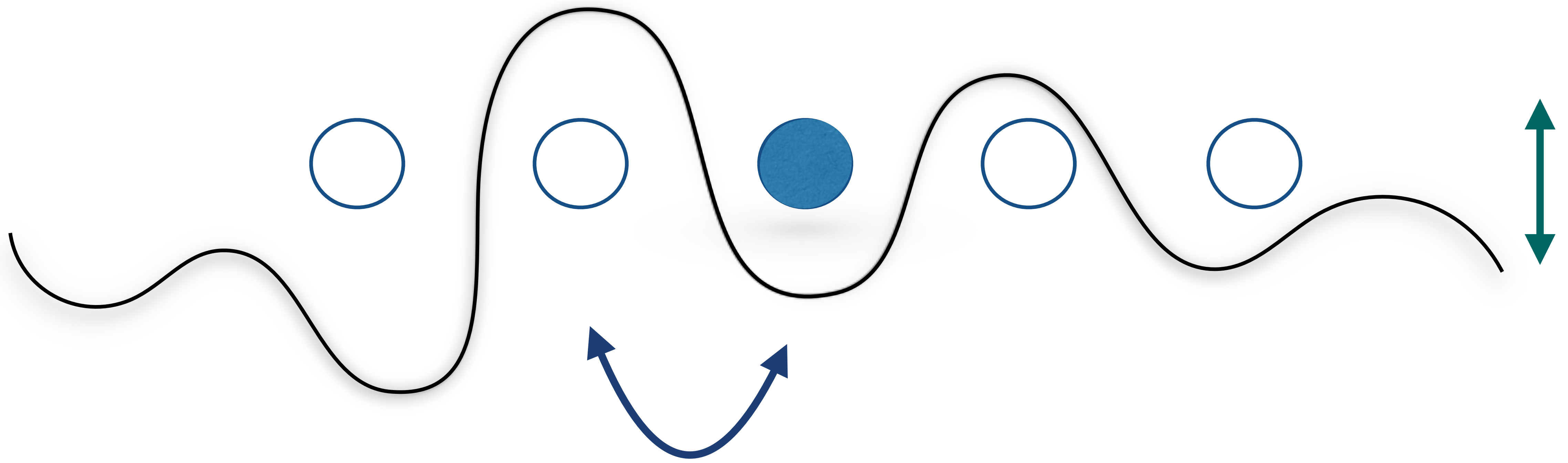


Joint work with:

Lea F. Santos (YU), Yevgeny Bar Lev (BGU), Jonathan Torres-Herrera (BUAP), Francisco Pérez-Bernal (UH)

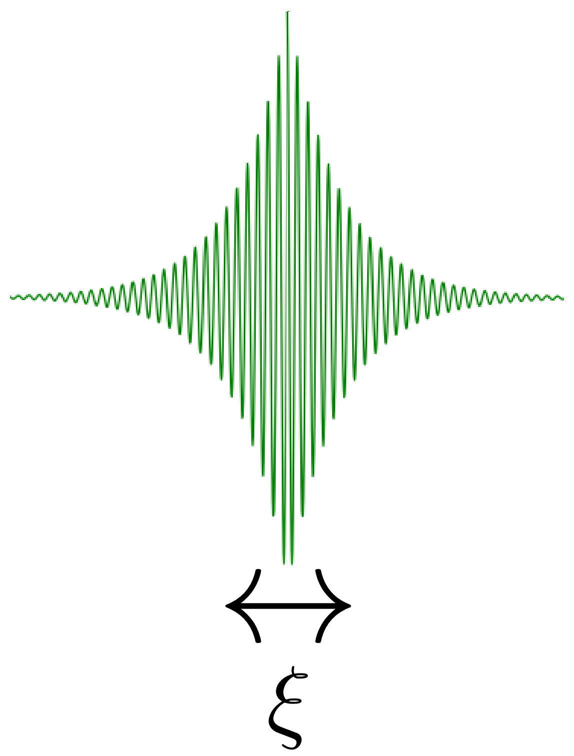
Anderson localization

- Closed
- Onsite random potential
- Noninteracting



$$h_i \in [-W, W]$$

$$H_A = \sum_i t(c^\dagger_{i+1} c_i + \text{h.c.}) + \sum_i h_i n_i$$

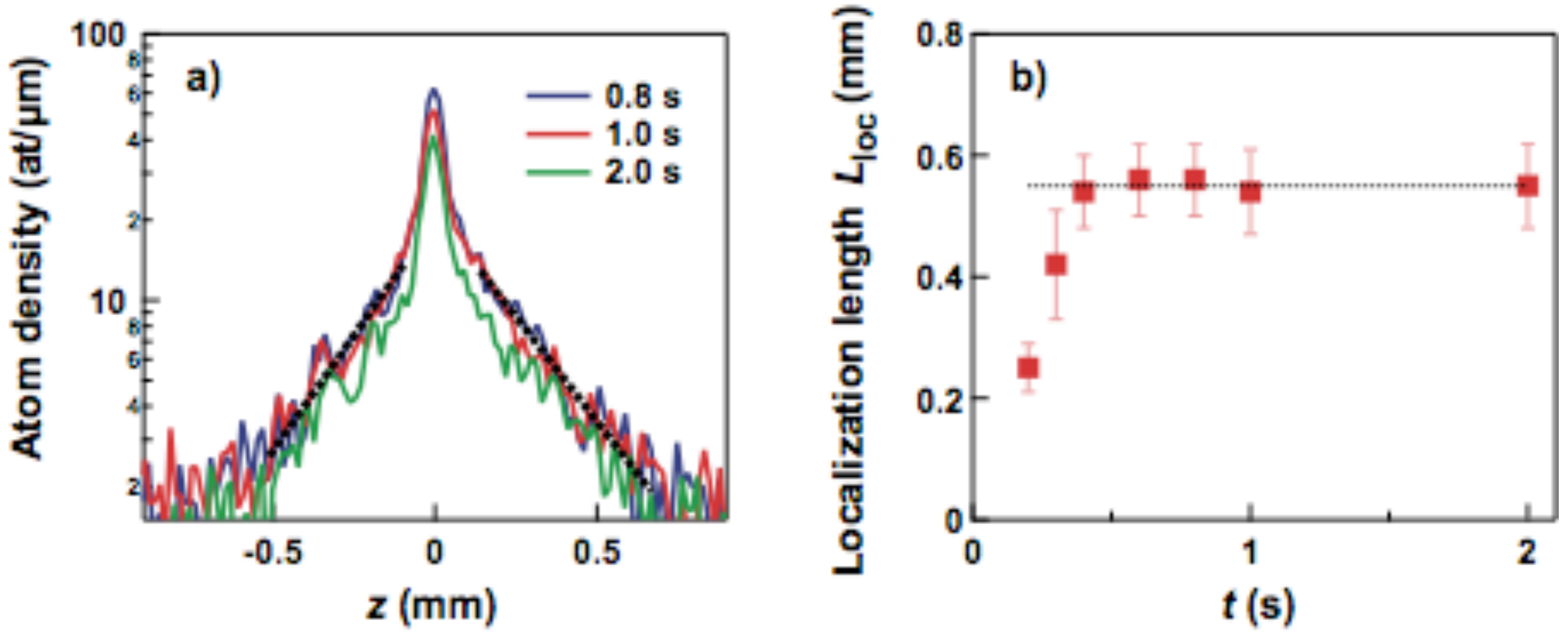


$$|\psi(x)| \sim e^{-x/\xi}$$

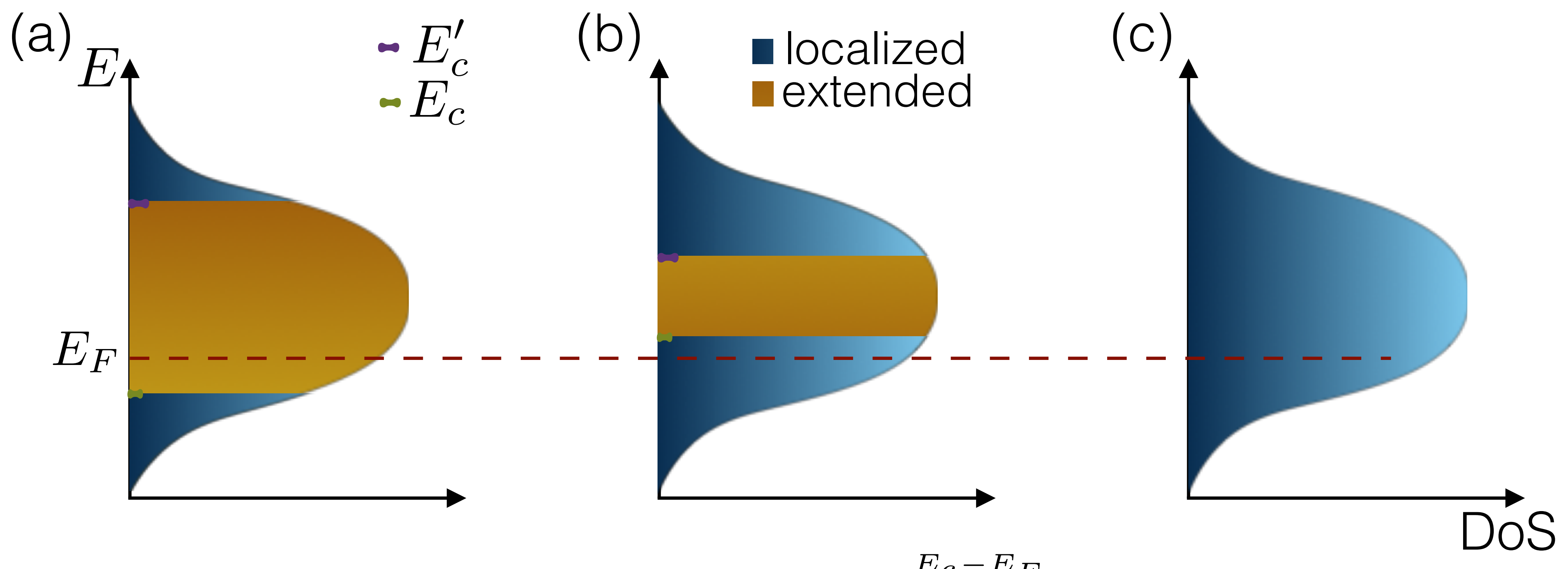
In 1D and 2D: Wavefunctions are exponentially localized for any disorder strength.

