Participation Ratio and Survival Probability in the Dicke model.

Sergio Lerma-Hernández
(Universidad Veracruzana, Mexico)

A (growing) collaboration

Jorge Hirsch
(Instituto de Ciencias Nucleres-UNAM)
Miguel Bastarrachea-Magani
(now in Universidad Autónom Metropolitana-México)
Jorge Chávez-Carlos
(postdoc, Instituto Ciencias Física-UNAM)
Lea Santos
(Physics Department, Yeshiva University, NY, USA)
E.Jonathan Torres-Herrera
(Instituto de Física, BUAP)
David Villaseñor
(PhD student UNAM)
Saúl Pilatowsky-Cameo
(Bachelor in Physics, UNAM)

How do the Hamiltonian eigenstates participate in building a given (coherent) state?

Why is this interesting?

- The energy components (together the energy values) determine the evolution of the states.

- Allows to study in detail equilibration process and obtain valuable insights valid for closed quantum systems

- Why do we use the Dicke Hamiltonian?
- Why do we use coherent states as initial states?
- How do we measure the participation of the Hamiltonian eigenstates?

Why do we use the Dicke Hamiltonian?

$$H = \omega a^{\dagger} a + \omega_o J_z + \frac{\gamma}{\sqrt{N}} \left(a + a^{\dagger}\right)\left(J_+ + J_-\right)$$

Bosons energy — Two levels' energy splitting — Interaction term

$\hbar = 1$

$$J_z = \frac{1}{2}\sum_{i=1}^{N}\sigma_{iz} = \frac{1}{2}(N_e - N_g)$$

$$J_{\pm} = \frac{1}{2}\sum_{i=1}^{N}\sigma_{i\pm}$$

J=N/2

One of the simplest autonomous Hamiltonian (time independent) with regular and **chaotic** regimes. Excellent playground to test ideas.

Experimentally accesible on different platforms

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Dicke-model simulation via cavity-assisted Raman transitions

Zhiqiang Zhang, Chern Hui Lee, Ravi Kumar, K. J. Arnold, Stuart J. Masson, A. L. Grimsmo, A. S. Parkins, and M. D. Barrett
Phys. Rev. A **97**, 043858 – Published 25 April 2018

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Ultrastrong-coupling phenomena beyond the Dicke model

Tuomas Jaako, Ze-Liang Xiang, Juan José Garcia-Ripoll, and Peter Rabl
Phys. Rev. A **94**, 033850 – Published 27 September 2016

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Verification of a Many-Ion Simulator of the Dicke Model Through Slow Quenches across a Phase Transition

A. Safavi-Naini, R. J. Lewis-Swan, J. G. Bohnet, M. Gärttner, K. A. Gilmore, J. E. Jordan, J. Cohn, J. K. Freericks, A. M. Rey, and J. J. Bollinger
Phys. Rev. Lett. **121**, 040503 – Published 27 July 2018

Why do we use coherent states?

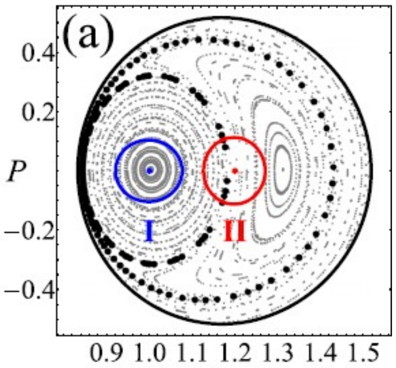
Coherent states

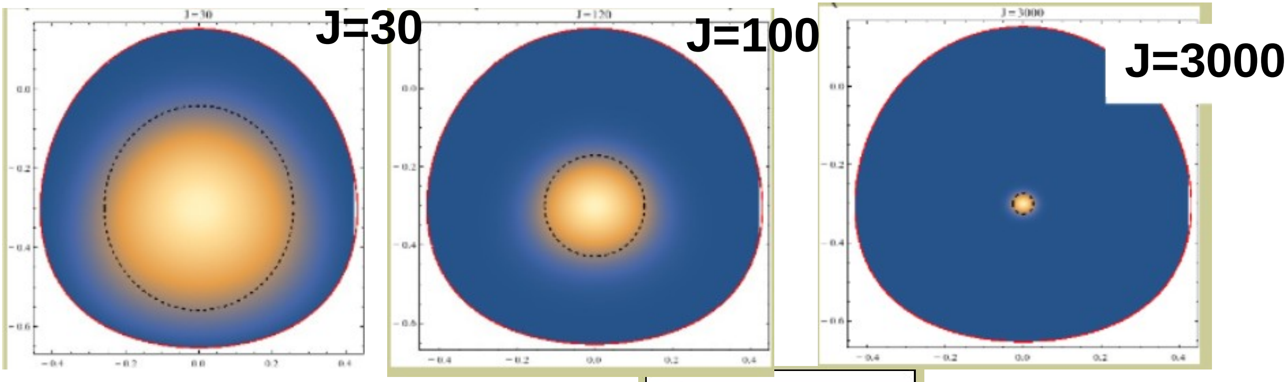
bosonic $|z\rangle = e^{-|z|^2/2} e^{z a^\dagger} |0\rangle_b$

