Relaxation of short spin chains: Implications on quantum chaos and control

Diego A. Wisniacki
UBA-Argentina

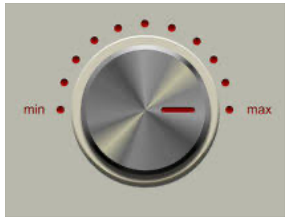
Let me start with some key ideas or topics we are interested in…

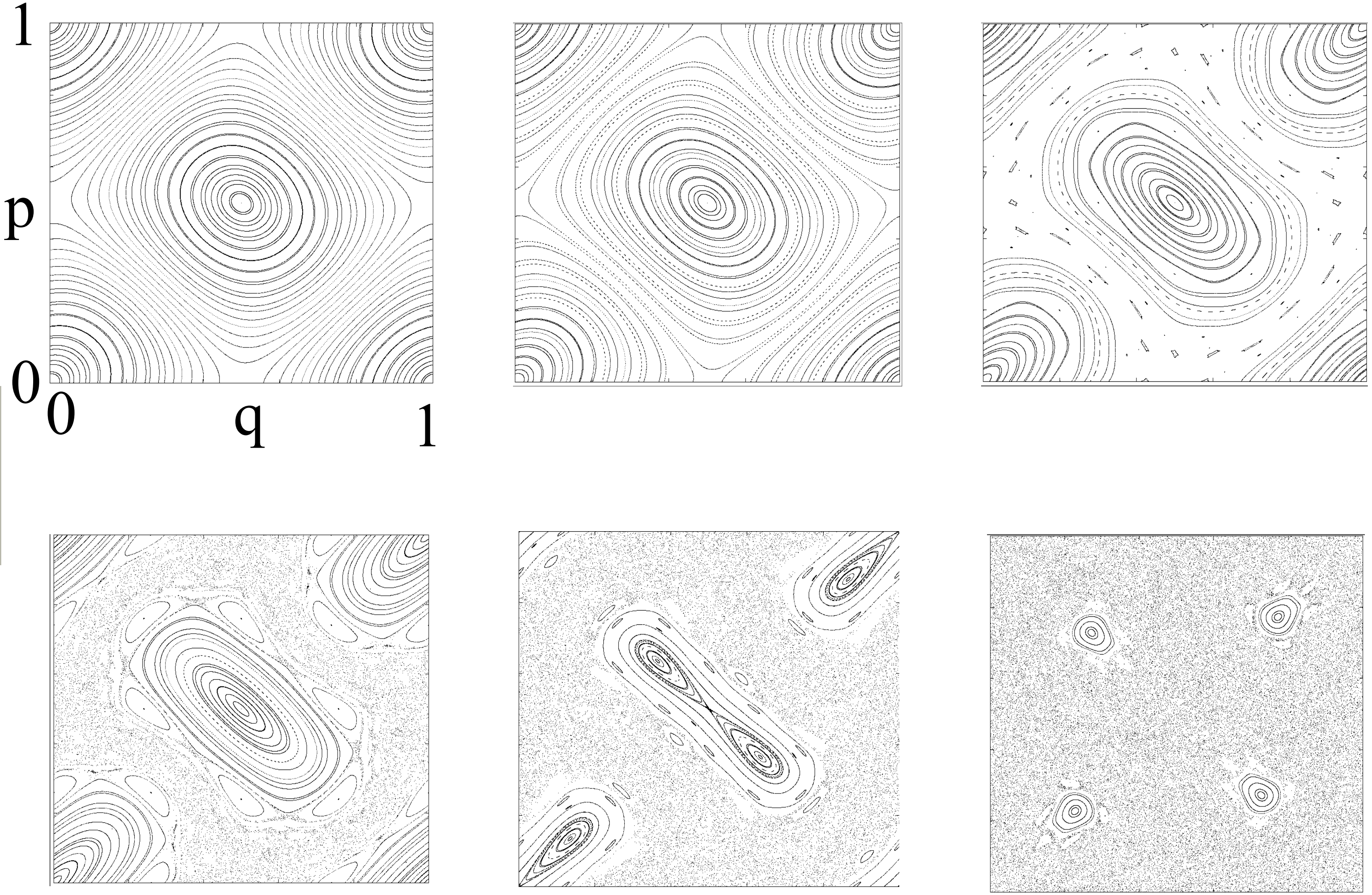
1

Integrability-to-chaos transition in quantum mechanics

Many-body systems without clear semiclassical limit

1





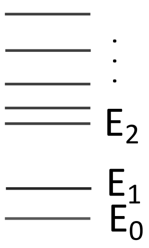
Integrability-to-chaos transition

1

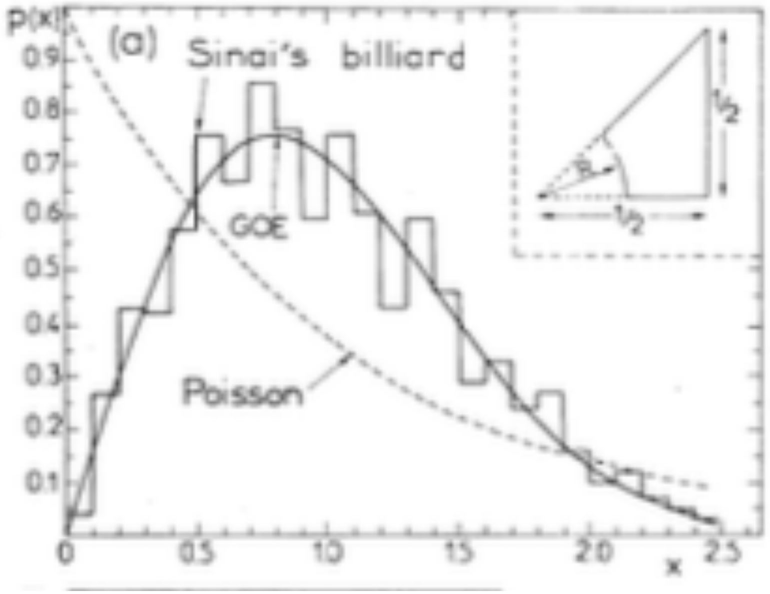
How can we gauge such a transition in quantum mechanics?

80’/90’ **Quantum chaos** or quantum manifestations of classical chaos

Distributions of eigenenergies: connection with RMT



Level spacings distributions



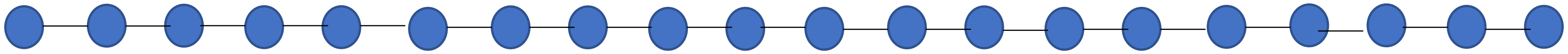
Nearest neighbour distribution

Bohigas et al 1982

1

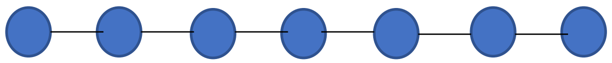
- How can we gauge such a transition in quantum mechanics?
- How can we connect spectral statistics with some dynamical properties? (mean value of some operator)

2



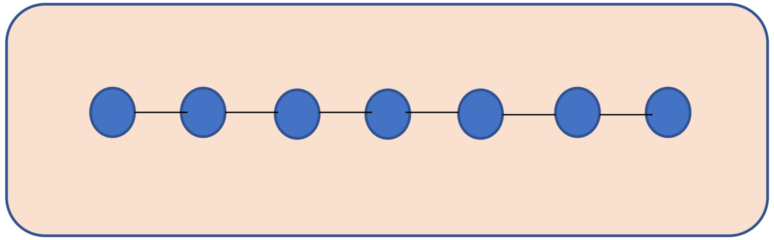
I want to know some quantum chaos measure of this chain

2



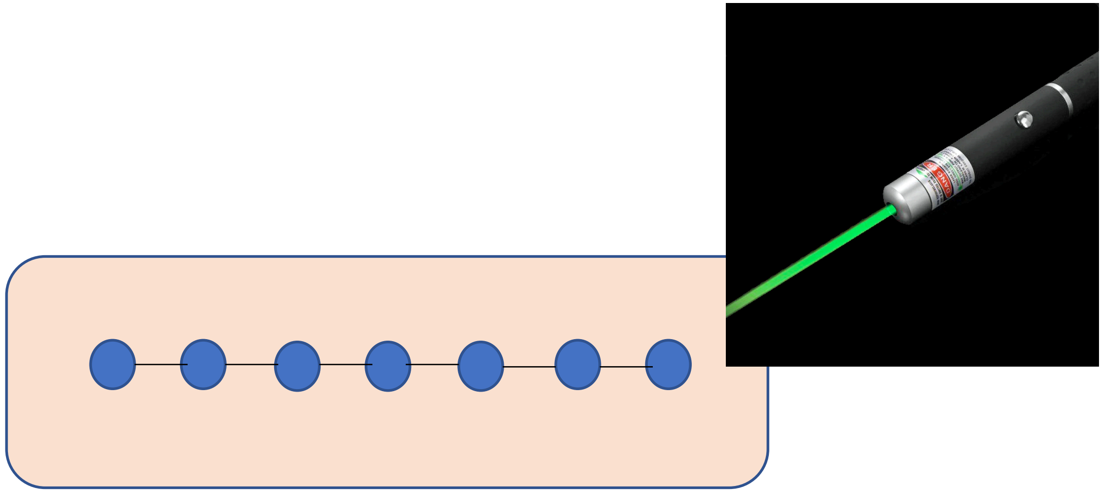
Can I learn something from this small system?

3



I want to control this system

3



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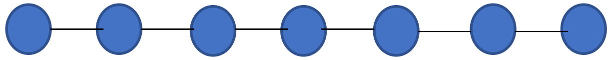
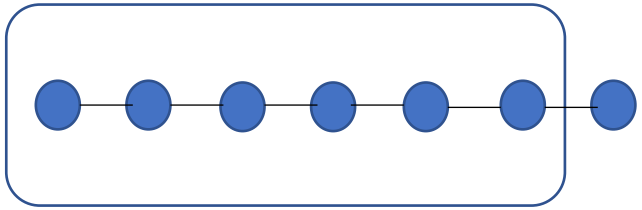


I am a good experimentalist so I can isolate my chain

3

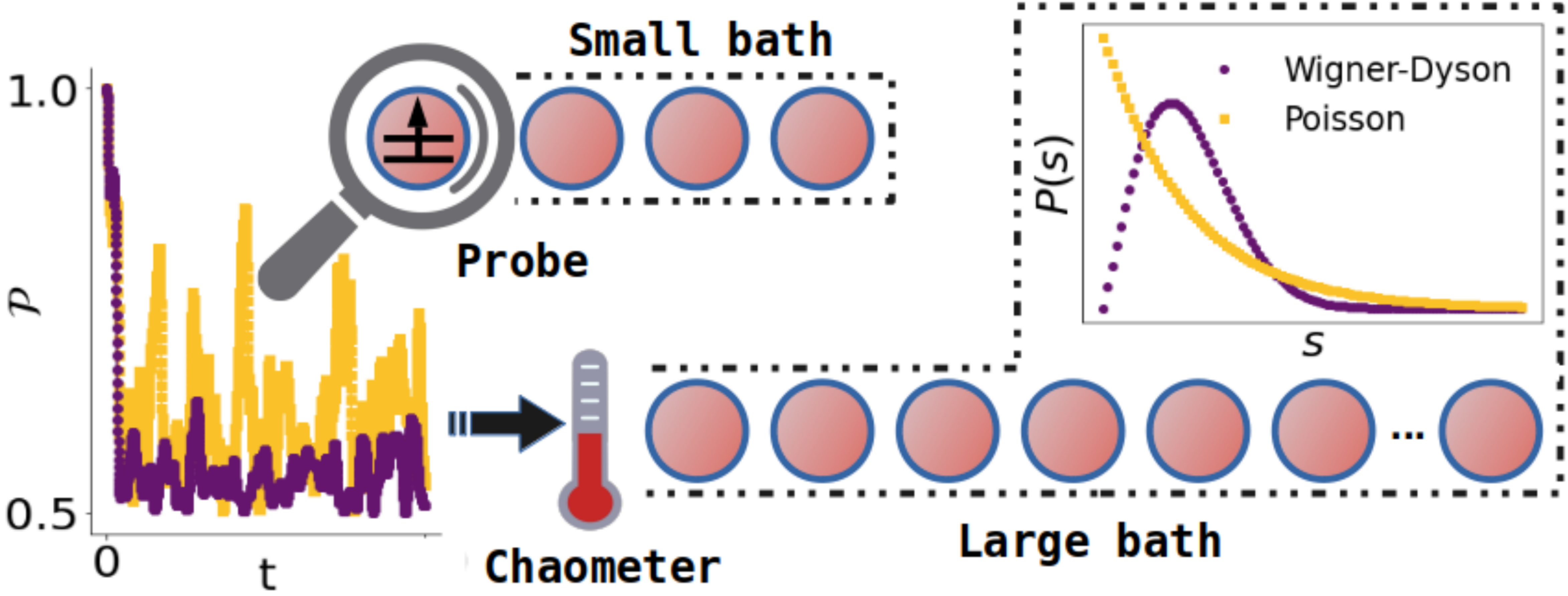
The environment of the system that I want to control is the remainder of the chain



Can we characterize such an ‘small’ environment?