In 3D: Metal-insulator transition

- Absence of diffusion
- Absence of thermalization



Nature 453, 2008

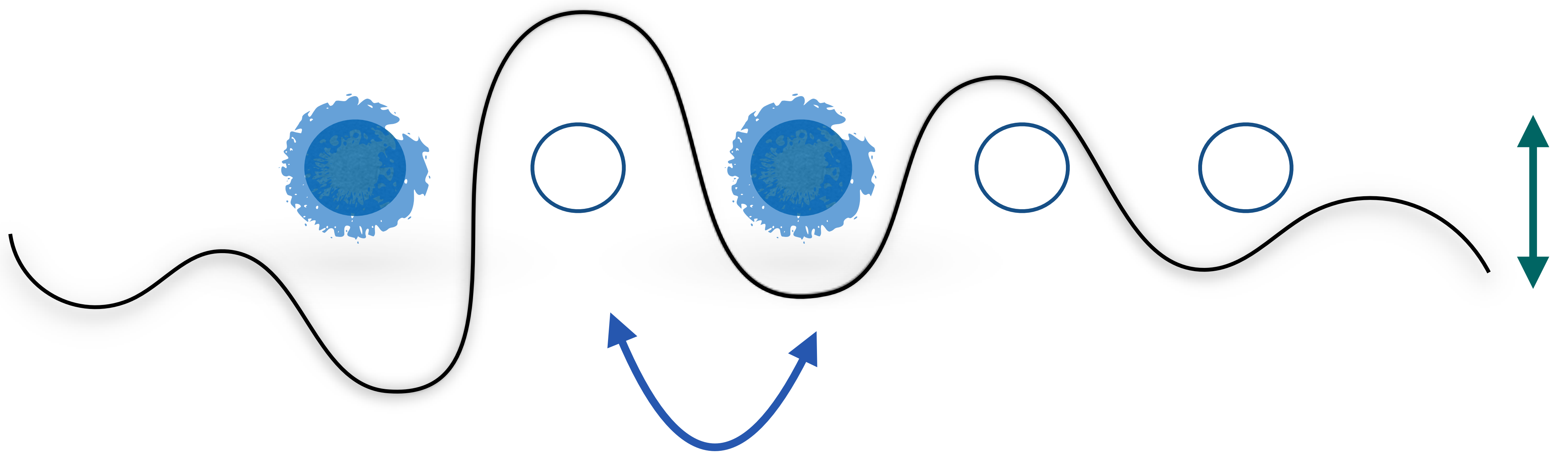


$\sigma(T \to 0) > 0$ $\quad$ $\sigma(T) \propto e^{-\frac{E_c - E_F}{T}}$ $\quad$ $\sigma(T) = 0 \ \forall T$

Anderson, 1958

Many-body localization (MBL)

- Closed
- Onsite random potential
- Interacting



$$H_A = \sum_i t(c_{i+1}^\dagger c_i + \mathrm{h.c.}) + \sum_i h_i n_i$$

$$h_i \in [-W, W]$$

Q: Does localization hold?

A: In ID, yes, but at sufficiently strong disorder.

$$H = H_A + V \sum_{\langle ij \rangle} n_i n_j$$

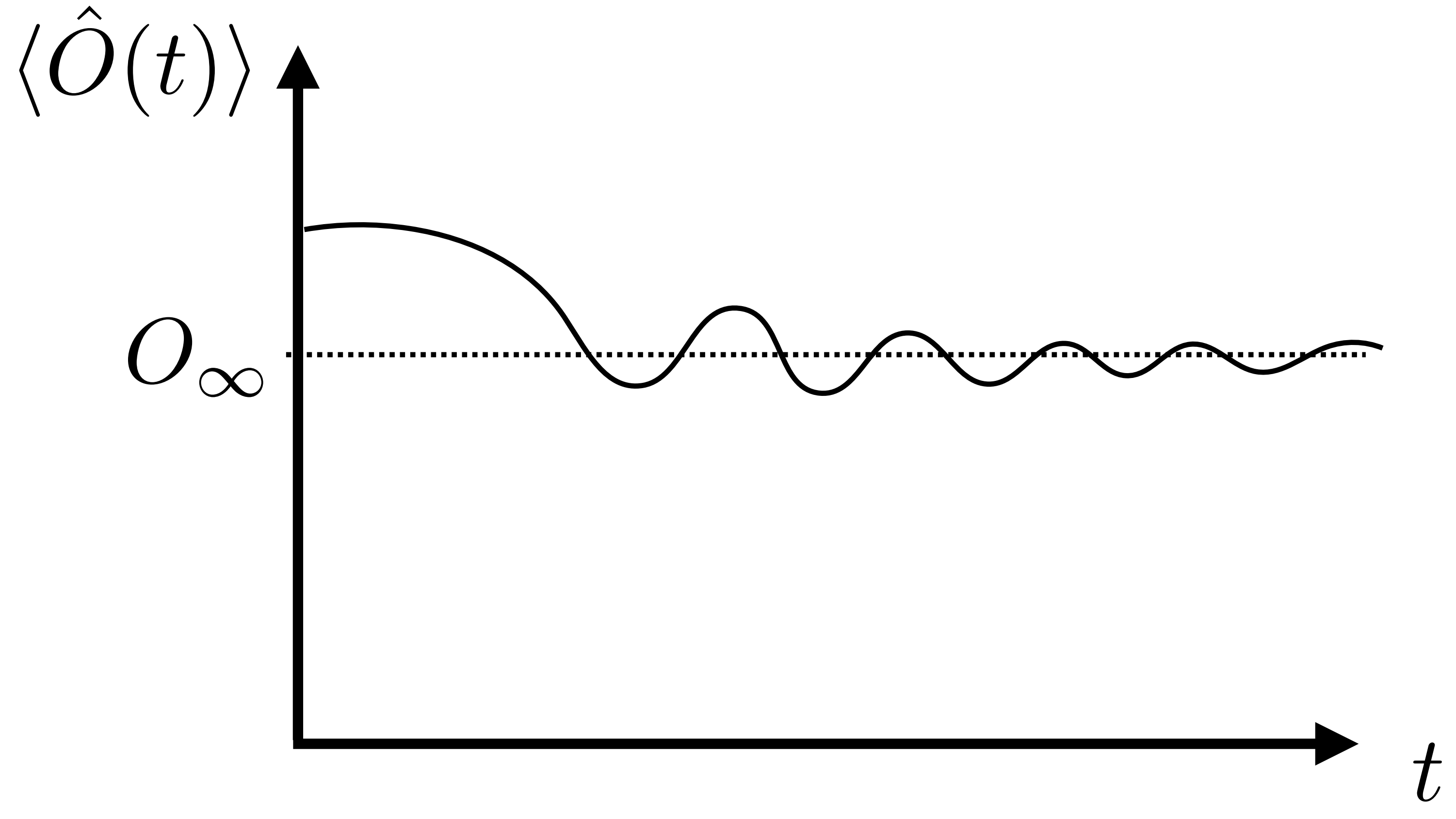
Eigenstate properties?

How are these reflected on the dynamics and long-time behavior?

Basko, Aleiner, Altshuler, Ann. Phys., 2006

Thermalization in a closed quantum system

$$\rho(t) = U\rho(0)U^\dagger \qquad H|\nu\rangle = E_\nu|\nu\rangle$$



$$\langle\hat{O}(t)\rangle \equiv \langle\psi(t)|\hat{O}|\psi(t)\rangle = \sum_{\mu\nu} c_\mu^* c_\nu e^{-i(E_\nu - E_\mu)t} O_{\mu\nu}$$

$$= \sum_\nu |c_\nu|^2 O_{\nu\nu} + \sum_{\mu\neq\nu} c_\mu^* c_\nu e^{-i(E_\nu - E_\mu)t} O_{\mu\nu}$$

$$\overline{\langle\hat{O}(t)\rangle} \equiv \lim_{T'\to\infty} \frac{1}{T'} \int_0^{T'} dt\ \hat{O}(t) = \sum_\nu |c_\nu|^2 O_{\nu\nu} = O_\infty$$

When do local observables reach thermal equilibrium?