$z, \alpha \in \mathcal{C}$

pseudospin $|\alpha\rangle = \frac{1}{(1+|\alpha|^2)^j} e^{\alpha J_+} |jm = -j\rangle$

1) Maximally localized quantum states in phase space

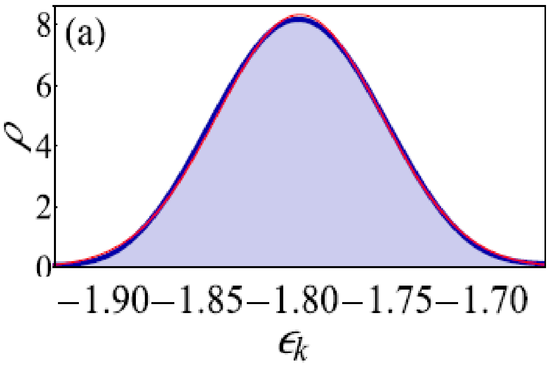




$\hbar_{eff} = 1/J$

Husimi function $\mathcal{Q}(\alpha, z) = |\langle z, \alpha | \Psi \rangle|^2$

2) They are also well localized in energy space



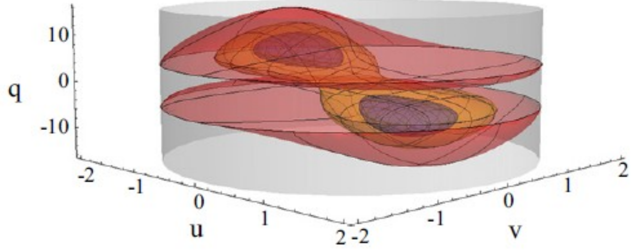
Smoothed Local Density of States (LDOS):

Gaussian distribution (except for states close to critical energies, GS or ESQTP)

3) They allow to build a classical Hamiltonian (stationary phase approximation)

$$\hbar_{eff} = 1/J$$

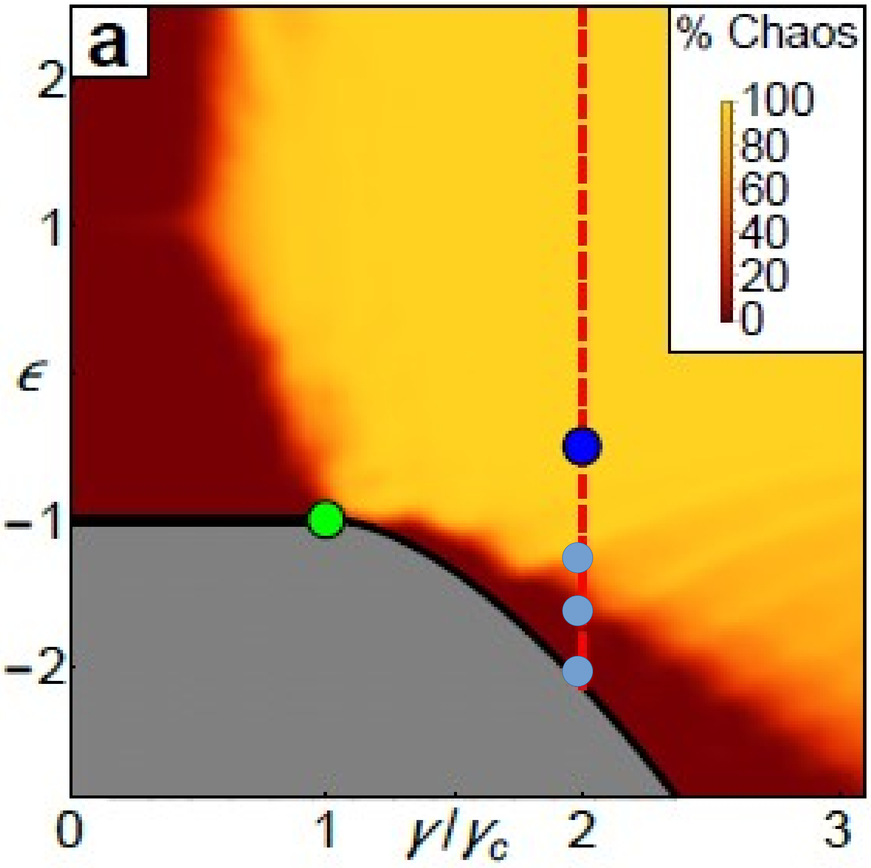
$$H_c = \langle \alpha, z | H | \alpha, z \rangle = \frac{\omega}{2}\left(p^2 + q^2\right) + \omega_o J_z + \frac{2\gamma}{\sqrt{j}} q J_x$$