What is the relation with the control?

Main idea of our work

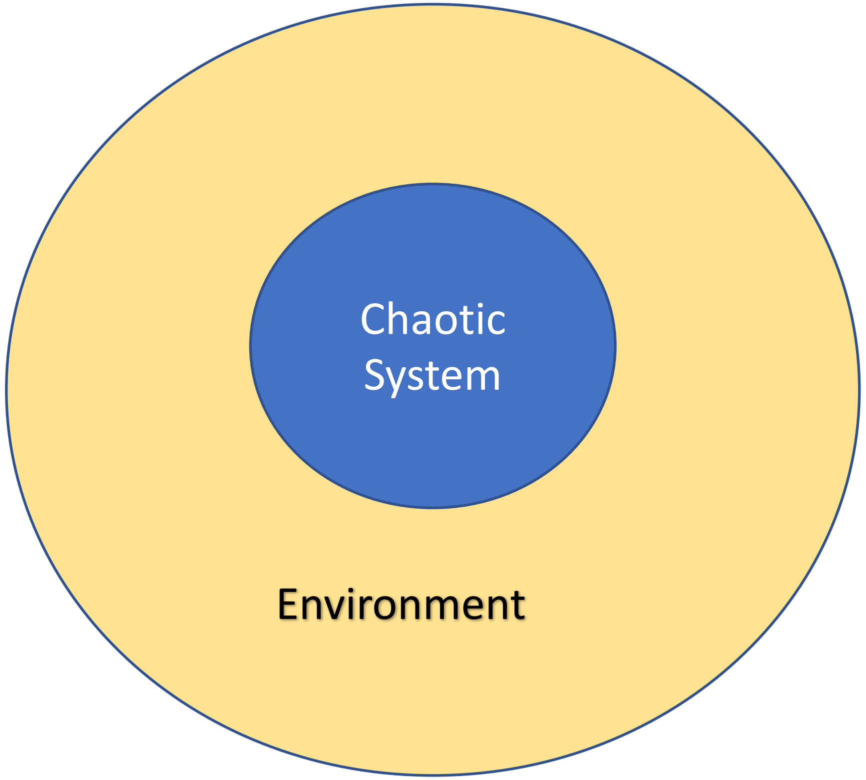


1.0
0.5
$\mathcal{P}$
0
t
Small bath
Probe
Chaometer
Large bath
Wigner-Dyson
Poisson
P(s)
s

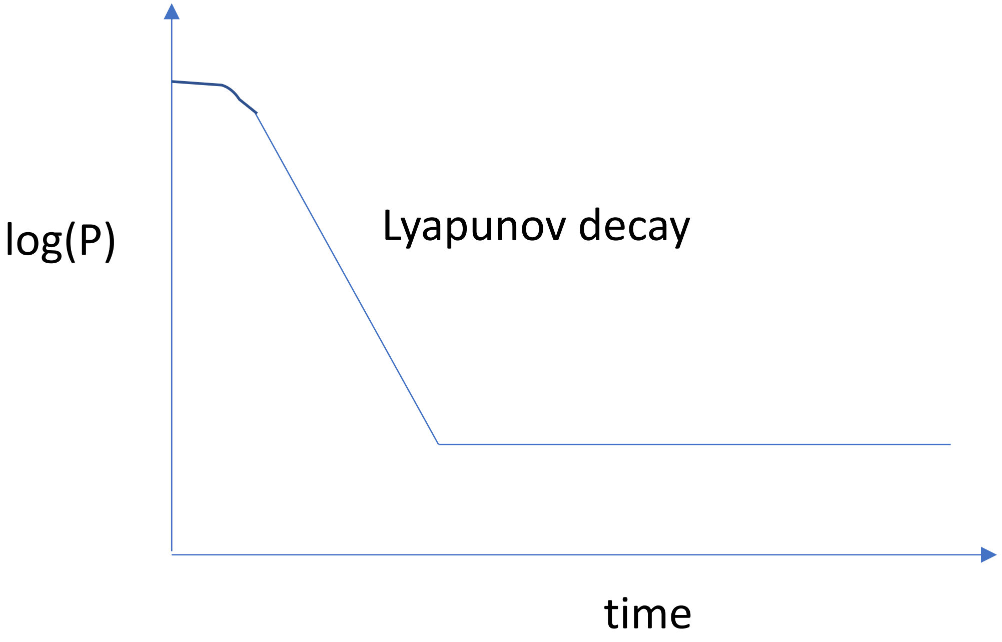
Long time of the purity (Quantum chaos meter and relation with control).

90’ Chaos and decoherence

Zurek, Paz, Pattanayak and others



- Short times
- Non-chaotic environments



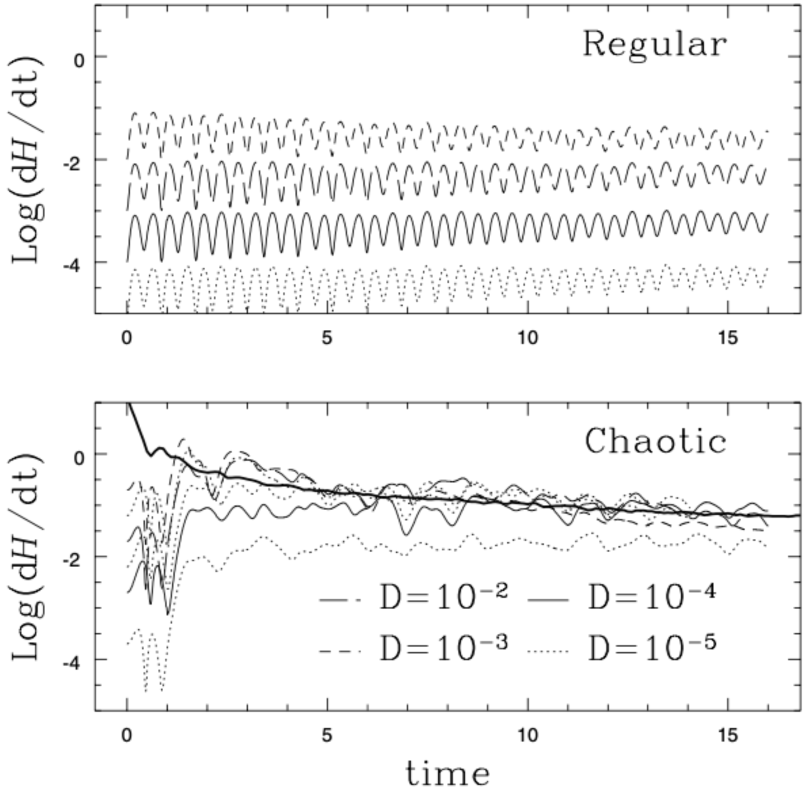
VOLUME 85, NUMBER 16 PHYSICAL REVIEW LETTERS 16 OCTOBER 2000

Decoherence and the Rate of Entropy Production in Chaotic Quantum Systems

Diana Monteoliva* and Juan Pablo Paz†
Departamento de Física "J.J. Giambiagi," FCEN, UBA, Pabellón 1, Ciudad Universitaria, 1428 Buenos Aires, Argentina
(Received 17 March 2000)

We show that for an open quantum system which is classically chaotic (a quartic double well with harmonic driving coupled to a sea of harmonic oscillators) the rate of entropy production has, as a function of time, two relevant regimes: For short times it is proportional to the diffusion coefficient (fixed by the system-environment coupling strength). For longer times (but before equilibration) there is a regime where the entropy production rate is fixed by the Lyapunov exponent. The nature of the transition time between both regimes is investigated.

PACS numbers: 05.45.Mt, 03.65.Bz, 03.65.Sq



Questions:

- Can small environment tell something about the case of big environment?
- What is the effect of this small environment to the system?
- What is the relation with control?

$$H = \sum_{k=1}^{L} \left(h_x \hat{\sigma}_k^x + h_z \hat{\sigma}_k^z \right) - \sum_{k=1}^{L-1} J_k \hat{\sigma}_k^z \hat{\sigma}_{k+1}^z$$

Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW X 7, 031011 (2017)

Measuring Out-of-Time-Order Correlators on a Nuclear Magnetic Resonance Quantum Simulator

Jun Li,[1] Ruihua Fan,[2,3] Hengyan Wang,[3] Bingtian Ye,[3] Bei Zeng,[4,5,2,*] Hui Zhai,[2,6,†] Xinhua Peng,[7,8,9,‡] and Jiangfeng Du[7,8]
[1]*Beijing Computational Science Research Center, Beijing 100193, China*
[2]*Institute for Advanced Study, Tsinghua University, Beijing 100084, China*
[3]*Department of Physics, Peking University, Beijing 100871, China*
[4]*Department of Mathematics and Statistics, University of Guelph, Guelph N1G 2W1, Ontario, Canada*
[5]*Institute for Quantum Computing, University of Waterloo, Waterloo N2L 3G1, Ontario, Canada*
[6]*Collaborative Innovation Center of Quantum Matter, Beijing 100084, China*
[7]*Hefei National Laboratory for Physical Sciences at Microscale and Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China*
[8]*Synergetic Innovation Centre of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei, Anhui 230026, China*
[9]*College of Physics and Electronic Science, Hubei Normal University, Huangshi, Hubei 435002, China*
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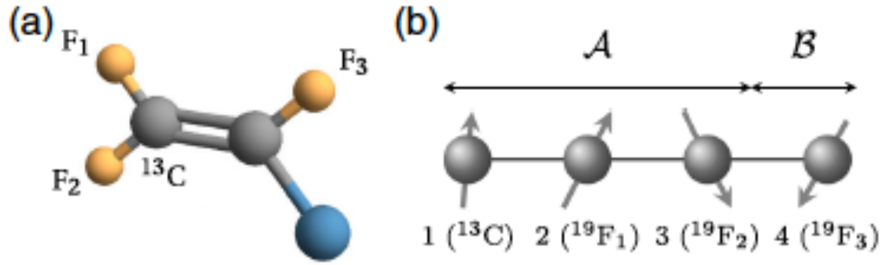
Subject Areas: Quantum Physics, Quantum Information, Statistical Physics