$$\overline{\langle\hat{O}(t)\rangle} = \langle\hat{O}(t)\rangle_{\mathrm{TH}}$$

Nonequilibrium dynamics

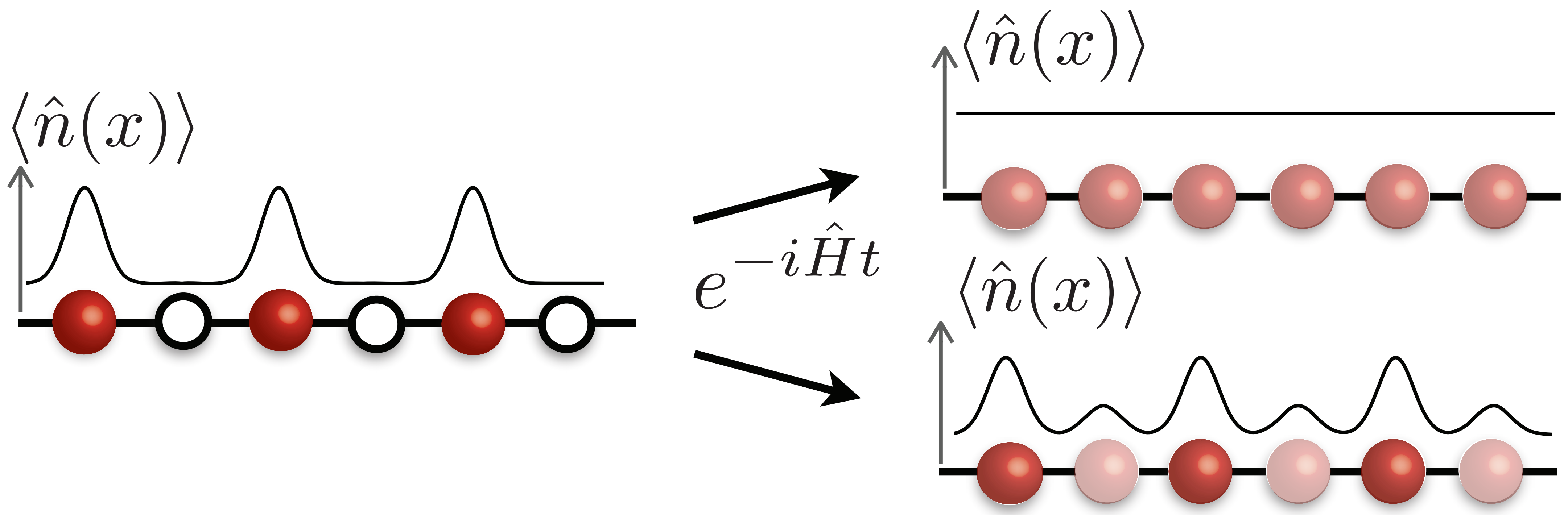
Time-independent Hamiltonian

$t = 0 \longrightarrow t$

$H_0 \qquad H(\lambda) = H_0 + \lambda V \quad ; \quad [H, H_0] \neq 0$

$|\Psi_0\rangle \qquad |\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle$

- Global quench



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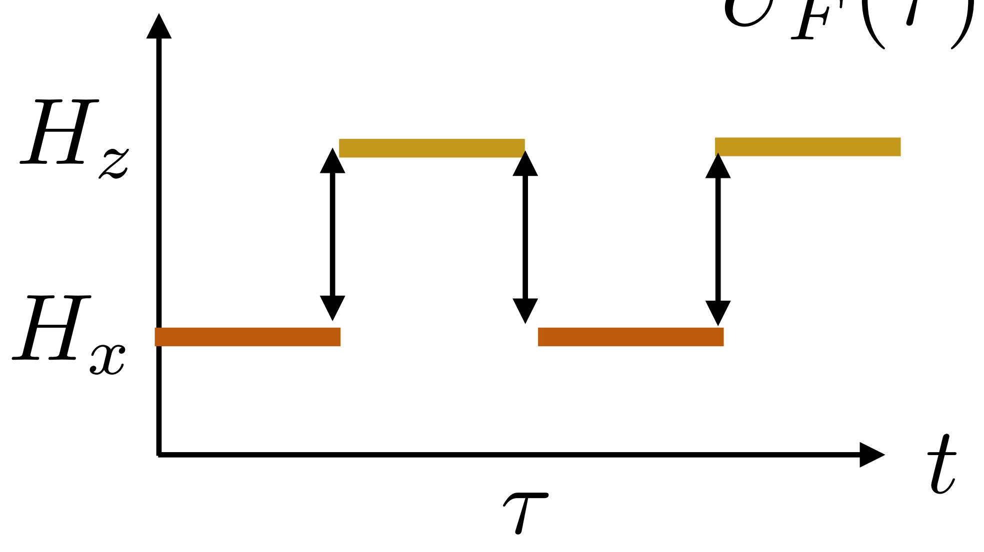
Time-dependent Hamiltonian

- Periodic driving

$$H(t) = H(t+\tau)$$

$$U_F(\tau) = \mathcal{T} \int_0^{\tau} dt\, e^{-itH(t)}$$

$$U_F(\tau) = e^{-iH_x\tau} e^{-iH_z\tau}$$



Models of MBL in a 1D lattice

Spin-1/2 Heisenberg XXZ

$$H = J_\perp \sum_{i=1}^{L} \left(\hat{\sigma}_i^x \hat{\sigma}_{i+1}^x + \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y\right) + J_z \sum_{i=1}^{L} \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + \frac{1}{2} \sum_{i=1}^{L} h_i \hat{\sigma}_i^z \qquad h_i \in [-W, W]$$

Flip-flop term — Ising interaction — Onsite random potential (Zeeman splitting)

Spinless fermions

$$H = -\frac{J}{2} \sum_i c_i^\dagger c_{i+1} + \text{h.c.} + V \sum_i^L \left(n_i - \frac{1}{2}\right)\left(n_{i+1} - \frac{1}{2}\right) + \sum_i^L h_i \left(n_i - \frac{1}{2}\right)$$

Hopping term — n.n interactions — Onsite random potential

Jordan-Wigner transformation

$$\sigma_i^+ = e^{-i\pi \sum_{k=1}^{i-1} n_k} c_i^\dagger \quad ; \quad \sigma_i^- = e^{i\pi \sum_{k=1}^{i-1} n_k} c_i \quad ; \quad \sigma_i^z = 2n_i - 1$$

Many-body localization transition

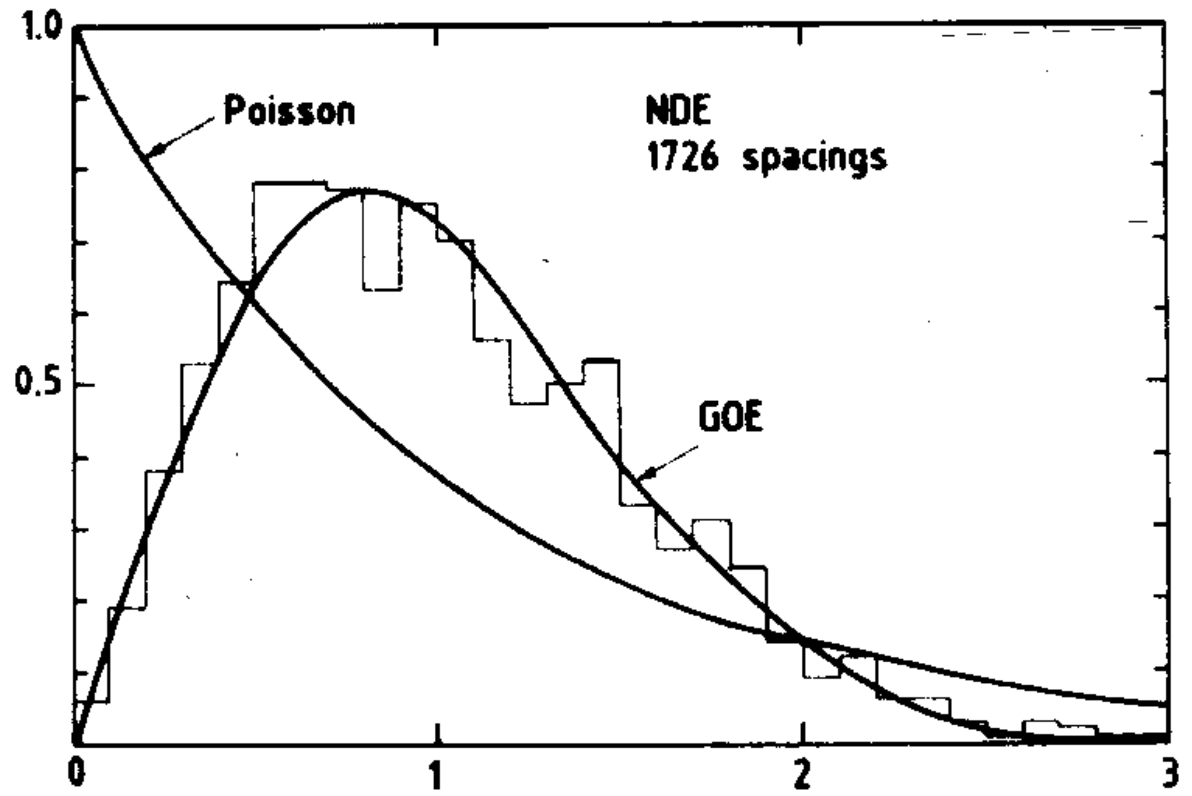
Eigenstate properties

Not a conventional phase transition

(finite energy density)

Jonathan's talk

Thermal	MBL
Extended eigenstates (metal)	Localized (insulator)
Volume-law entanglement $S \propto L$	Area-law entanglement $S \propto \mathrm{const}$
Wigner-Dyson (level repulsion)	Poisson $P(s) = e^{-s}$
$P(s) = A_\beta s^\beta e^{-B_\beta s^2}$	Local integrals of motion



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Bohigas et al., (1983)

Many-body localization transition

Dynamical properties

Not a conventional phase transition

(finite energy density)

Thermal | Prethermal | MBL ?