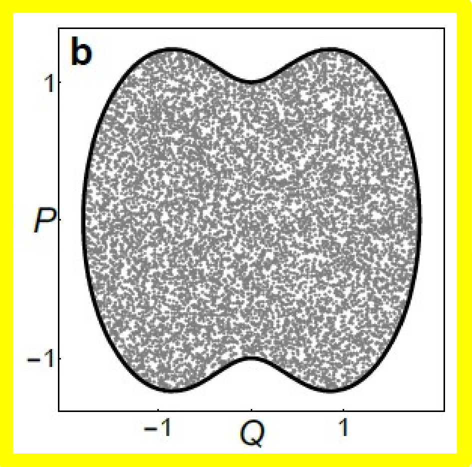


Energy surfaces

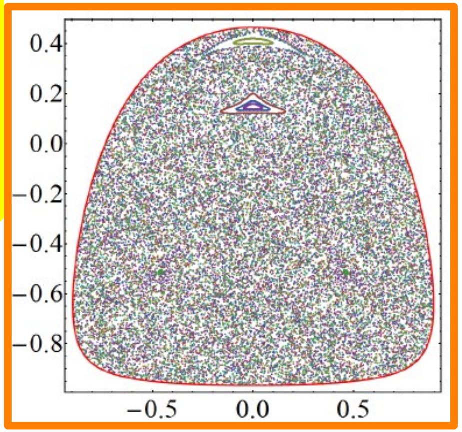
The study of this classical model is an excelent guide for exploring the quantum version

Regular to chaotic transition very well characterized...

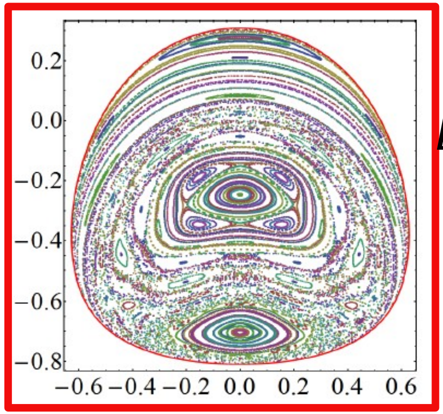




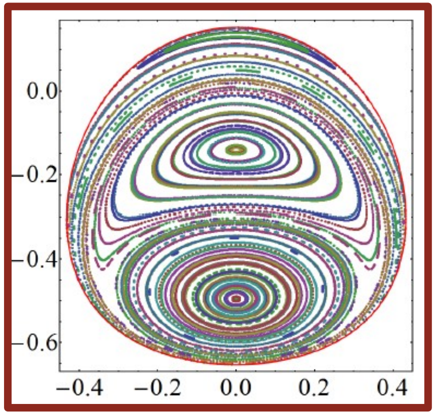
E/J=-0.5



E/J=-1.1



E/J=-1.5



E/J=-1.8

Physical Review E 94 (2), 022209 (2016)

How do we measure the participation of the Hamiltonian eigenstates?

Participation ratio

$$P_R = \frac{1}{\sum_k |c_k|^4}. \qquad c_k = \langle E_k | \Psi_o \rangle$$

P_R=1 if the state is a member of the basis

P_R=D if the state is evenly distributed along the D elements of the basis

$$|\Psi, t\rangle = e^{-i\hat{H}_D t}|\Psi_o\rangle = \sum_k c_k e^{-iE_k t}|E_k\rangle$$

$$\lim_{t\to\infty} \langle SP \rangle = IPR = \frac{1}{PR}$$

Asymptotic value of the **survival probability**

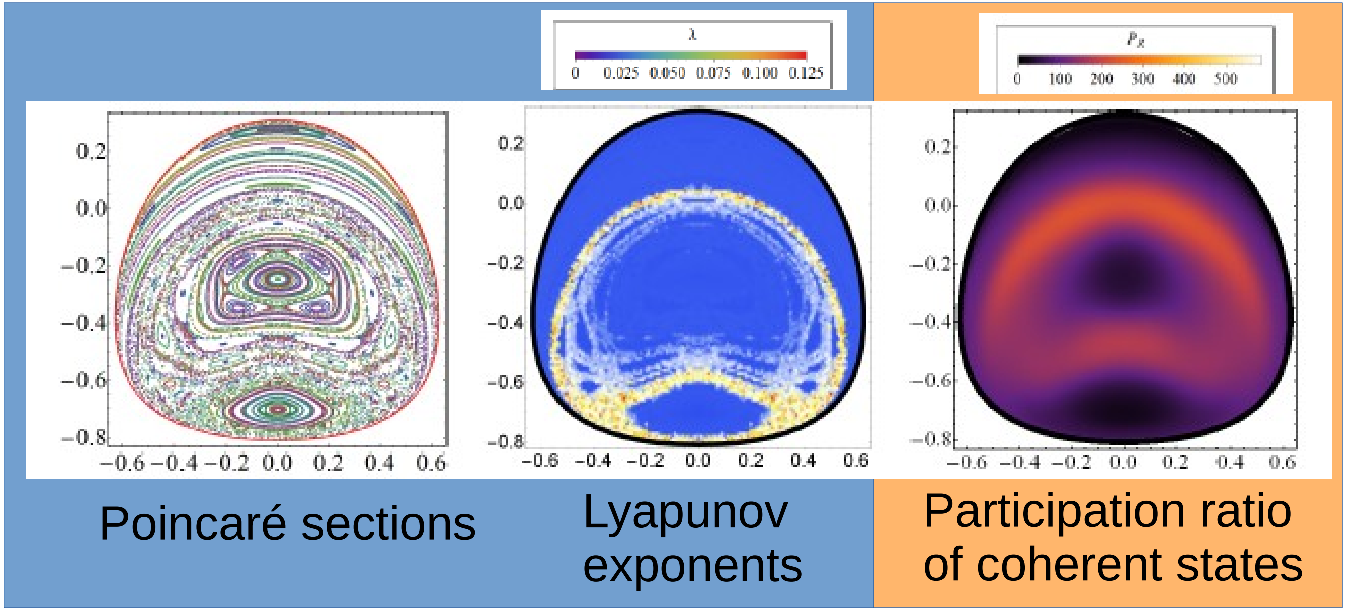
$$\mathrm{SP}(t) = |\langle \Psi(0)|\Psi(t)\rangle|^2 = \left| \sum_k |c_k|^2 e^{-iE_k t} \right|^2 = \sum_{k\neq k'} |c_k|^2 |c_{k'}|^2 e^{-i(E_k - E_{k'})t} + \frac{1}{P_R}$$