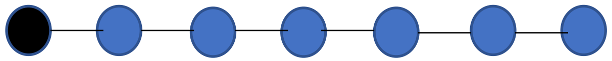
$$|\psi(0)\rangle = |\psi_1\rangle|\psi_2\rangle...|\psi_L\rangle$$

$$|\psi_k\rangle = \cos\left(\frac{\theta_k}{2}\right)|\uparrow\rangle + e^{i\phi_k}\sin\left(\frac{\theta_k}{2}\right)|\downarrow\rangle$$

$$\theta_k \in [0, \pi)$$

$$\phi_k \in [0, 2\pi)$$

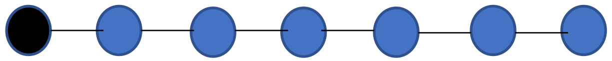
random direction
in Bloch sphere

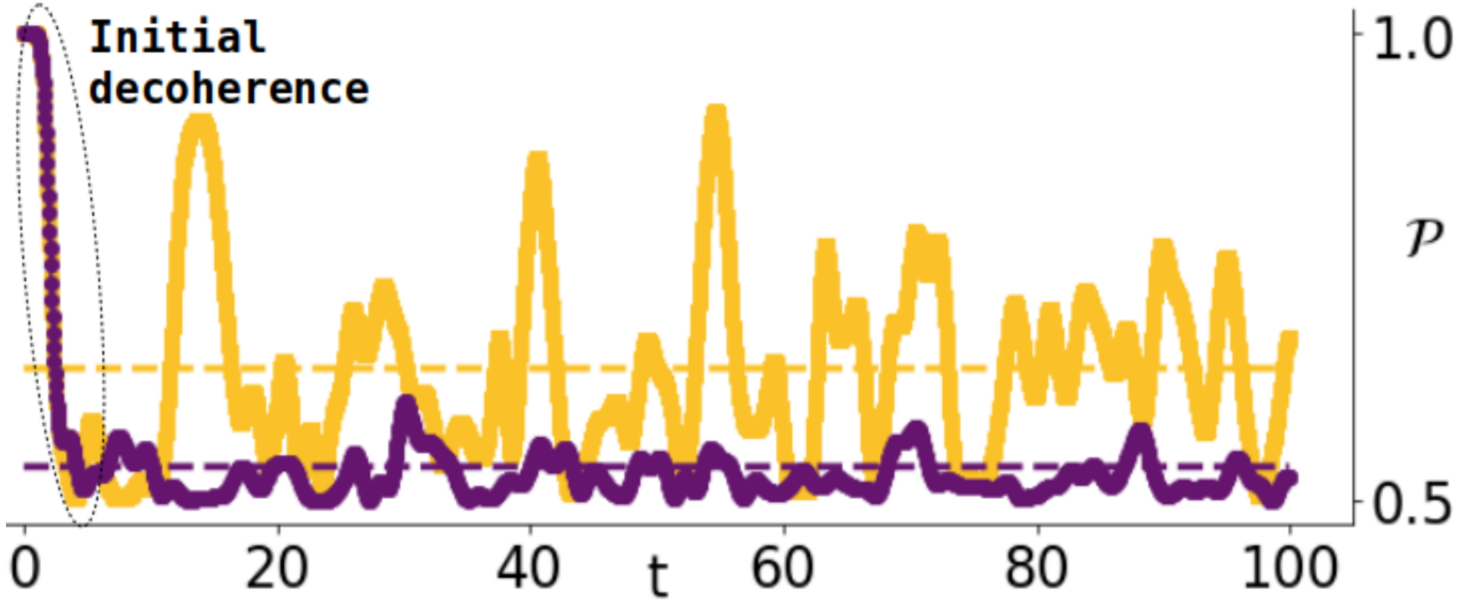


$$\mathcal{P}(t) = Tr[\tilde{\rho}^2(t)]$$

reduced density matrix

$$\overline{\mathcal{P}} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{T}\int_0^T Tr[\tilde{\rho}_i^2(t)]dt\right)$$

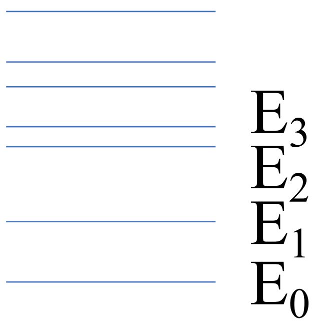


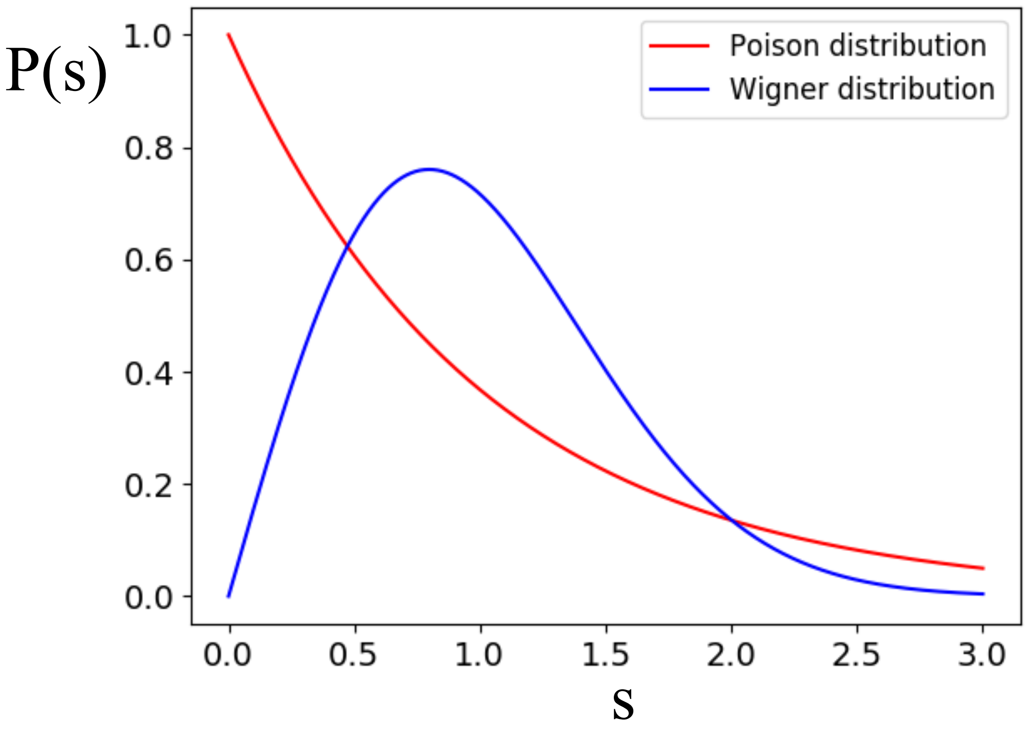


$$\overline{\mathcal{P}}_{Norm} = \frac{\overline{\mathcal{P}} - \min\left(\overline{\mathcal{P}}\right)}{\max\left(\overline{\mathcal{P}}\right) - \min\left(\overline{\mathcal{P}}\right)} \qquad \left(0 \leq \overline{\mathcal{P}}_{Norm} \leq 1\right)$$

Nearest neighbor distribution

$s_n = E_{n+1} - E_n$





r distribution:

$s_n = E_{n+1} - E_n$

$s_{n+1} = E_{n+2} - E_{n+1}$

$r_n = s_{n+1}/s_n$ → $min(1/r_n, r_n)$

being the ratio between the two nearest neighbor

*) The distribution of min(1/*r*, *r)* has the advantage of not requiring an energy unfolding

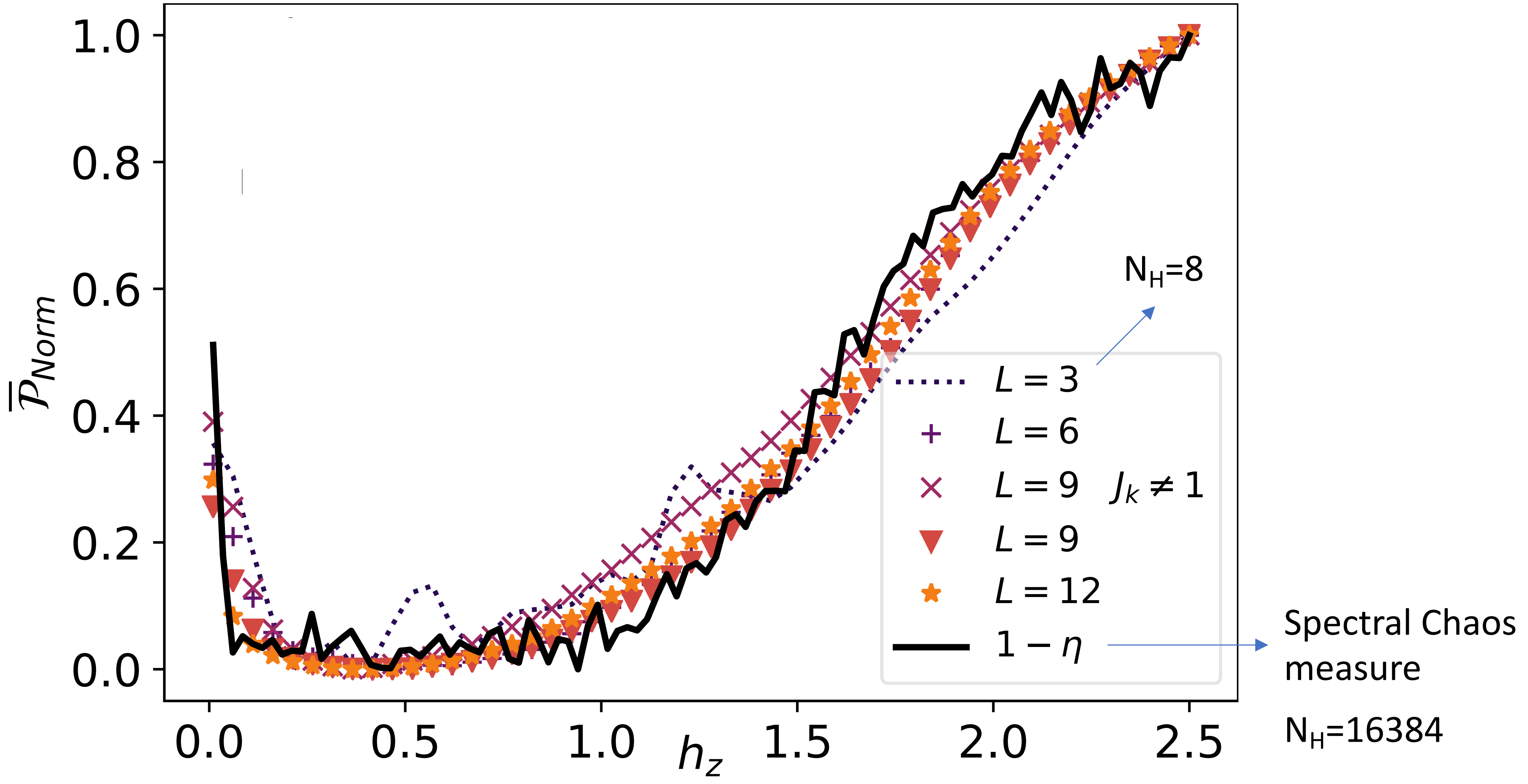
**)The mean value min(1/*r*,*r*) attains a minimum $I_P \approx 0.386$ when the statistics is Poissonian and $I_{WD} \approx 0.586$ when is Wigner-Dyson. Therefore, we can define the quantity:

$$\eta \equiv \frac{\overline{\min(1/r, r)} - I_P}{I_{WD} - I_P}$$

→

$\eta = 1$ chaotic

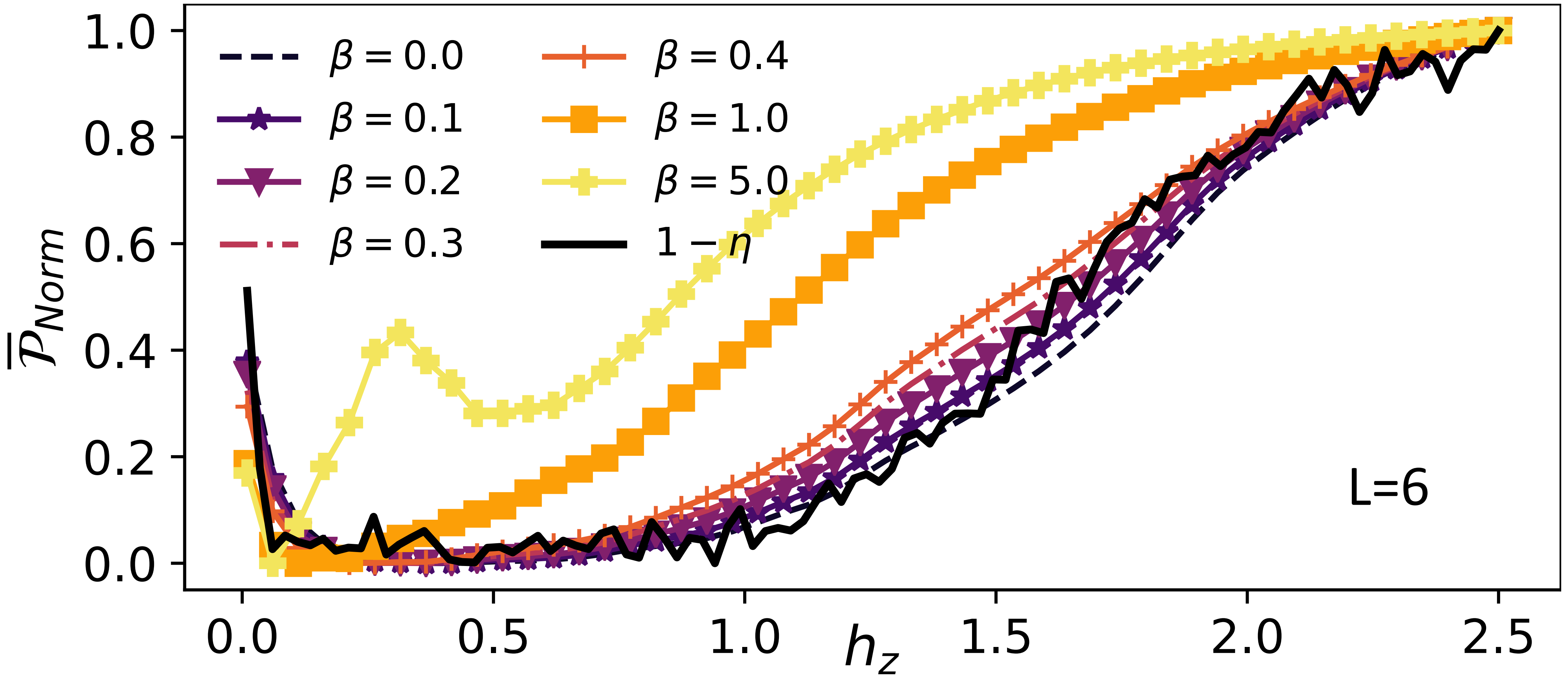
$\eta = 0$ Integrable



for η L= 14 spins (D= 16384) was selected and only the odd subspace was taken into account (D_{even}≈8192)

Important: no symmetry considerations were taken into account for the analysis of the purity!!!