Entanglement entropy

Ballistic

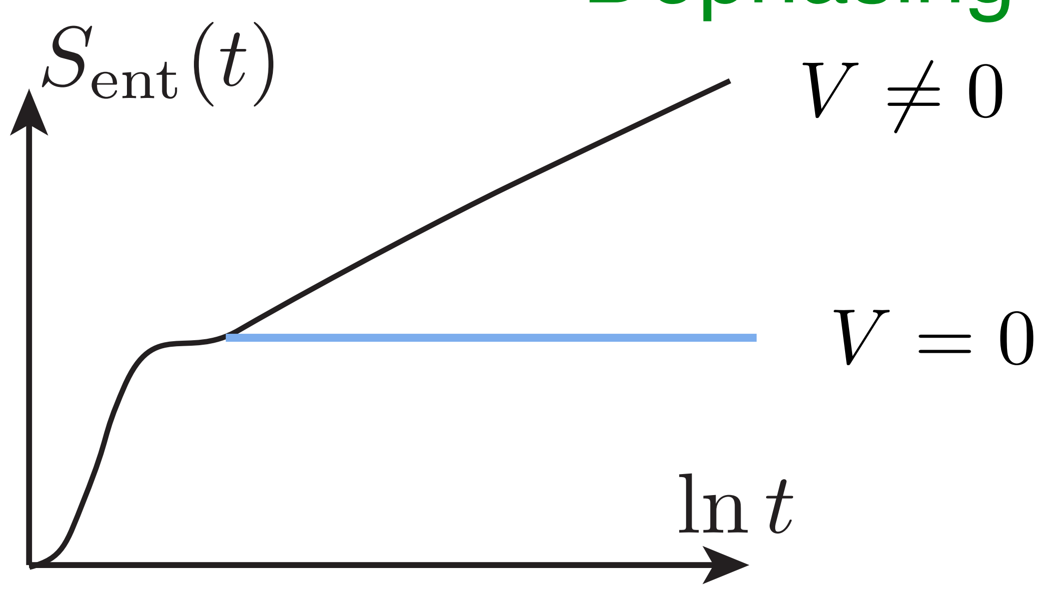
$S(t) \sim t$

Logarithmic growth

$S(t) \sim \xi \log(Vt)$

Dephasing

(b)



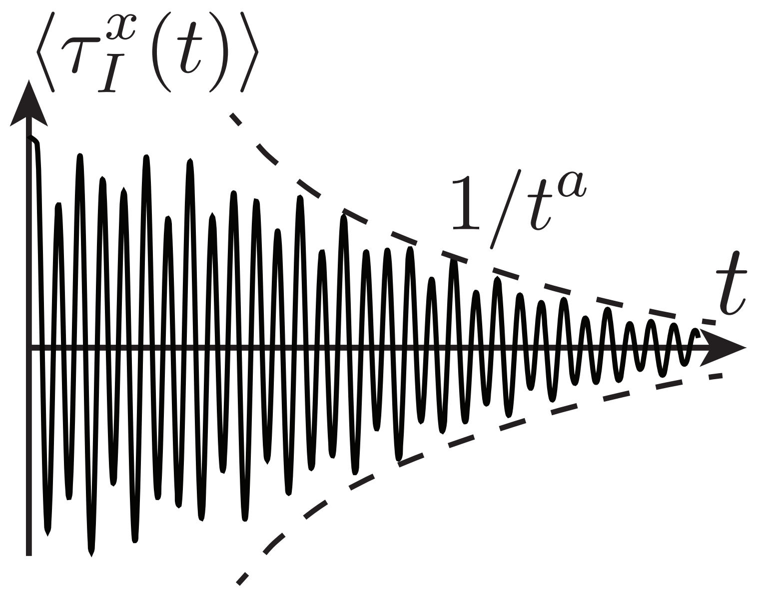
Local observables

Exponential relaxation

Power-law relaxation

$\langle \Psi(t)|\mathcal{O}|\Psi(t)\rangle \sim t^{-\alpha}$

(c)



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How long do realistic systems take to equilibrate/ thermalize?

Q: How to define the equilibration/thermalization time?

Q: How does the equilibration time depends on the **model, observables, and initial state**?

Model: Disordered one-dimensional spin-1/2 Heisenberg chain (realistic and in some sense generic)

$$H = J \sum_{i=1}^{L} \mathbf{S}_i \cdot \mathbf{S}_{i+1} + \sum_{i=1}^{L} h_i S_i^z, \qquad \mathbf{S}_i = (S_i^x, S_i^y, S_i^z) \qquad h_i \in [-W, W]$$

$W = 0.5$

Initial states $|\Psi_0\rangle = |\uparrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow\dots\rangle$ in the middle of the spectrum

Chaotic/thermal regime

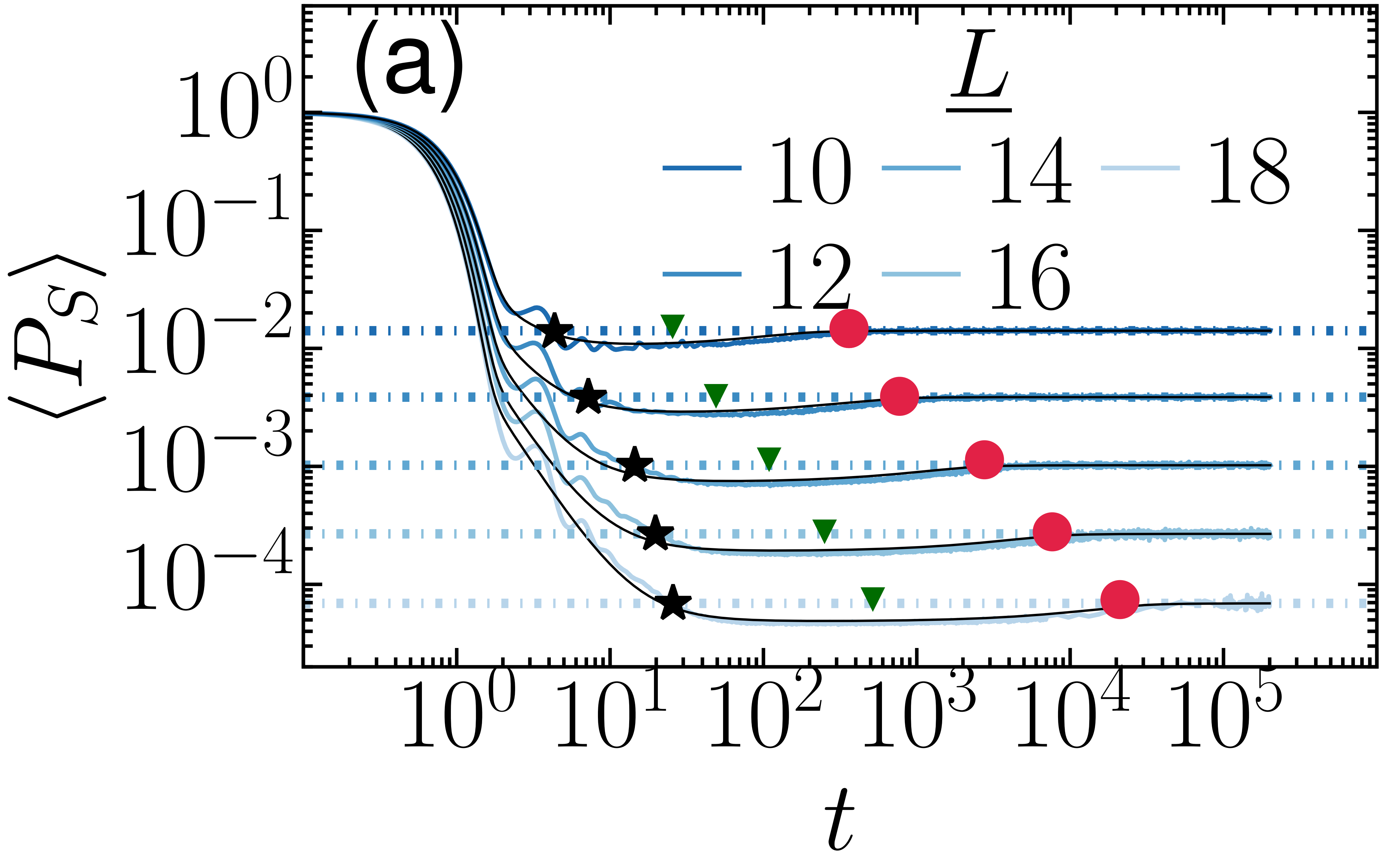
$$S_T^z = \sum_i^L S_i^z$$

Extensive conservation laws, necessary for spin transport.

Correlation hole: manifestation of spectral correlations in the late-time dynamics

Survival probability $P_S(t) = |\langle \Psi(0)|\Psi(t)\rangle|^2$

$$\langle P_S(t)\rangle = \langle \sum_{\alpha\neq\beta} |C_\alpha|^2 |C_\beta|^2 e^{-i(E_\alpha - E_\beta)t}\rangle + \langle \sum_\alpha |C_\alpha|^4 \rangle$$



Spectral form factor $\langle \sum_{\alpha\neq\beta} e^{-i(E_\alpha - E_\beta)t}\rangle$