One of the simplest observable

RESULTS

Quantum participation ratio is sensitive to classical structures

Classical results | Quantum results



Weak chaos, mixed phase space (regular and chaotic coexisting regions)

E/J=-1.5

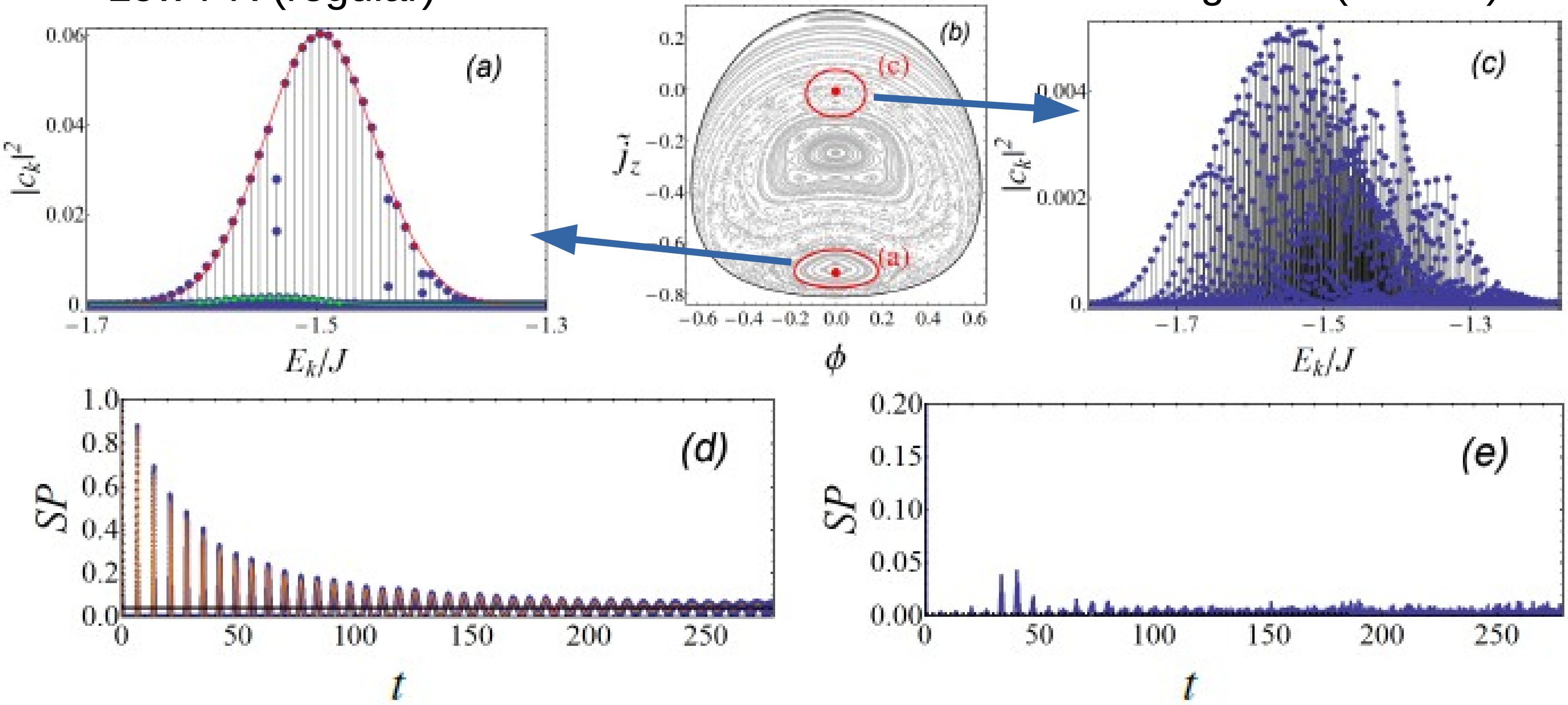
Physical Review E 93 (2), 022215(2016)

AIP Conf.Proc. 1950 (1), 030002(2018)

This is also reflected in the components distribution (LDOS)and the dynamics of the survival probability