What is the dependence of the results with the temperature?

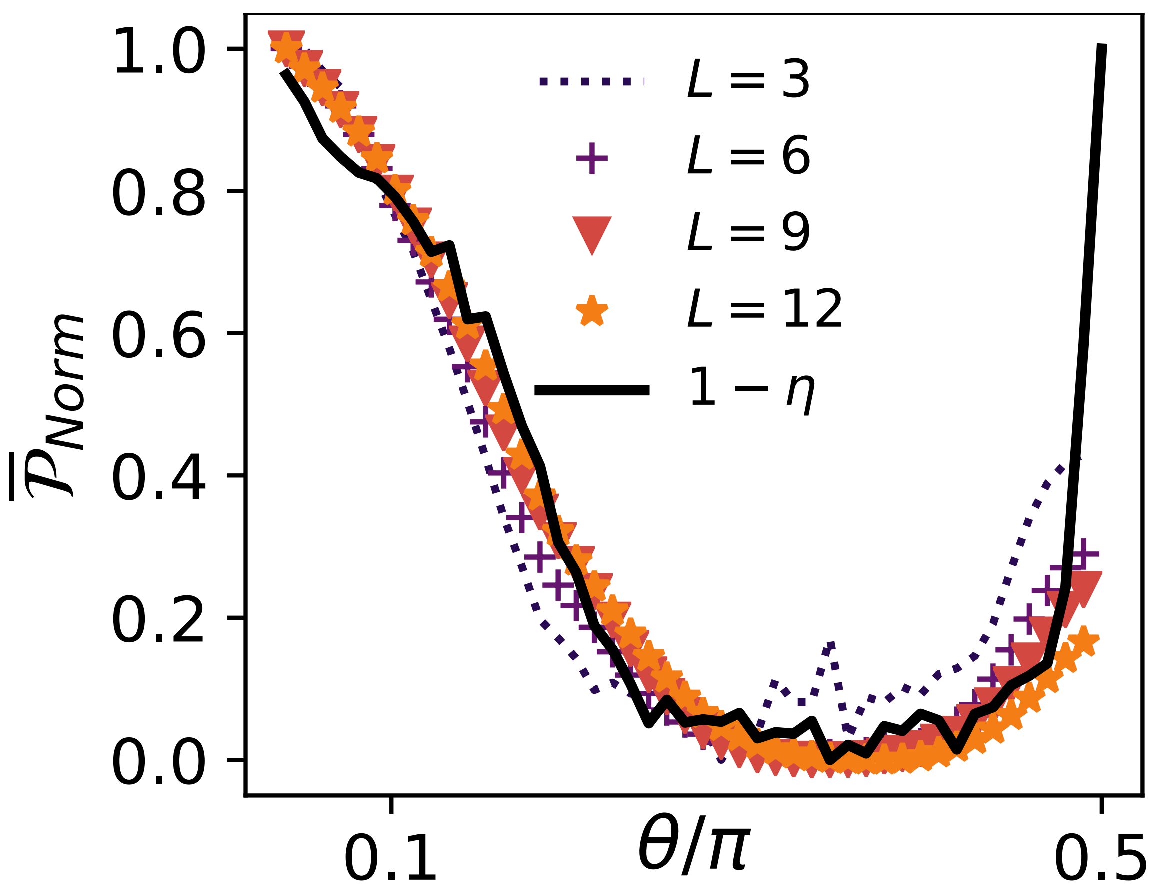


$\beta = 0.0$
$\beta = 0.1$
$\beta = 0.2$
$\beta = 0.3$
$\beta = 0.4$
$\beta = 1.0$
$\beta = 5.0$
$1 - \eta$
L=6
$\overline{\mathcal{P}}_{Norm}$
h_z
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0
2.5

Universality?

Ising with tilted magnetic field

$$H(\theta) = B \sum_{k=1}^{L} \left(\sin\theta \hat{S}_k^x + \cos\theta \hat{S}_k^z \right) + J \sum_{k=1}^{L-1} \hat{S}_k^z \hat{S}_{k+1}^z$$

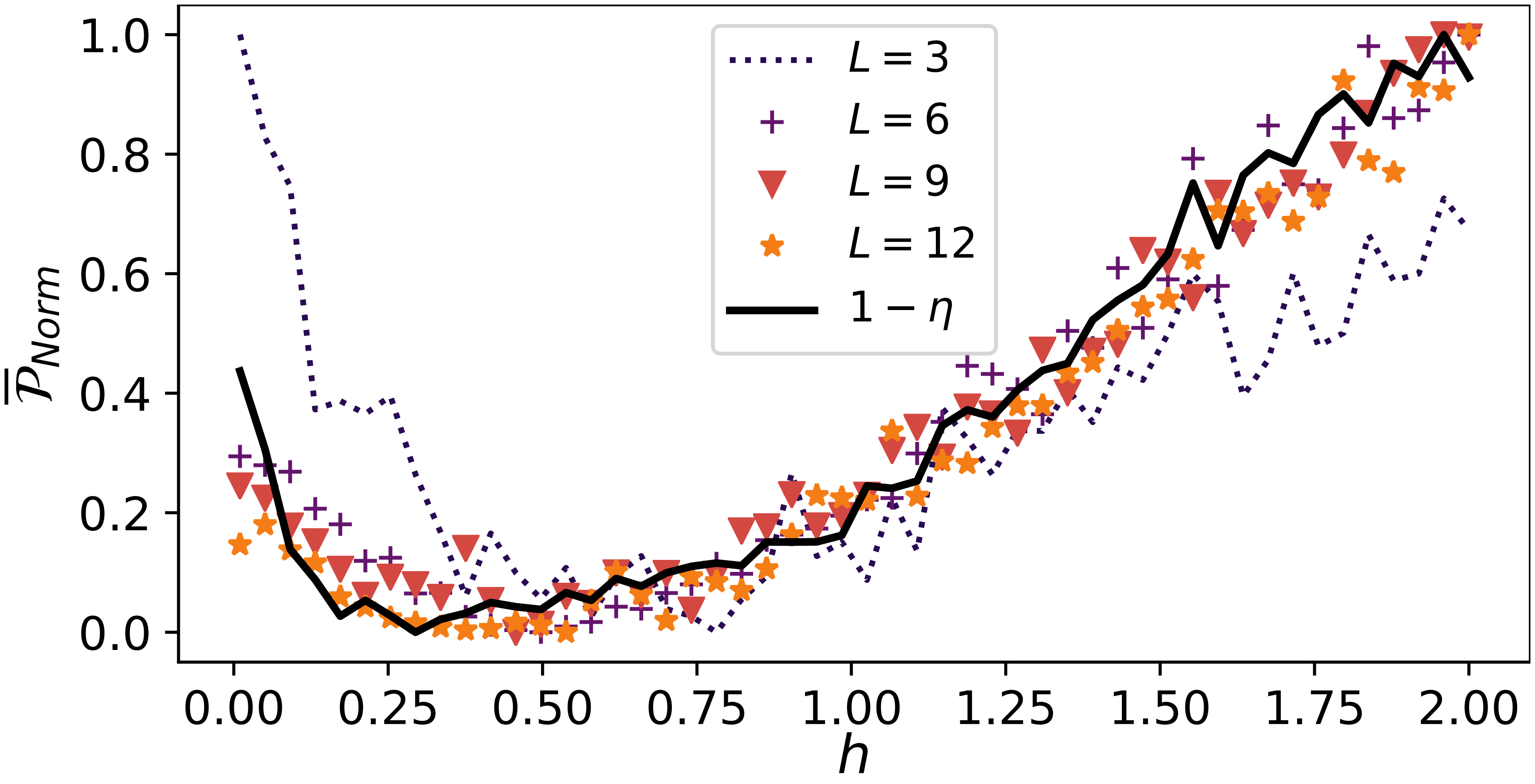


1.0
0.8
0.6
0.4
0.2
0.0
$\overline{\mathcal{P}}_{Norm}$
$L = 3$
$L = 6$
$L = 9$
$L = 12$
$1 - \eta$
0.1
θ/π
0.5

Heisenberg with random magnetic field

$$H = \sum_{k=1}^{L-1} \left(\hat{S}_k^x \hat{S}_{k+1}^x + \hat{S}_k^y \hat{S}_{k+1}^y + \hat{S}_k^z \hat{S}_{k+1}^z \right) + \sum_{k=1}^{L} h_k^z \hat{S}_k^z$$

h_k^z is a set of random variables at each site, uniformly distributed in the interval [-h,h]


$L = 3$
$L = 6$
$L = 9$
$L = 12$
$1 - \eta$
$\overline{\mathcal{P}}_{Norm}$
h
0.0
0.2
0.4
0.6
0.8
1.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