● Heisenberg time

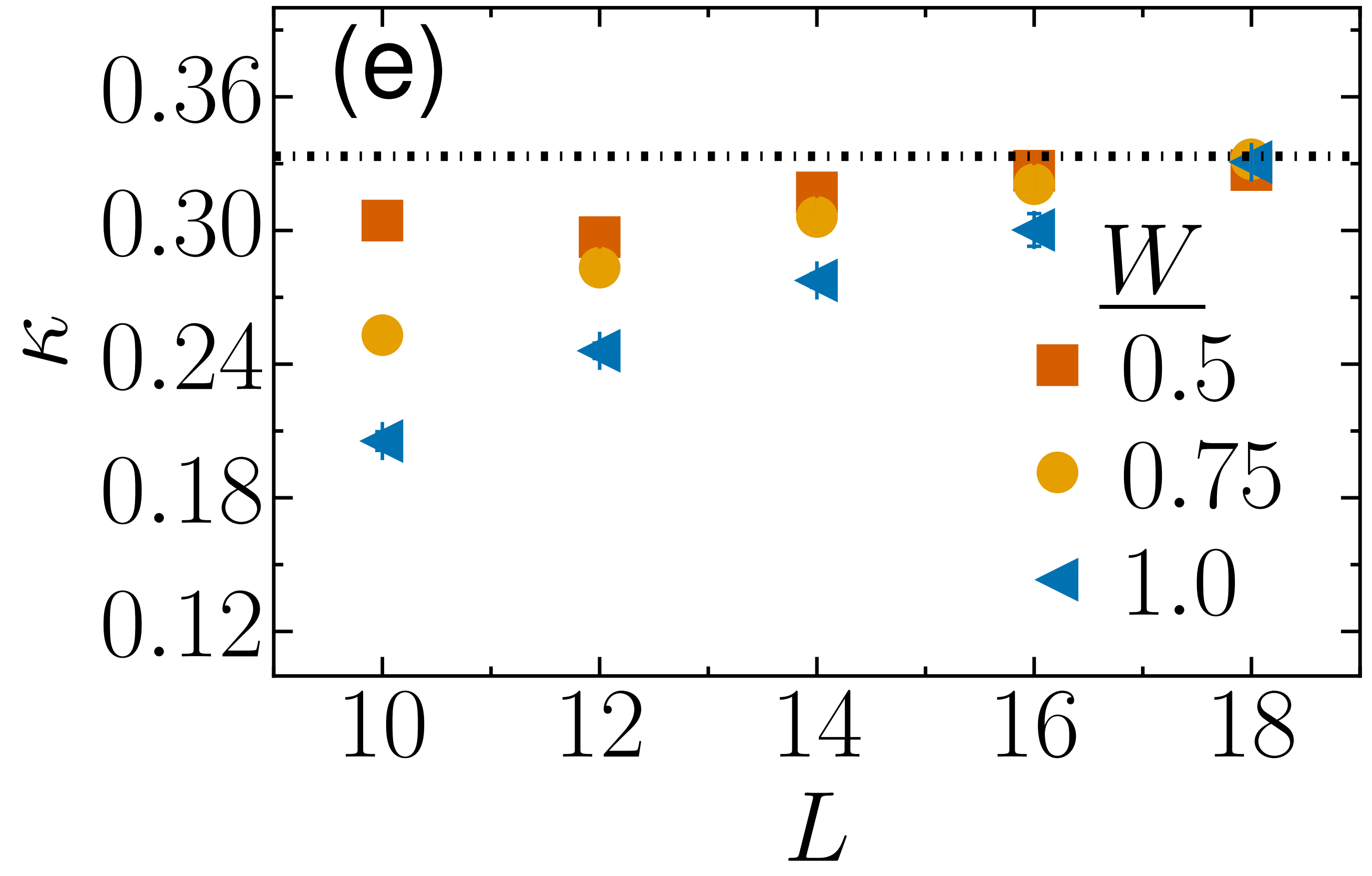
$$t_H \propto 1/\Delta \propto D \propto e^L$$

▼ Minimum of the correlation hole

Q: Is the correlation hole robust?

······· Random matrices

$$\kappa = \frac{\langle \overline{O}\rangle - \langle O\rangle_{\min}}{\langle \overline{O}\rangle}$$



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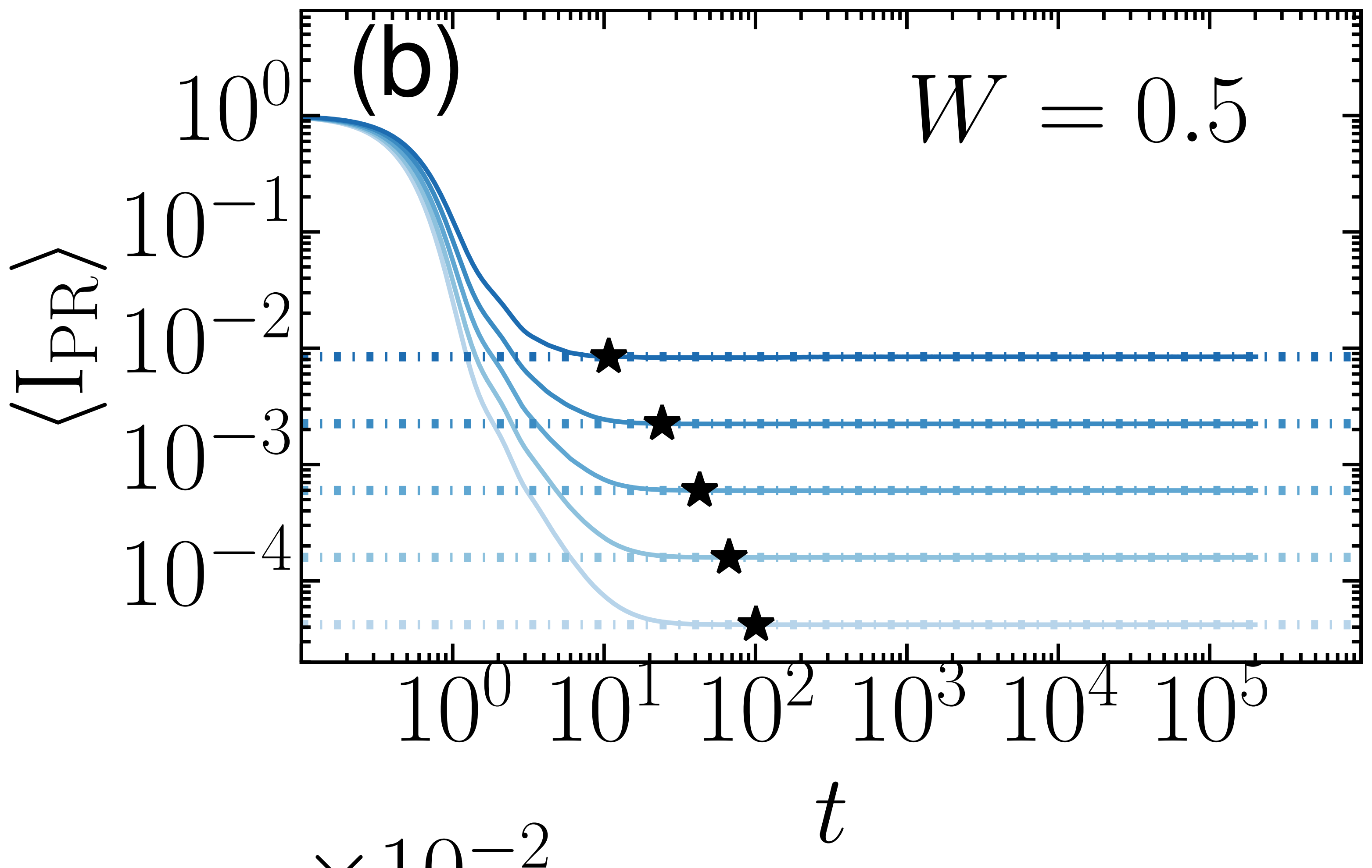
PTRSL A 375, 20160434 (2017)

PRB 99, 174313 (2019)

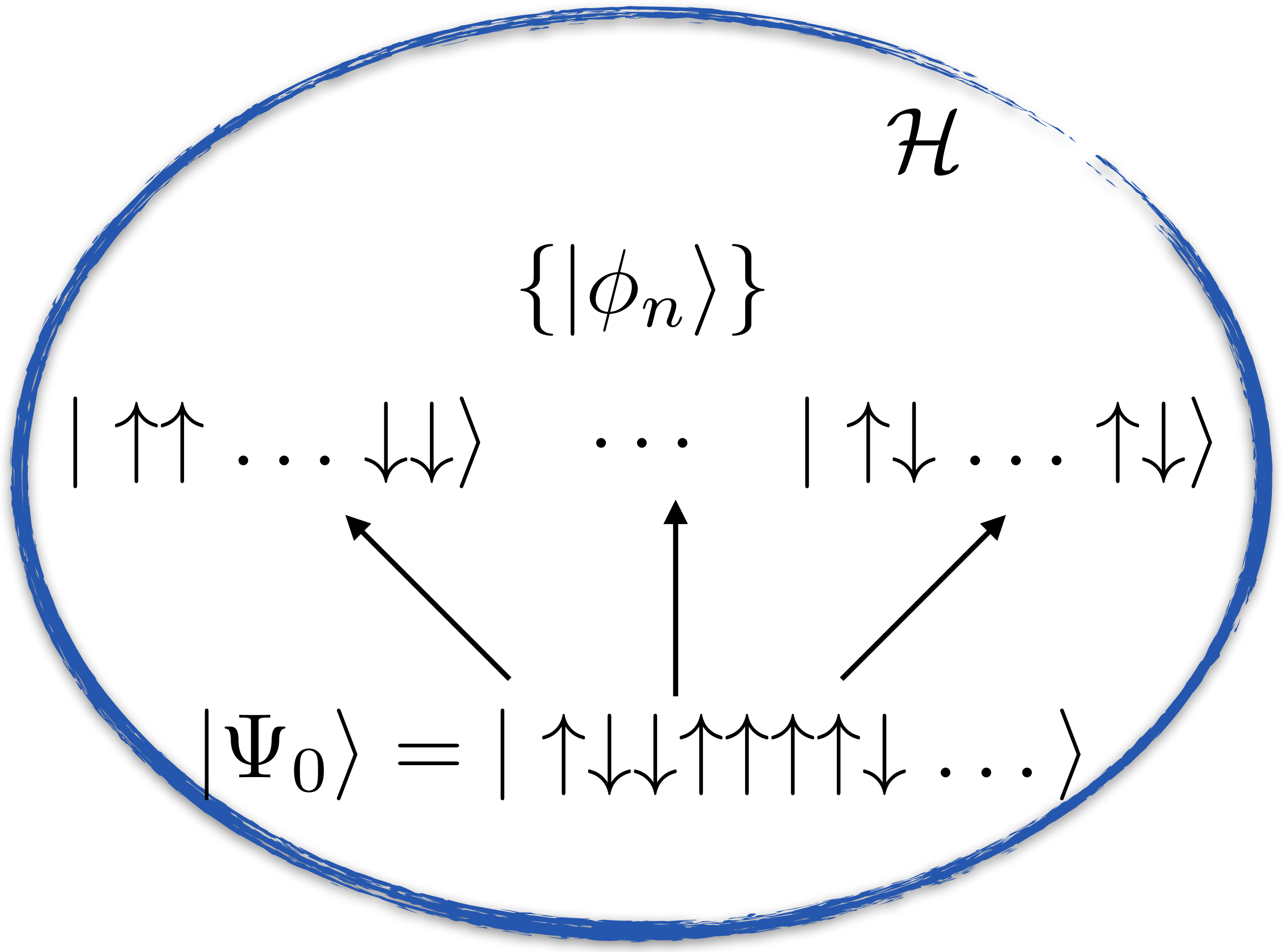
Correlation hole: manifestation of spectral correlations in the late-time dynamics