Low PR (regular)

High PR (chaotic)

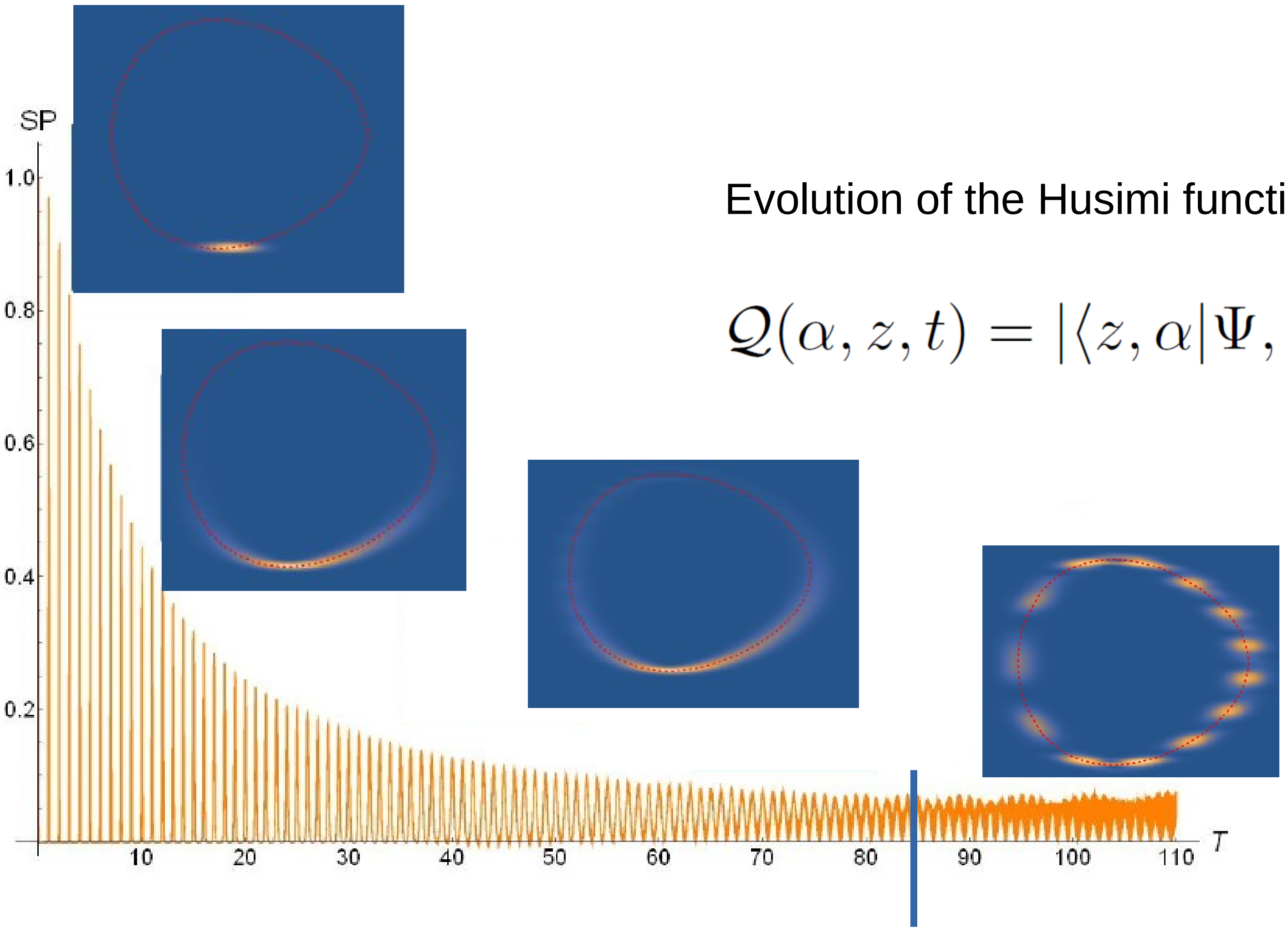


Periodical and decaying revivals

Journ of Phys. A 51 (47), 475302 (2018)