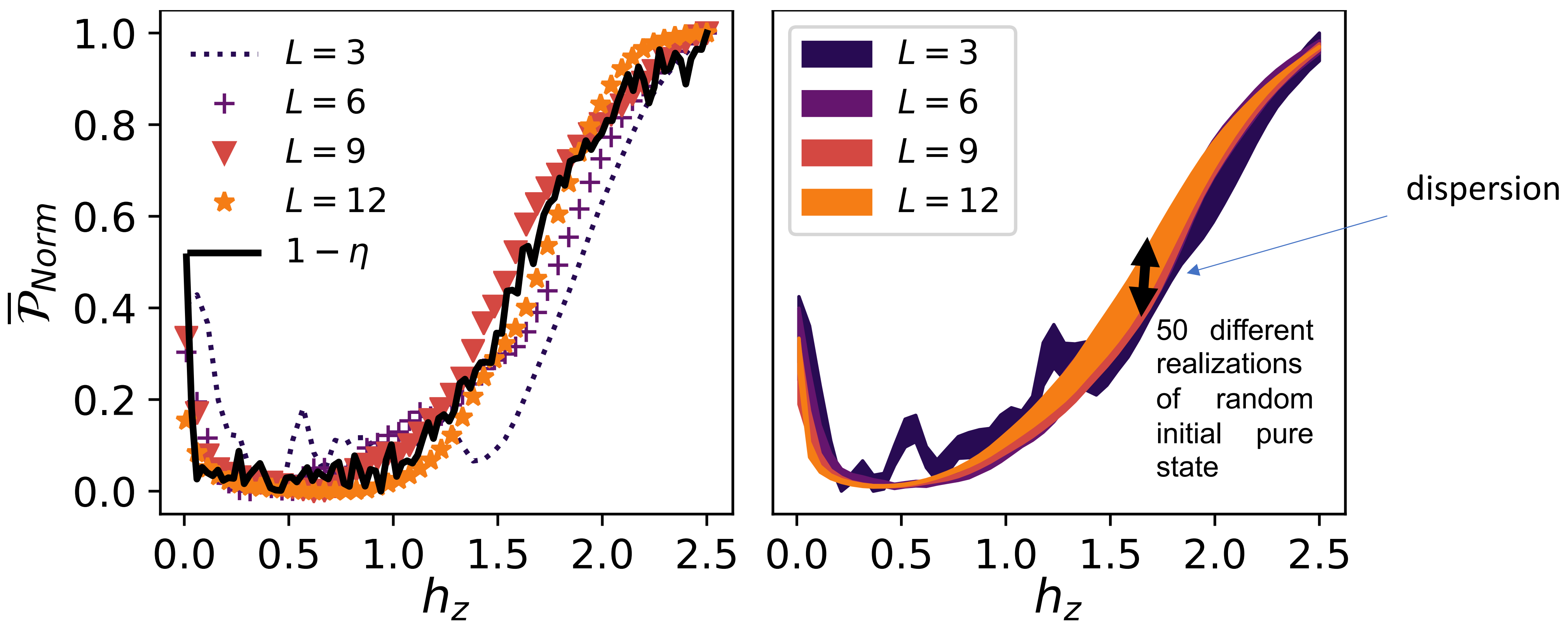
Perturbed XXZ model

$$H(\lambda) = H_0 + \lambda H_1$$

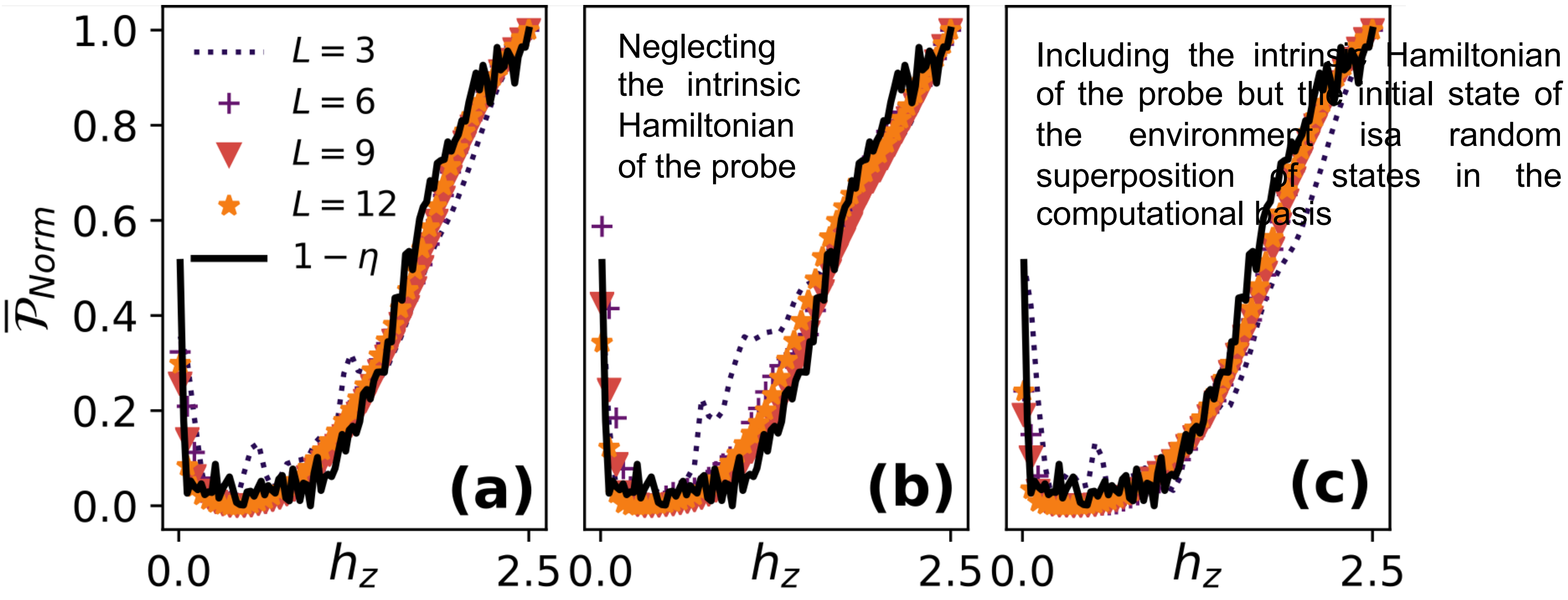
$$H_0 = \sum_{k=1}^{L-1} \left(\hat{S}_k^x \hat{S}_{k+1}^x + \hat{S}_k^y \hat{S}_{k+1}^y + \mu \hat{S}_k^z \hat{S}_{k+1}^z \right)$$

$$H_1 = \sum_{k=1}^{L-2} \left(\hat{S}_k^x \hat{S}_{k+2}^x + \hat{S}_k^y \hat{S}_{k+2}^y + \mu \hat{S}_k^z \hat{S}_{k+2}^z \right)$$

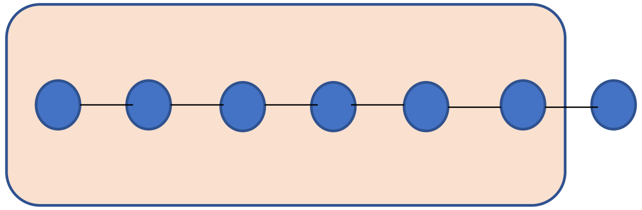
Average over different realizations vs a single realization



Different sensing setups



Control



external control

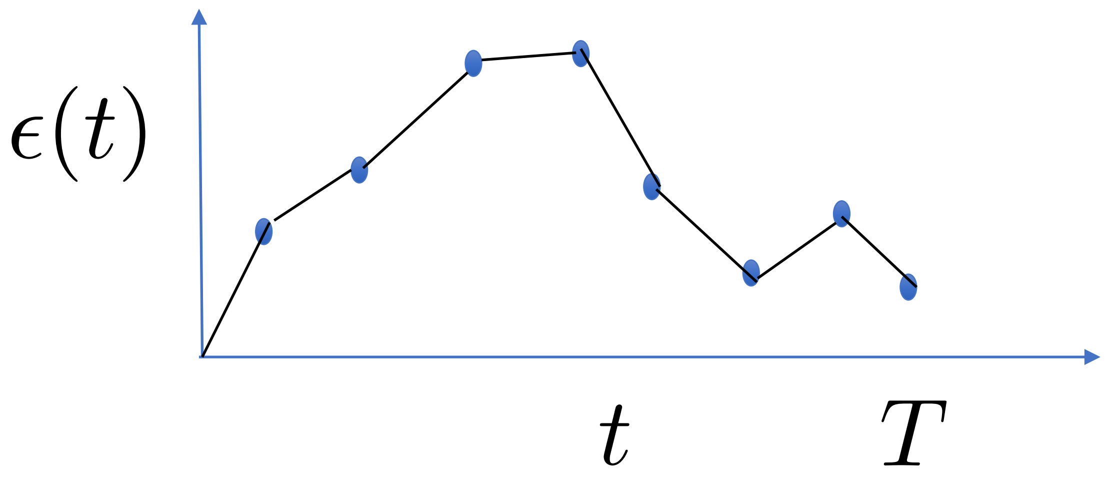
$$H = \sum_{k=1}^{L} (h_x \hat{\sigma}_k^x + h_z \hat{\sigma}_k^z) - \sum_{k=1}^{L-1} J_k \hat{\sigma}_k^z \hat{\sigma}_{k+1}^z + \epsilon(t) \hat{\sigma}_1^z$$

Protocols:

1) $|\psi(0)\rangle = |0\rangle \longrightarrow |\psi_{targ}\rangle = |1\rangle$

2) $|\psi(0)\rangle = |00\rangle \rightarrow |\psi_{targ}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

The initial state is random for each spin of the system and only one realization was considered.

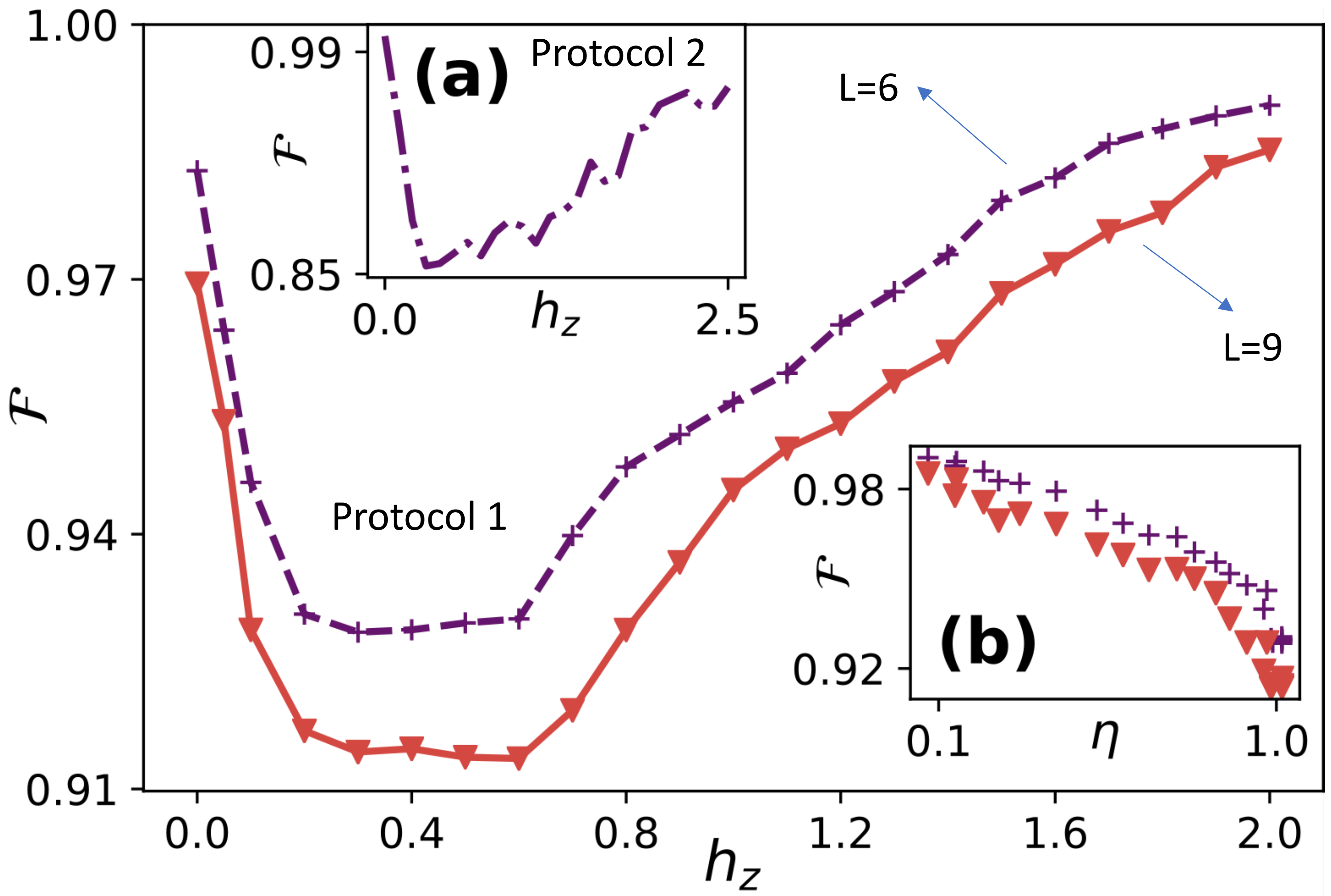


update the field iteratively by using information about the gradient of J(ε)

Algorithms to find ε(t)

$\epsilon_0(t) \longrightarrow \epsilon_1(t) \longrightarrow \qquad \longrightarrow \epsilon_n(t)$

seed



1.00
0.97
0.94
0.91
$\mathcal{F}$
0.0
0.4
0.8
1.2
1.6
2.0
h_z
Protocol 1
L=6
L=9
(a)
Protocol 2
0.99
0.85
$\mathcal{F}$
0.0
h_z
2.5
(b)
0.98
0.92
$\mathcal{F}$
0.1
η
1.0

Are other quantities that give the integrability-to-chaos transition in small systems?