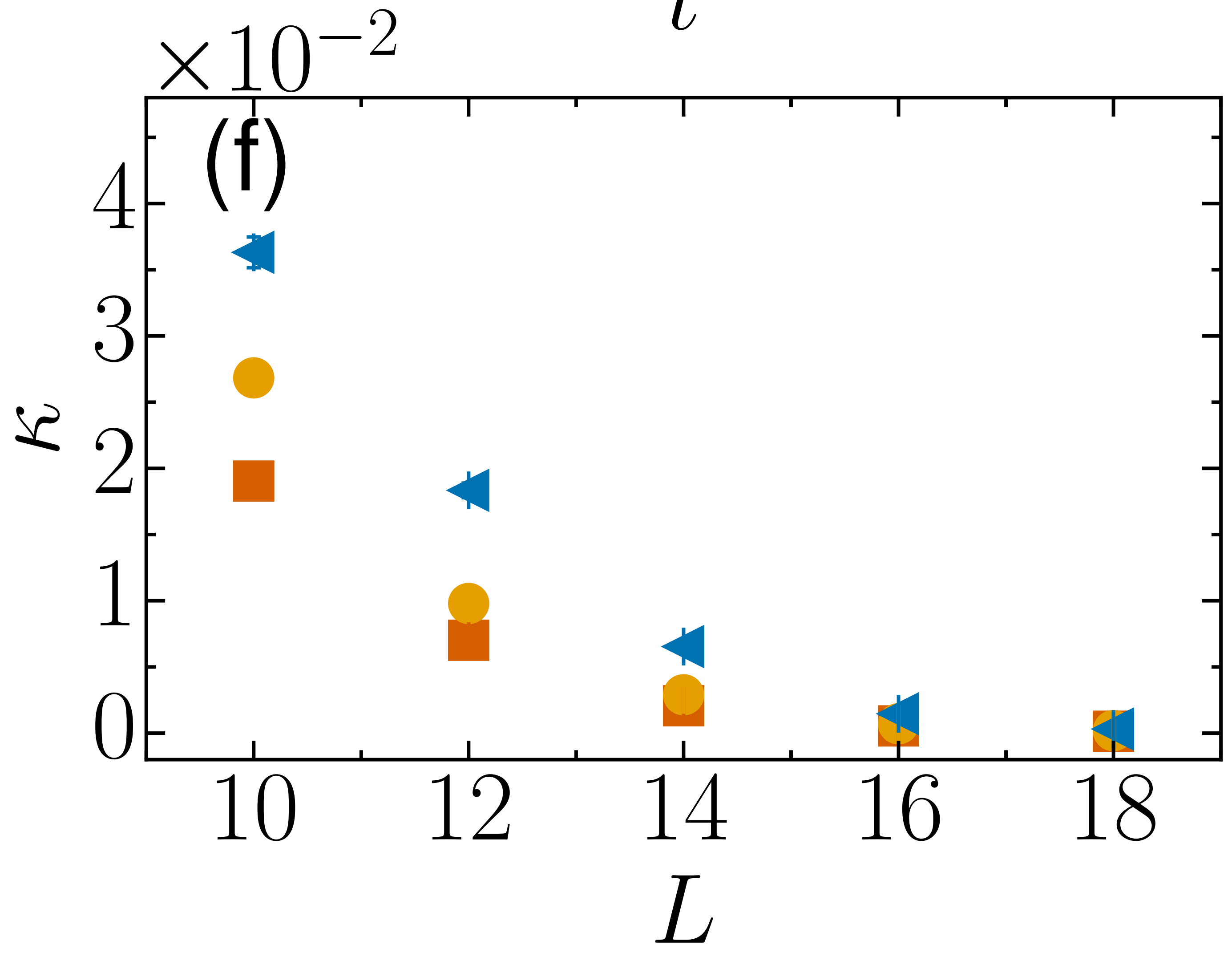
Inverse participation ratio

$$\mathrm{I_{PR}}(t) = \sum_n |\langle \phi_n | \Psi(t) \rangle|^4$$



Spreading of the initial state in Hilbert space





$$\kappa = \frac{\langle \overline{O} \rangle - \langle O \rangle_{\min}}{\langle \overline{O} \rangle}$$

The correlation hole is barely visible

Decays exponentially with system size

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Correlation hole: manifestation of spectral correlations in the late-time dynamics

What about few-body observables?

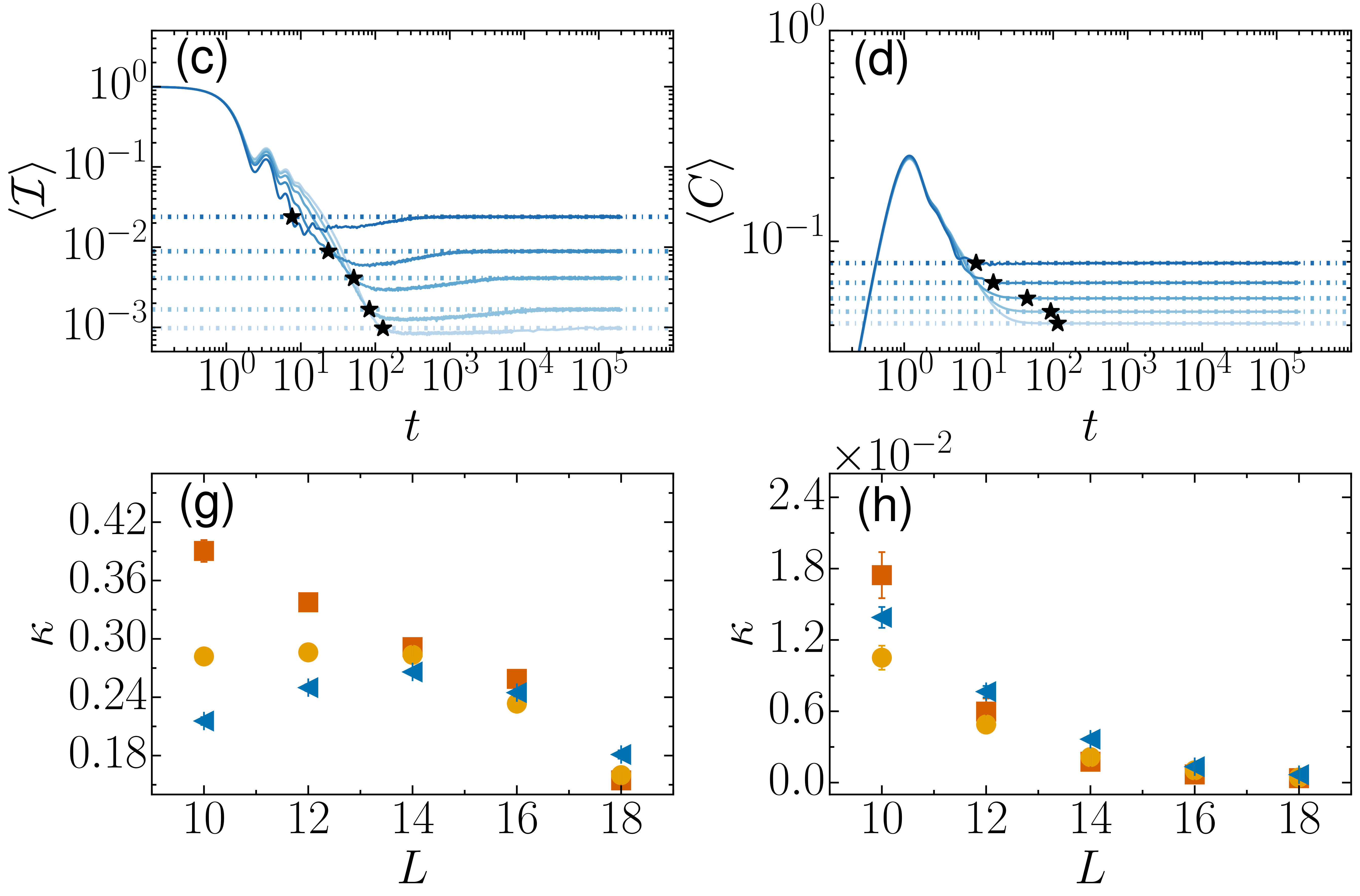
$$C(t) = \frac{4}{L}\sum_{i=1}\left[\left\langle \Psi(t)|\hat{S}_i^z \hat{S}_{i+1}^z|\Psi(t)\right\rangle - \langle \Psi(t)|\,\hat{S}_i^z\,|\Psi(t)\rangle \langle \Psi(t)|\,\hat{S}_{i+1}^z\,|\Psi(t)\rangle\right]$$

$$\mathcal{I}(t) = \frac{4}{L}\sum_{i=1}^{L} \langle \Psi_0|\,\hat{S}_i^z\,(0)\,\hat{S}_i^z\,(t)\,|\Psi_0\rangle$$

Transport correlators

The correlation hole vanishes in the thermodynamic limit

$$\kappa = \frac{\langle \overline{O}\rangle - \langle O\rangle_{\min}}{\langle \overline{O}\rangle}$$



Our results suggest that there might be a more appropriate definition of equilibration/thermalization time