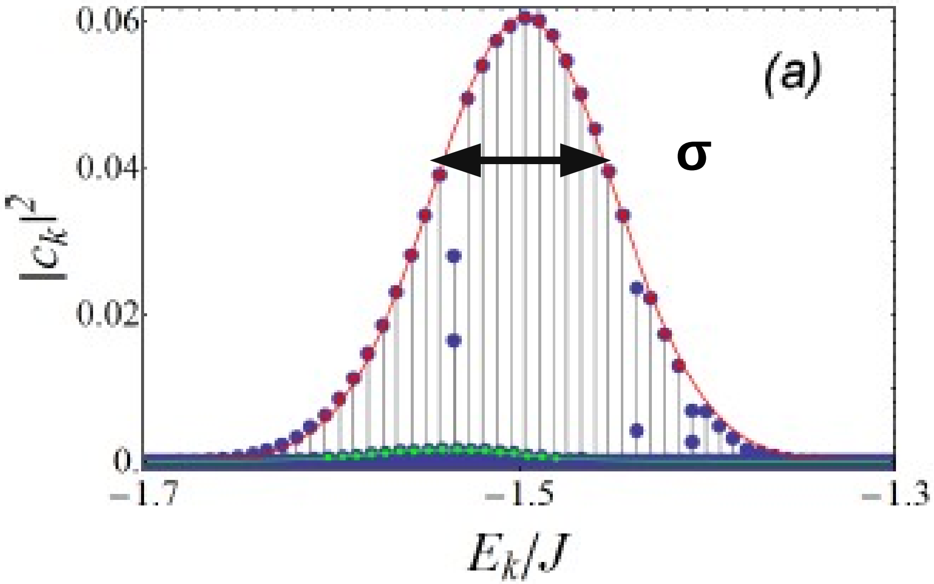
Evolution of the Husimi function

$$\mathcal{Q}(\alpha, z, t) = |\langle z, \alpha | \Psi, t \rangle|^2$$



T_H Heisenberg time (discrete spectrum resolved)

Only a small fraction of energy states participates



Energies of the **participating** components:

- Gaussian distributed

$$|c_k|^2 \approx g_k \equiv A e^{-\frac{(E_k - \bar{E})^2}{2\sigma^2}}$$

- Follow a simple quantization rule

$$E_k = e_o + e_1 k + e_2 k^2$$

$$e_1 \approx \omega_{cl} \qquad e_2 \approx \frac{\omega_{cl}}{2J}\frac{d\omega_{cl}}{d\epsilon}$$