OTOCS: out-of-time-order correlators

Larkin and Ovchinnikov 69 Maldacena et al 2016

$$\mathcal{C}(t) = \left\langle\left\langle \left[\hat{A}_t, \hat{B}\right]^\dagger \left[\hat{A}_t, \hat{B}\right] \right\rangle\right\rangle$$

$$= \left\langle\left\langle \left| \left[\hat{A}_t, \hat{B}\right] \right|^2 \right\rangle\right\rangle$$

$$A_t = e^{iHt/\hbar} A e^{-iHt/\hbar}$$

$$\left\langle\left\langle \hat{O} \right\rangle\right\rangle = \frac{1}{Z} \operatorname{Tr}\left\{ e^{-\beta \hat{H}} \hat{O} \right\}$$ Thermal state

OTOCS: out of time ordered correlators

- Spread of quantum information or information scrambling
- Irreversibility (Loschmidt echo)
- Thermalization
- Bound to chaos $\lambda < 2\pi k_B T/\hbar$

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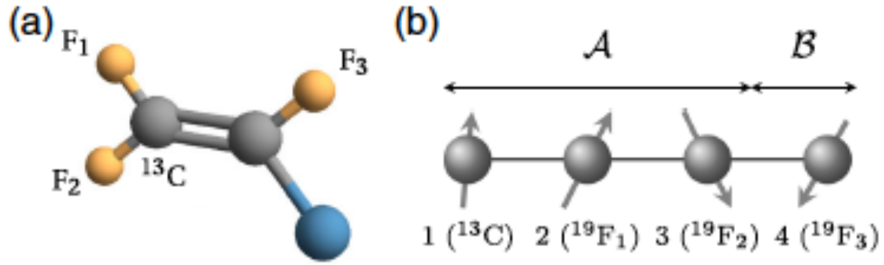
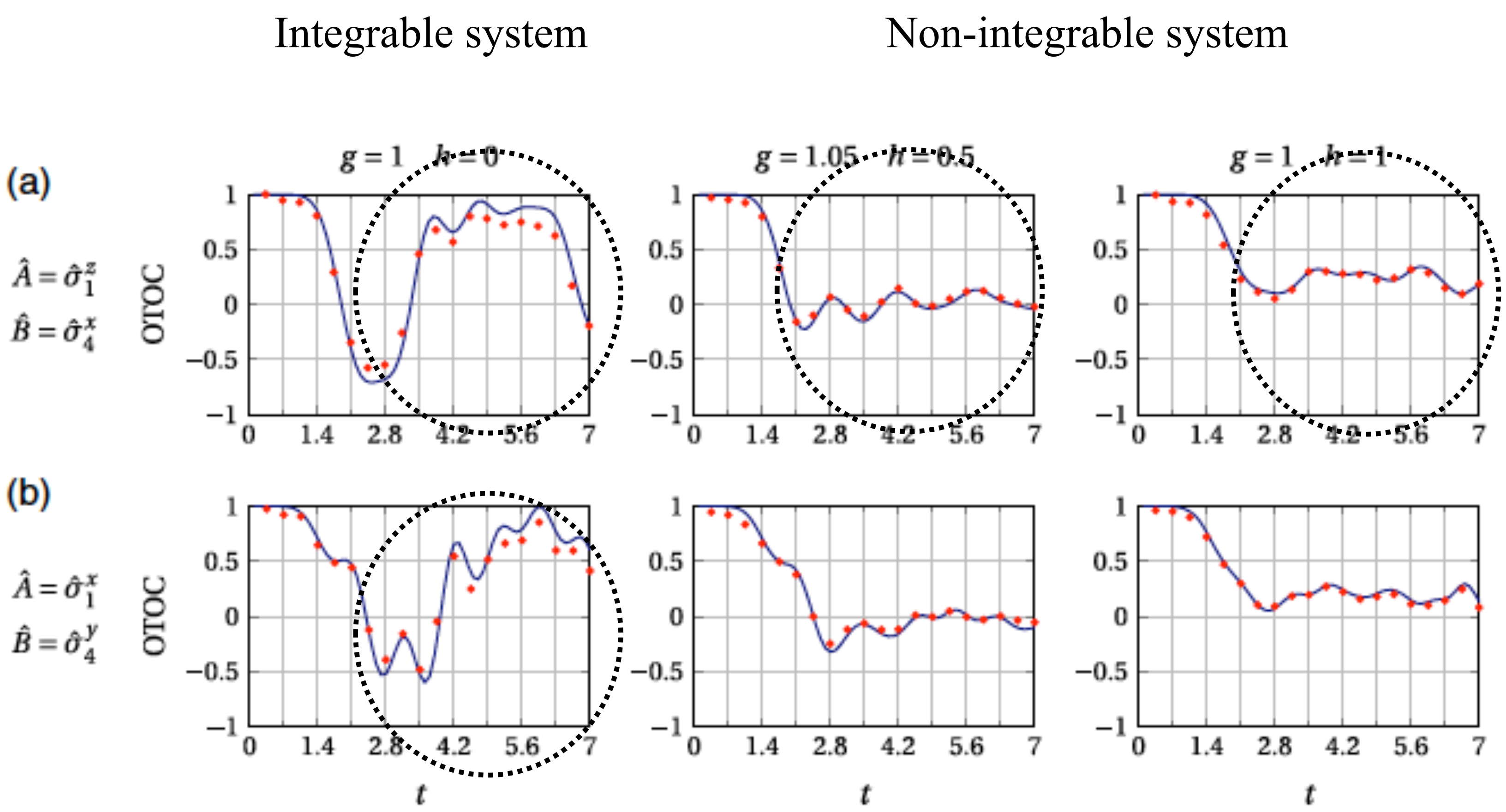
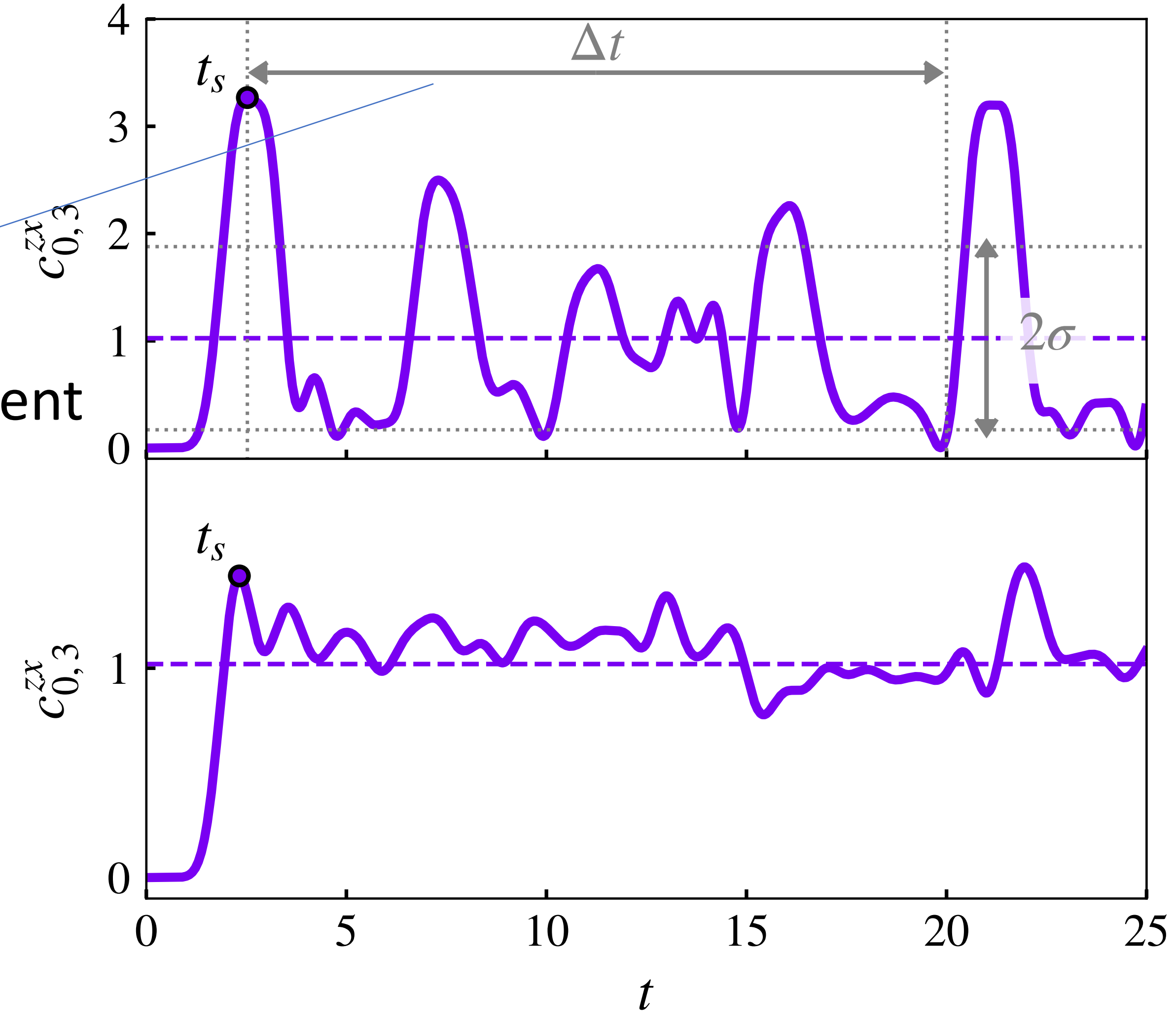


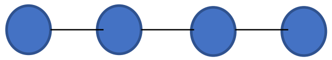


Integrable system
Non-integrable system
(a)
(b)
$\hat{A}=\hat{\sigma}_1^z$
$\hat{B}=\hat{\sigma}_4^x$
$\hat{A}=\hat{\sigma}_1^x$
$\hat{B}=\hat{\sigma}_4^y$
$g=1 \quad h=0$
$g=1.05 \quad h=0.5$
$g=1 \quad h=1$
OTOC
t