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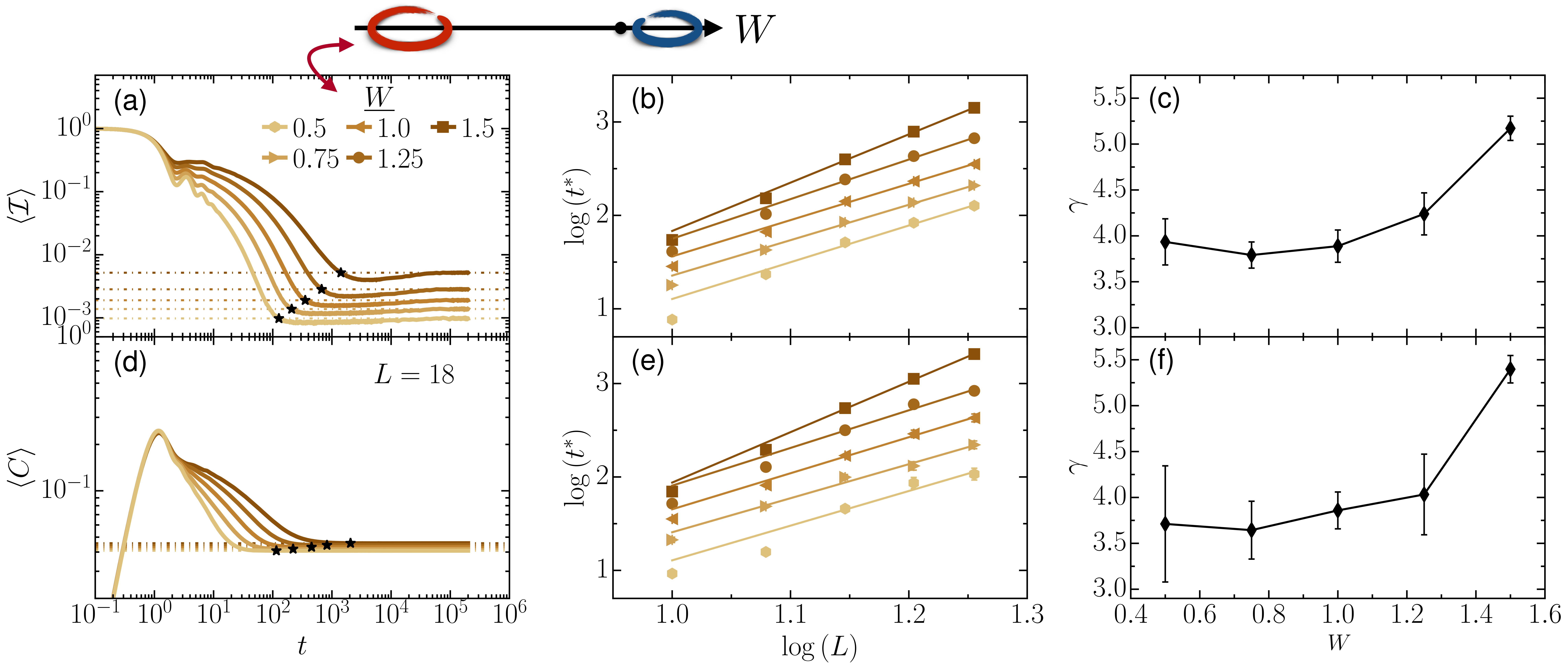
Equilibration time disregarding the correlation hole (few-body observables)



Scales polynomially with L

$$t^* \propto L^{\gamma}; \quad \gamma > 3$$

Parametrically larger than the diffusive scaling often assumed in studies of transport

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Conclusions and outlook:

- We studied the equilibration timescales in a many-body quantum system exhibiting the MBL transition, focusing in the chaotic (thermal) regime.
  - No assumptions, exact numerics.
  - Generic and experimentally feasible initial conditions.
  - Various observables, including few-body observables that are relevant for transport.
- We find that the correlation hole is only robust for the survival probability, while it is barely visible or vanishes in the thermodynamic limit for most observables, in particular, for the few-body observables.
  - Disregard the correlation hole and propose a new equilibration time.
  - Such equilibration time scales polynomially (rather than exponentially) with system system size.

    → More in agreement with transport properties of quantum thermalizing systems.
    But… parametrically away from the diffusive scaling.

OQ: Are our results general?
OQ: What is the phenomenology behind the observed scaling?

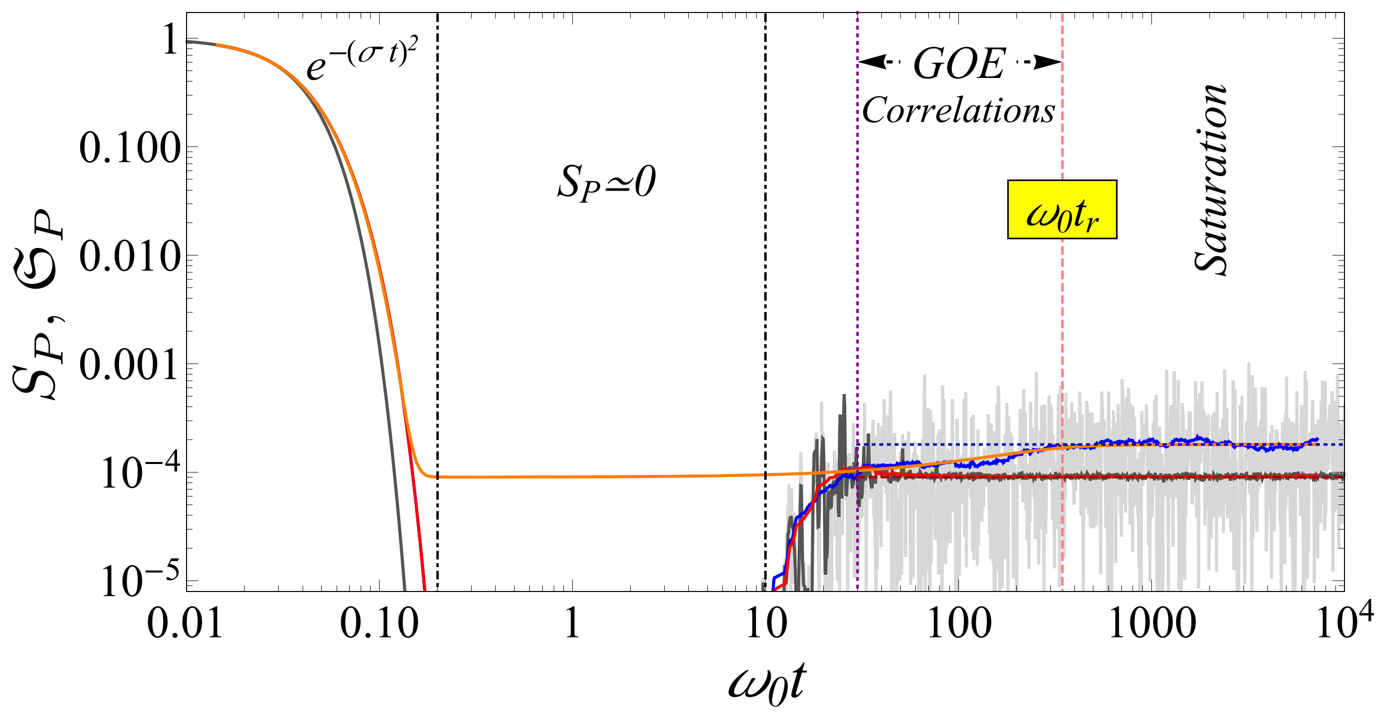
Outlook: Possible future directions and collaborations

- Disordered Bose-Hubbard model

$$H = \sum_{i=1}^{L} \left[-\frac{J}{2} \left(\hat{a}_i^\dagger \hat{a}_{i+1} + \text{h.c.} \right) + \frac{U}{2} \hat{n}_i \left(\hat{n}_i - 1 \right) + h_i \hat{n}_i \right] \qquad \hat{n}_i = \hat{a}_i^\dagger \hat{a}_i \qquad U > 0 \qquad h_i \in [-W, W]$$

- Models with semiclassical limit

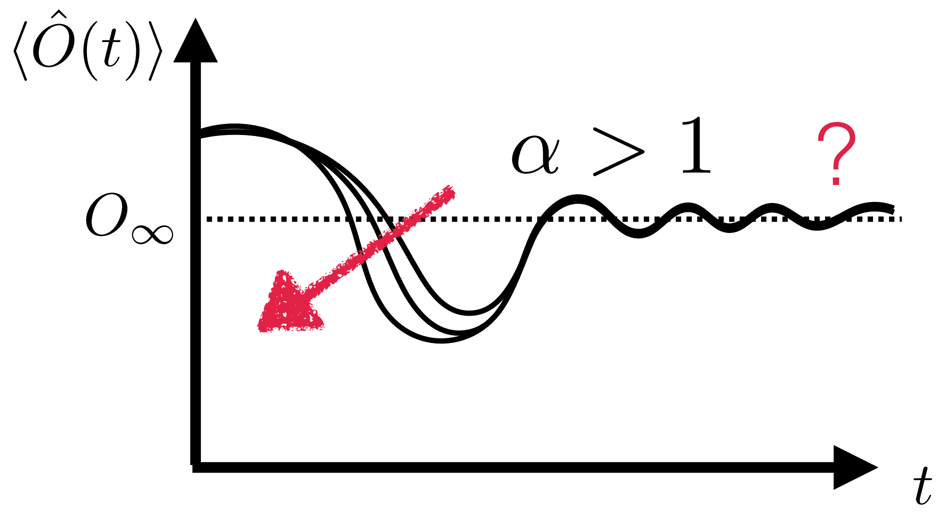
Quantum-classical correspondence



NJP 22, 063036 (2020)

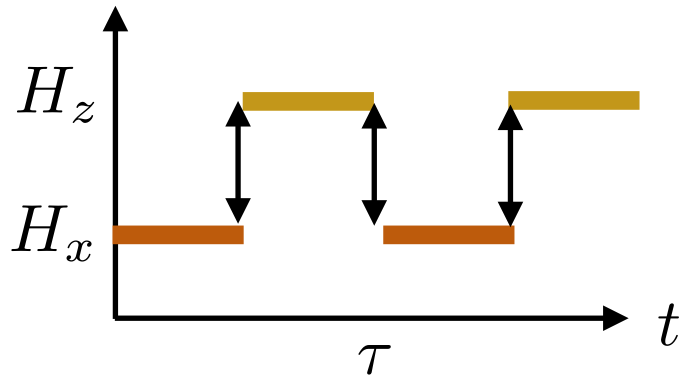
- Long-range models

$$H = B \sum_i \sigma_i^z + \sum_{i<j} \frac{J}{|i-j|^\alpha} \sigma_i^x \sigma_{i+1}^x$$