$$e_2 << e_1$$

e_2 measures the (small) deviation from a harmonic spectrum

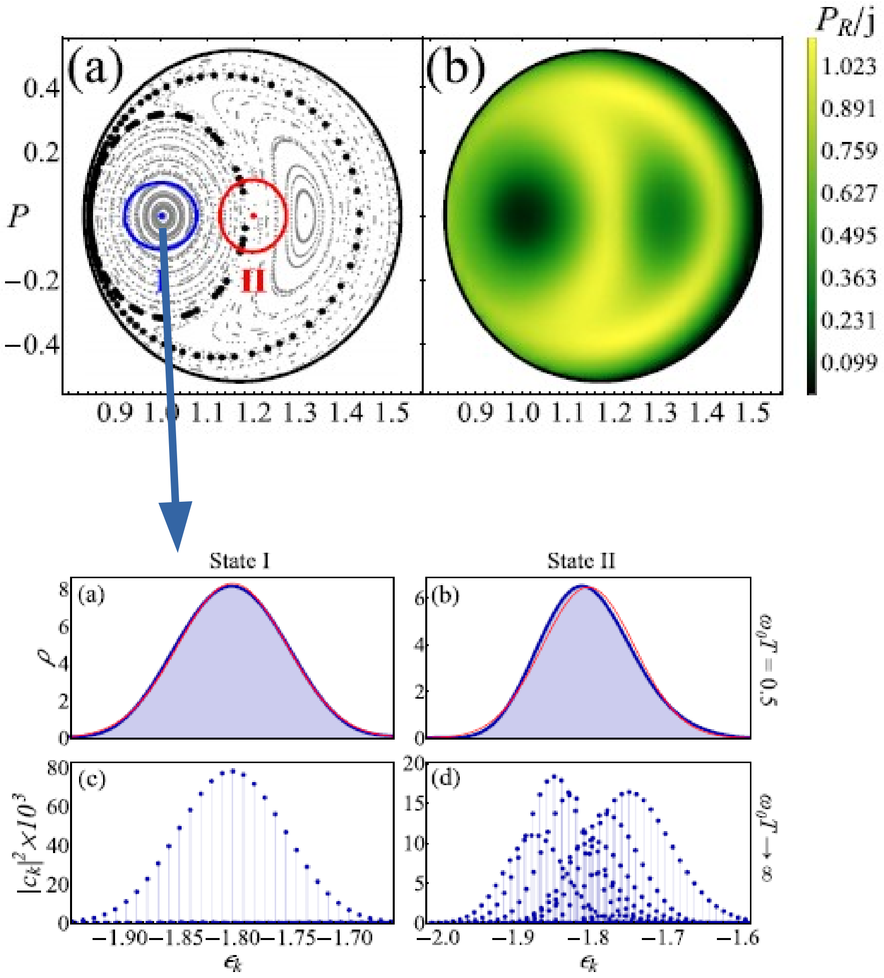
Analytical expression

$$SP(t) \approx \frac{\omega_1}{2\sigma\sqrt{\pi}} \Theta_3(x, y)$$

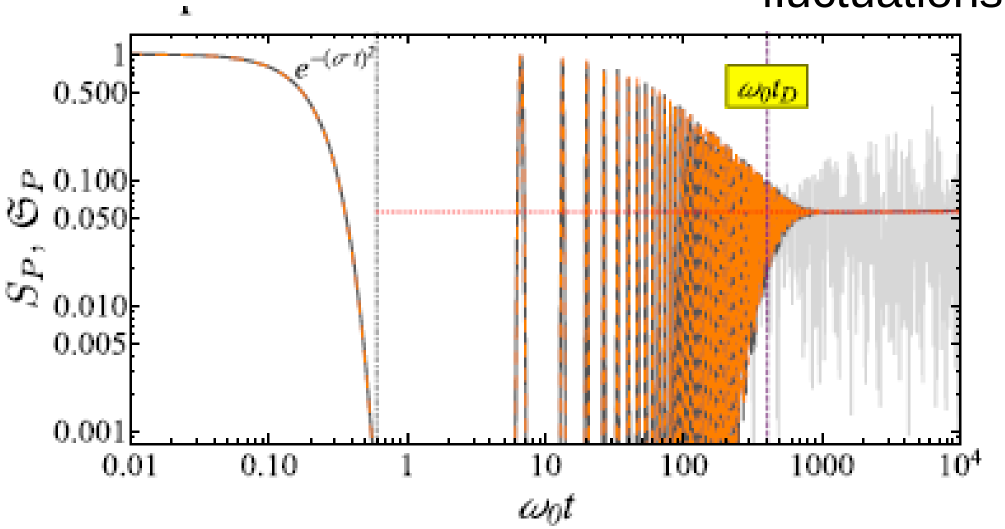
In terms of Jacobi theta function

Decay time

$$t_D \equiv \frac{\omega_1}{\sigma |e_2|}$$



Quantum fluctuations

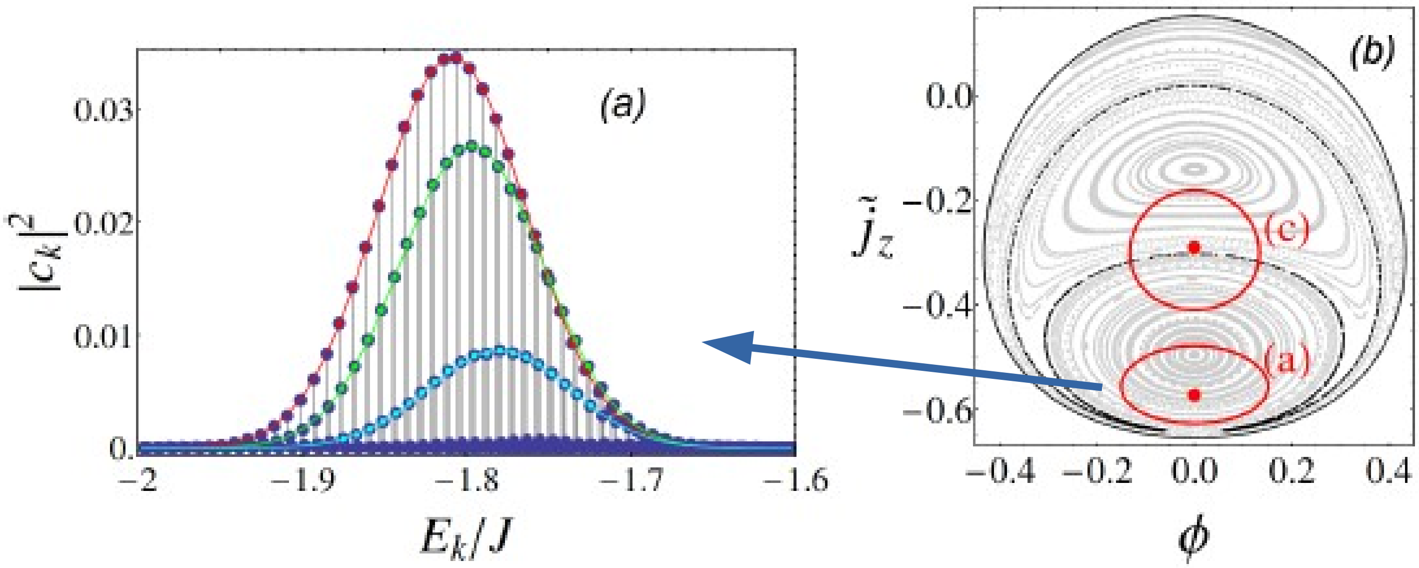


The analytical expression matches perfectly

the survival probability up to the Heisenberg time

and is equal to the ***classical Survival probability*** (Truncated Wigner Approximation)

In more general regular cases, the initial coherent state triggers the two degrees of freedom