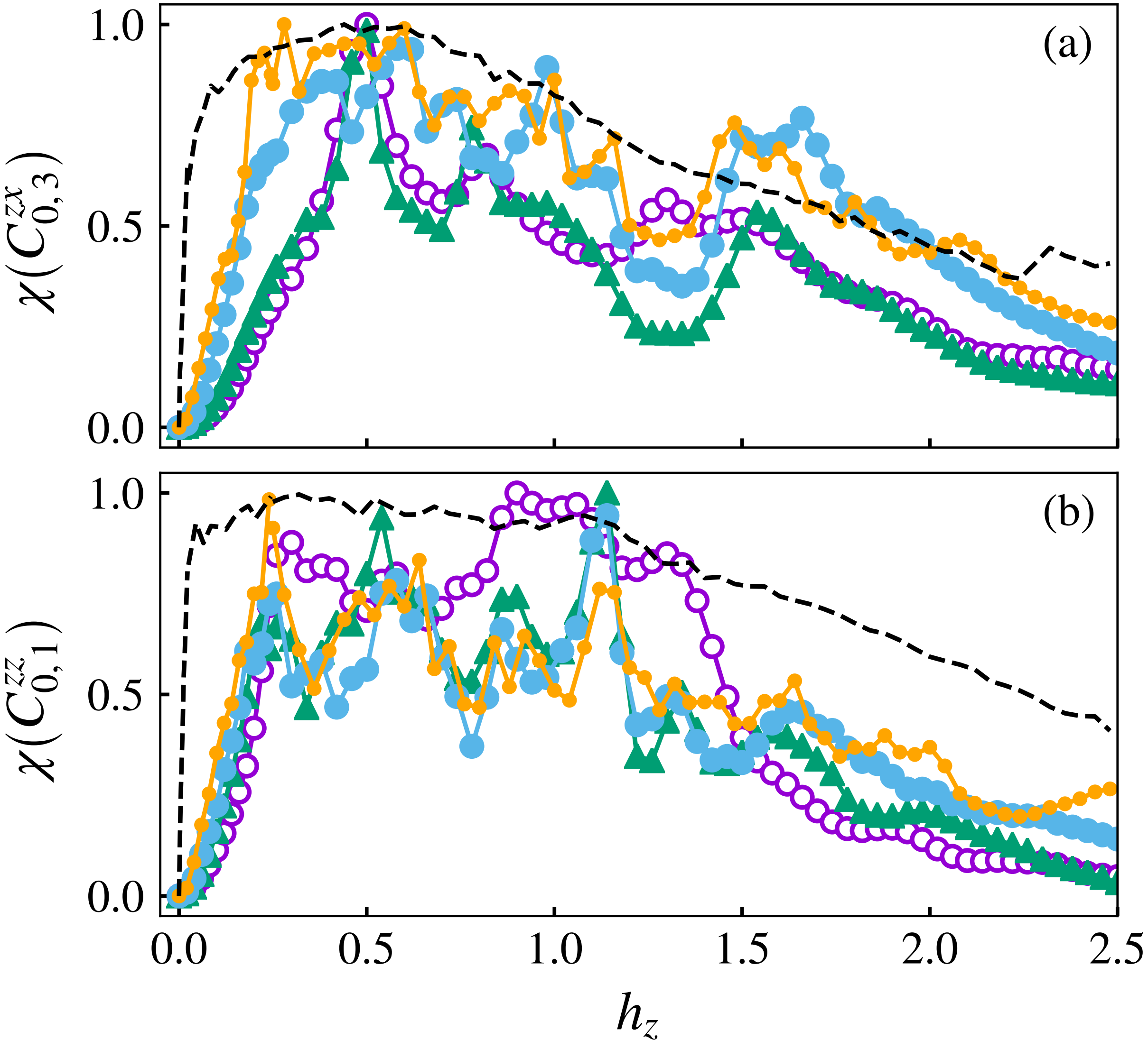
OTOC long-time measures



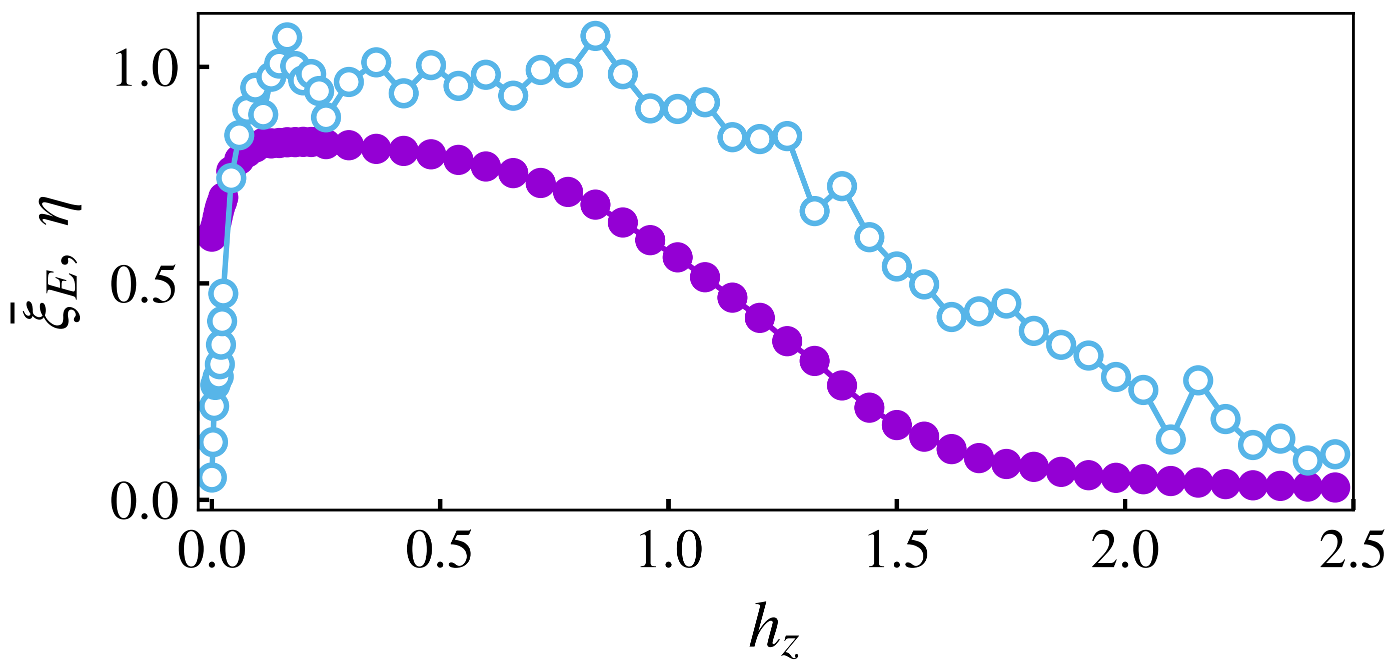
We consider the experiment window!!!!



of the Hilbert space: 16



of the Hilbert space: 4096



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Signatures of quantum chaos transition in short spin chains

Emiliano M. Fortes[1], Ignacio García-Mata[2], Rodolfo A. Jalabert[3] and Diego A. Wisniacki[1]

[1] *Departamento de Física "J. J. Giambiagi" and IFIBA, FCEyN, Universidad de Buenos Aires 1428 Buenos Aires, Argentina*
[2] *Instituto de Investigaciones Físicas de Mar del Plata (IFIMAR), Facultad de Ciencias Exactas y Naturales, Universidad Nacional de Mar del Plata and CONICET - 7600 Mar del Plata, Argentina*
[3] *Université de Strasbourg, CNRS, Institut de Physique et Chimie des Matériaux de Strasbourg, UMR 7504 F-67000 Strasbourg, France*

Final remarks

- Long time regime of d the purity
  describes the integrability-to-chaos transition.

- Small systems has information about the integrability-to-chaos transition. Why?

Schiulaz, M., Távora, M., & Santos, L. F. Quantum Sci. Technol. 3 (2018)
G. Zisling, L. F. Santos,2 and Y. Bar Lev, Arxiv (2020)

- It is better to control quantum systems with ‘integral’ local environment.

Quantum chaos, relaxation and control in extremely short spin chains

Nicolás Mirkin[1,*] and Diego Wisniacki[1]

[1] *Departamento de Física "J. J. Giambiagi" and IFIBA, FCEyN, Universidad de Buenos Aires, 1428 Buenos Aires, Argentina*

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The environment of an open quantum system is usually modelled as a large and complex many-body quantum system. However, the question of how large and complex this many-body quantum system must be in order to generate a robust equilibration is an open key-point in the literature. In this work, by studying the reduced dynamics of a spin connected to a generic spin chain, we show that the physical mechanism responsible for generating stronger equilibration on the reduced spin relies on the chaotic behaviour of the chain. More interestingly, even in the case of an extremely short spin chain composed of two spins, by monitoring the relaxation dynamics of the reduced system we are able to reproduce the whole integrable to chaos transition. Finally, we discuss implications on quantum control experiments and show that quantum chaos reigns over the best degree of control achieved, even in small chains.

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[3] *Université de Strasbourg, CNRS, Institut de Physique et Chimie des Matériaux de Strasbourg, UMR 7504 F-67000 Strasbourg, France*

Control and quantum chaos @dfuba


Nicolas Mirkin


Martin Larocca

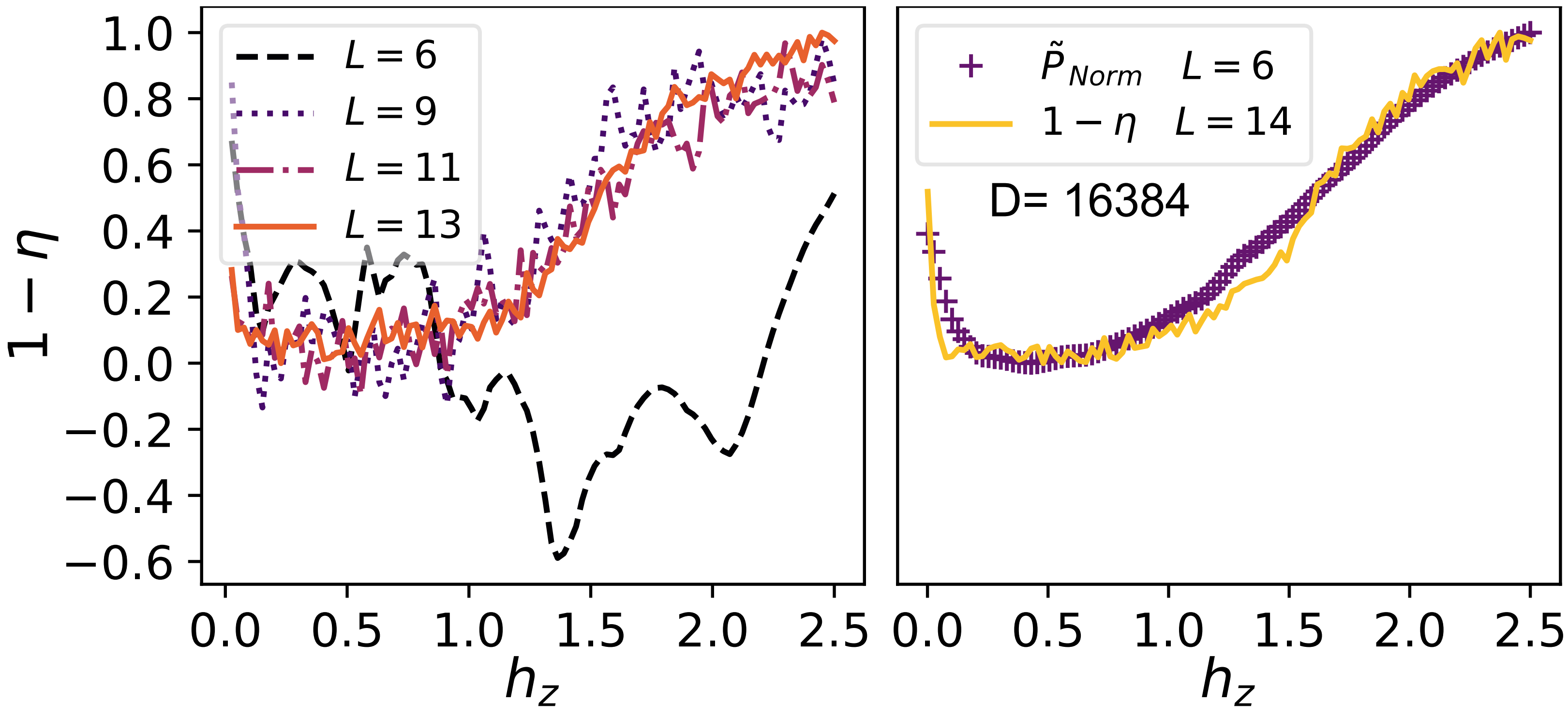

Emiliano Fortes


Tomás Nottenson


DAW

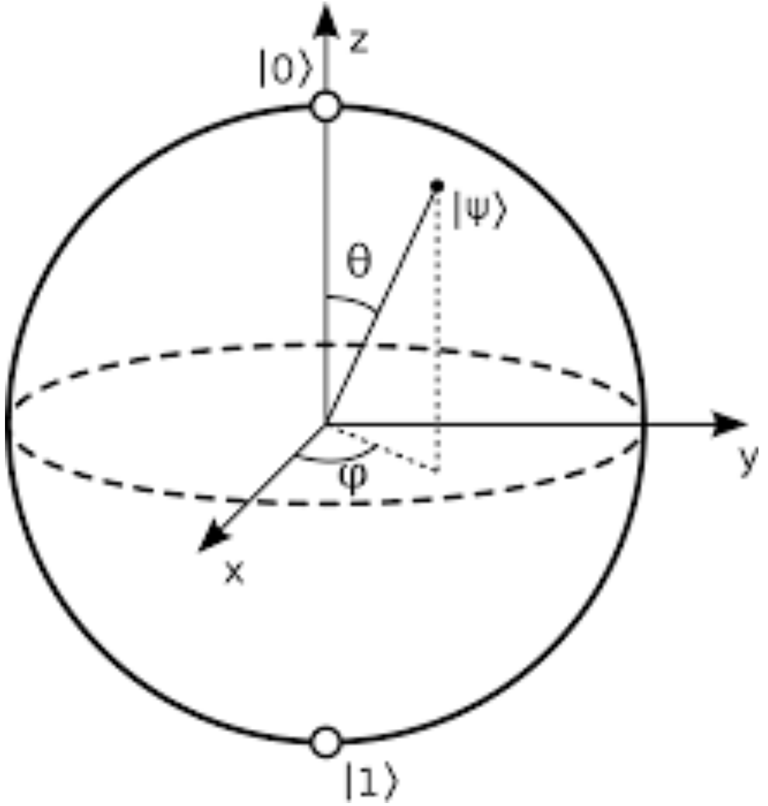
Thanks

Sensitivity of standard indicators to the dimension of Hilbert space



What is the connection with equilibration?