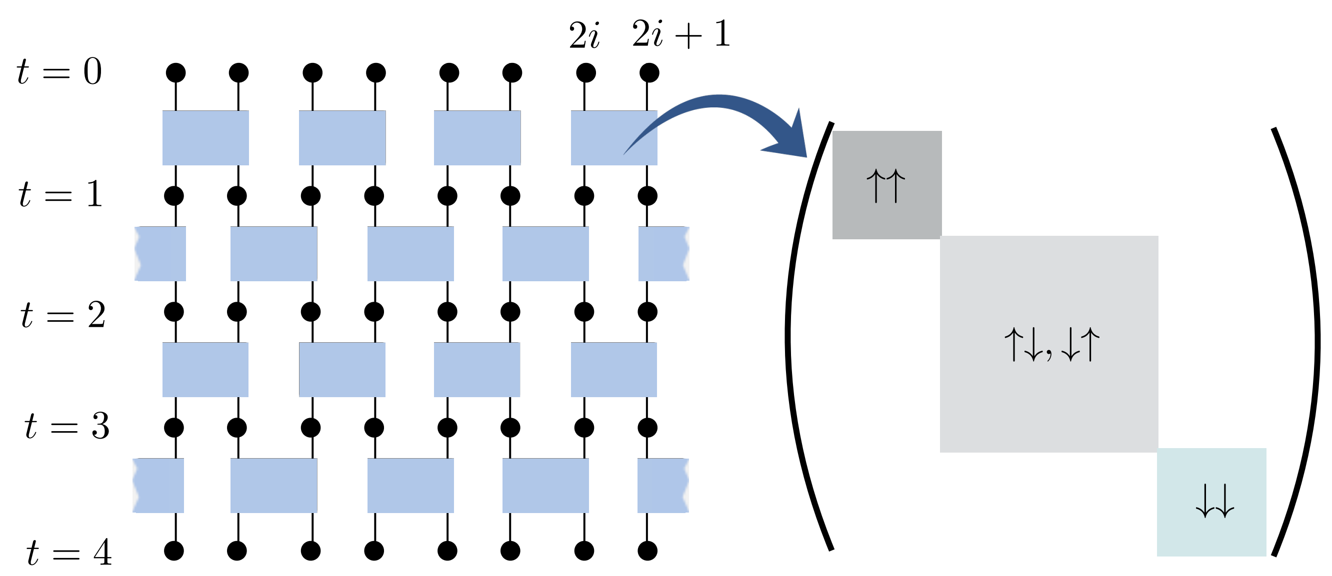
- Floquet systems and Random Unitary circuits

$$H(t) = H(t+\tau)$$



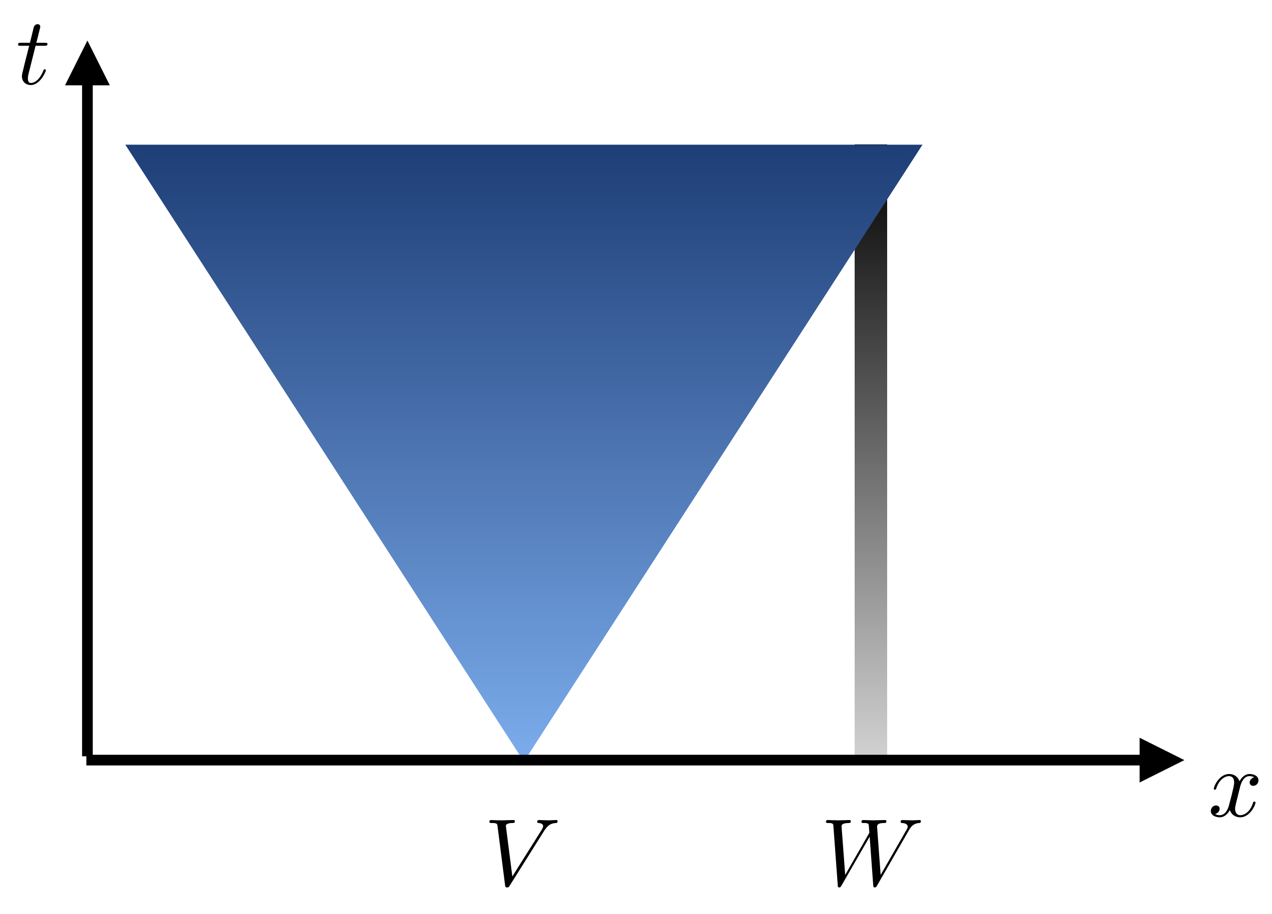
PRX 8, 031057 (2018)

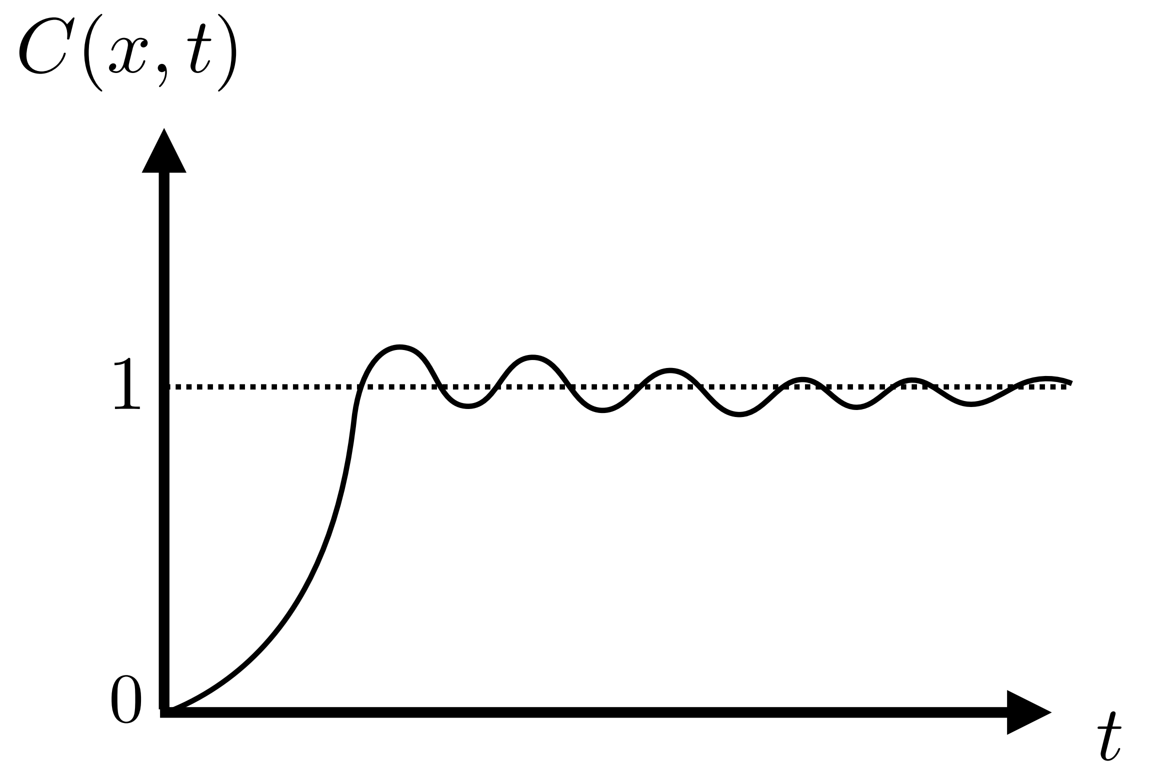
PRB 99, 161106 (2019)



Out-of-time ordered commutators (OTOCs)

$$C(x,t) \equiv \frac{1}{2}\langle [V(t),W]^{\dagger}[V(t),W]\rangle$$





- Much explored as detector of chaos.

  - Some works say it does detect chaos at long times (e.g. PRE 100, 042201 (2019))

  - We don't see that (to appear in arXiv soon)

Muchas gracias!