$$\mathcal{P}(t) = \mathrm{Tr}\left[\tilde{\rho}^2(t)\right]$$



$$\tilde{\rho} = 1/2(\mathrm{I} + \vec{r}.\vec{\sigma})$$

$$\vec{r} = (r_x, r_y, r_z)$$

$$r_i(t) = \mathrm{Tr}\left(\sigma_i \tilde{\rho}(t)\right)$$

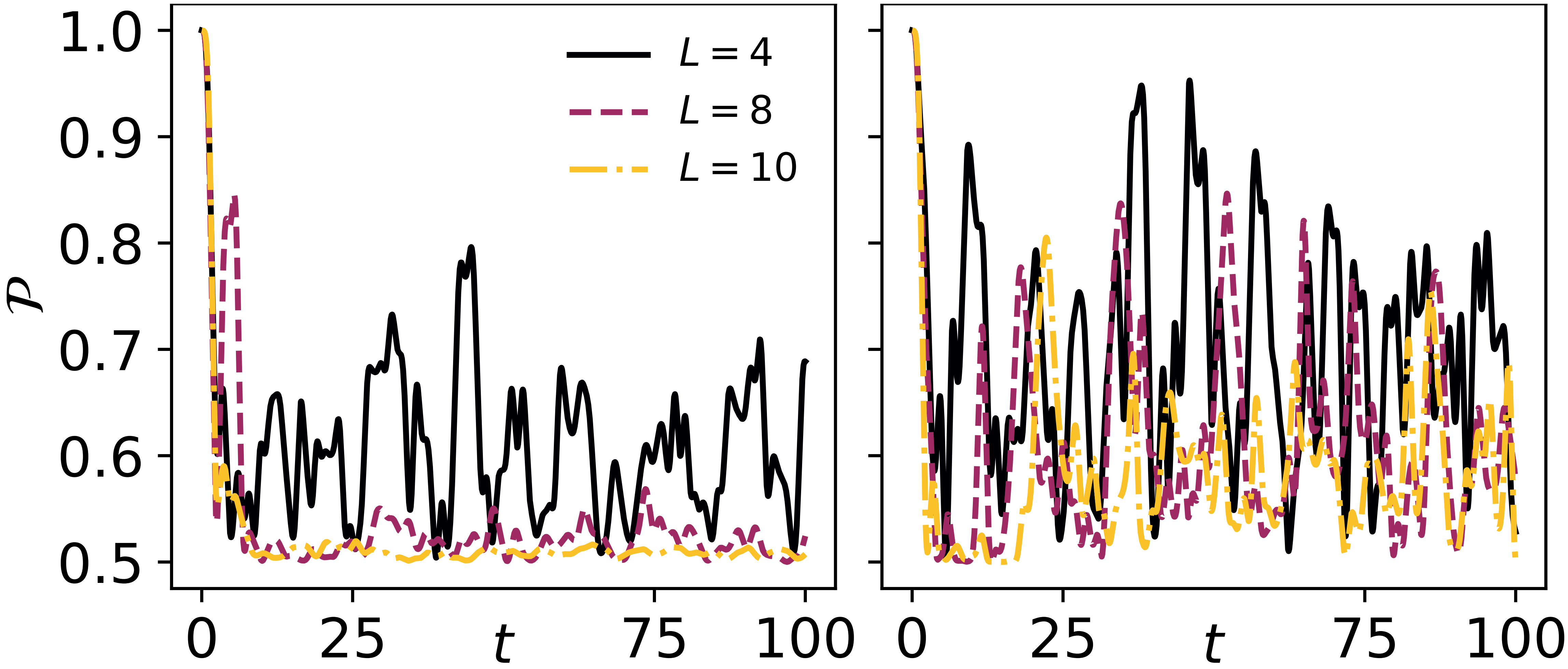
$$\mathcal{P}(t) = 1/2(1 + |\vec{r}(t)|^2)$$

$$A = \{\mathrm{I}, \sigma_x, \sigma_y, \sigma_z\}$$

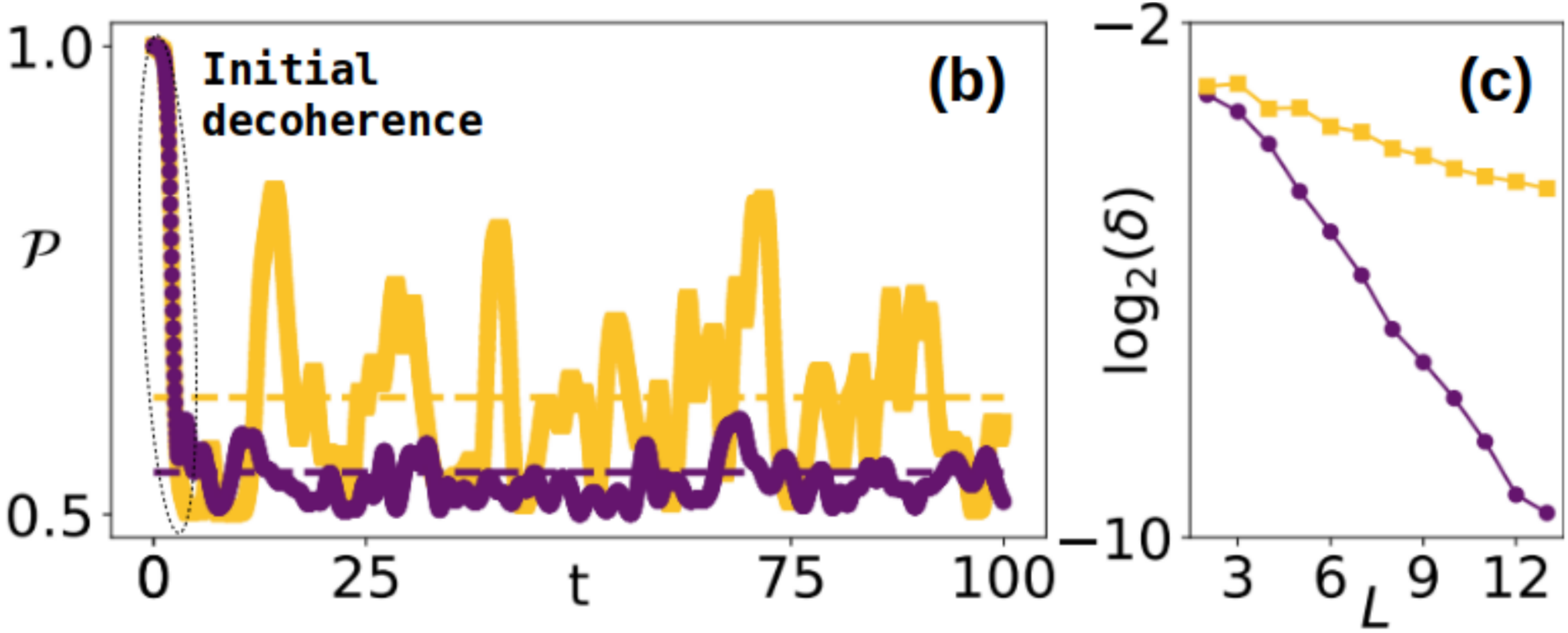
$$\lim_{t \to \infty} \mathrm{Tr}\left(\tilde{\rho}(t)\widehat{O}\right) = \mathrm{Tr}\left(\tilde{\rho}_\infty \widehat{O}\right) \forall \, \widehat{O} \in A$$

$$\tilde{\rho}_\infty = \frac{\mathrm{I}}{2} \qquad \mathrm{Tr}\left(\sigma_i \tilde{\rho}_\infty\right) = 0 \, \forall i \in \{x, y, z\}$$

$$\Rightarrow \quad \mathcal{P}_\infty = 1/2$$

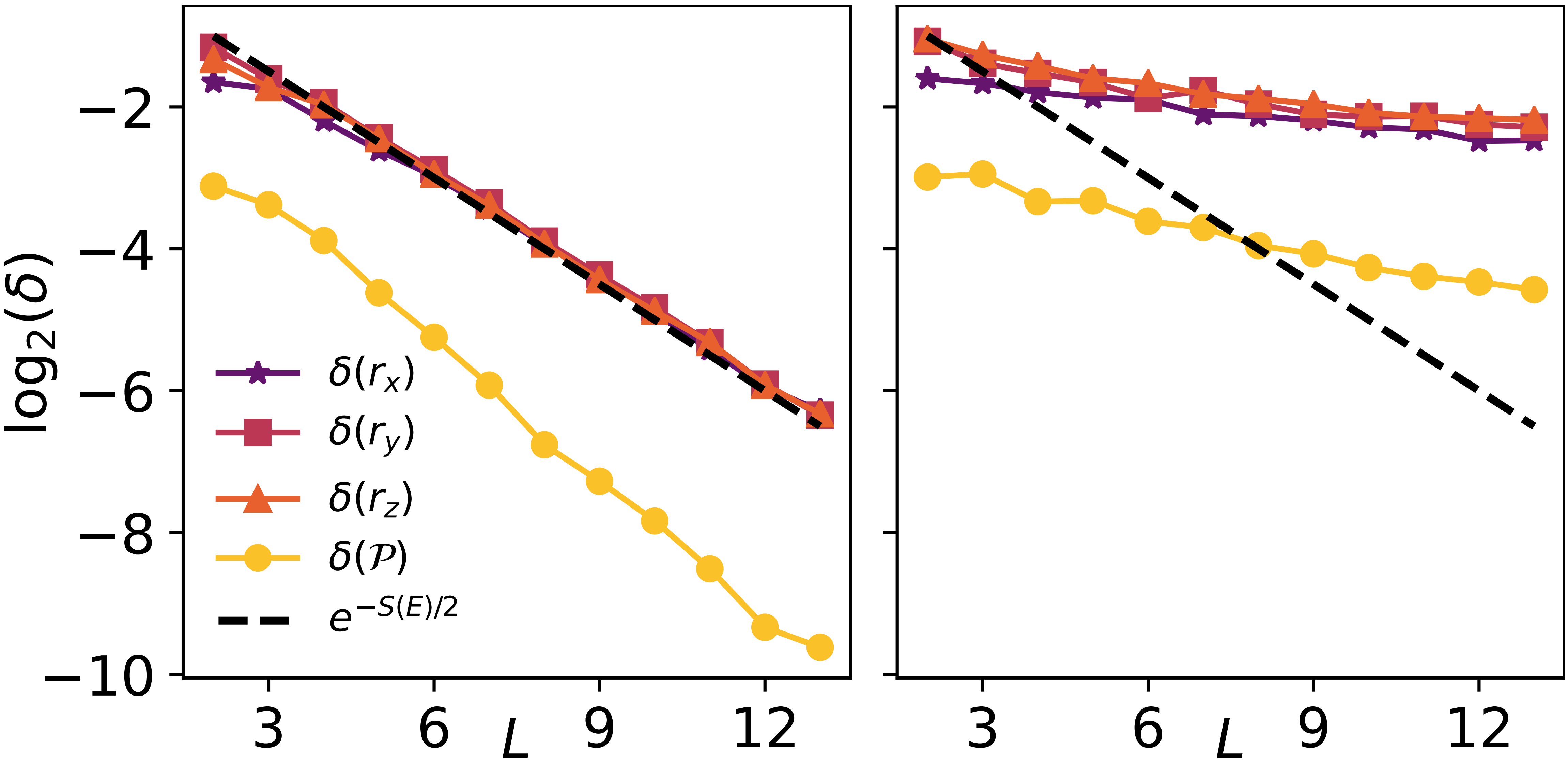


1.0
0.9
0.8
0.7
0.6
0.5
$\mathcal{P}$
$L = 4$
$L = 8$
$L = 10$
0
25
t
75
100
0
25
t
75
100



Fluctuations of P

$$\delta(P) = \sqrt{\langle P(t)^2 \rangle - \langle P(t) \rangle^2}$$



$\log_2(\delta)$
−2
−4
−6
−8
−10
3
6
9
12
L
$\delta(r_x)$
$\delta(r_y)$
$\delta(r_z)$
$\delta(\mathcal{P})$
$e^{-S(E)/2}$