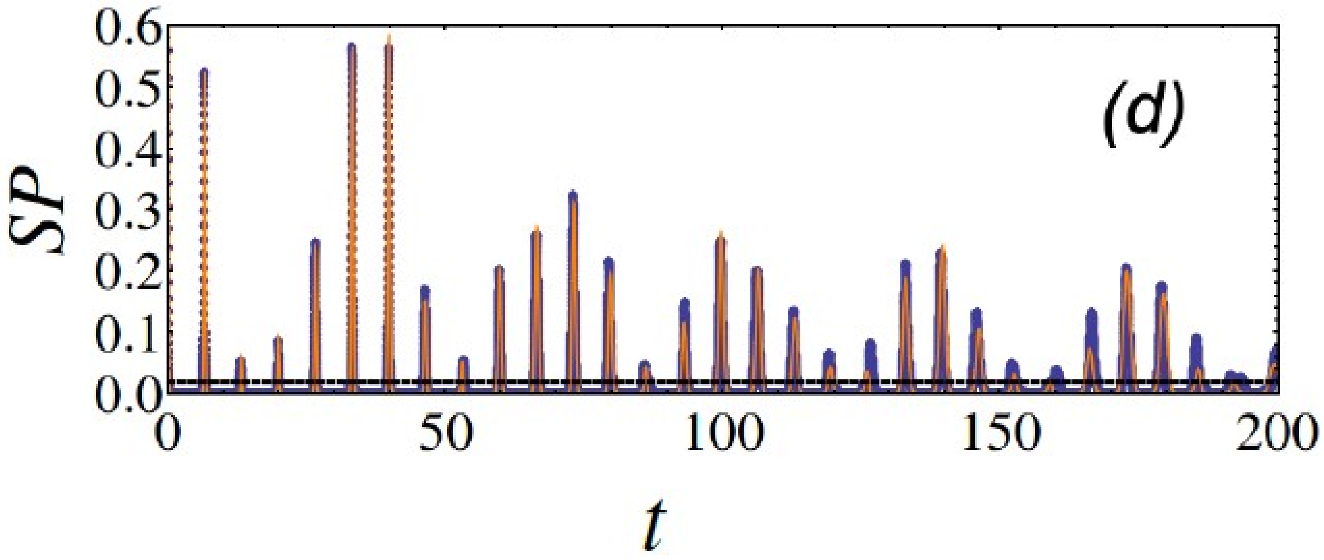
E/J=-1.8

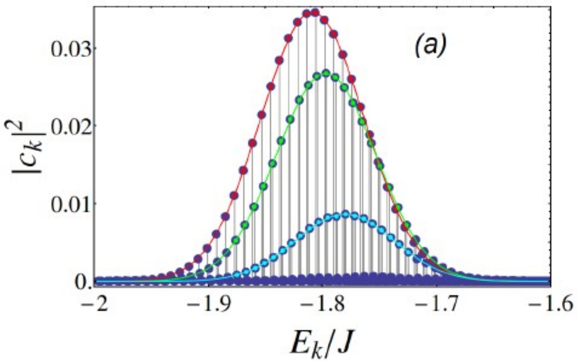
Several energy sequences with Gaussian distributed components

$$\mathrm{SP}(t) = \sum_i \mathrm{SP}^{(i)}(t) + \sum_{i<j} \mathrm{SP}_I^{(ij)}(t)$$

Interference terms between different energy sequences $\mathrm{SP}_I^{(ij)}(t)$

...and also describes very well the survival probability





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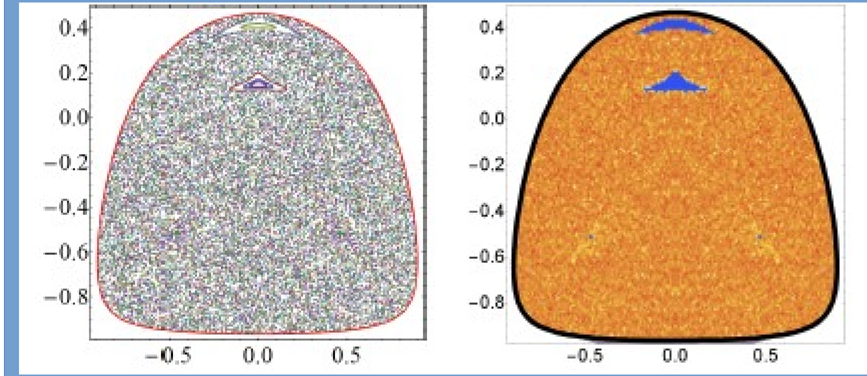
$$\mathrm{SP}(t) = \sum_i \mathrm{SP}^{(i)}(t) + \sum_{i<j} \mathrm{SP}_I^{(ij)}(t)$$

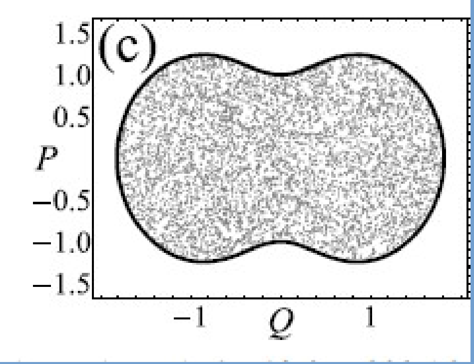
Interference terms between different energy sequences $\mathrm{SP}_I^{(ij)}(t)$

Chaotic region

The participation ratio reveals some structures **apparently** absent in the classical limit

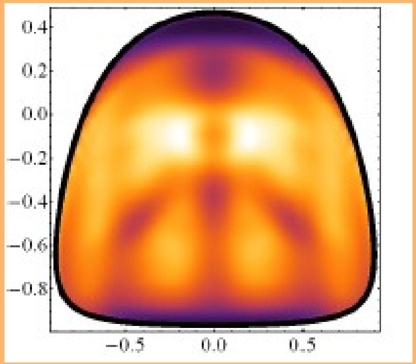
Classical results

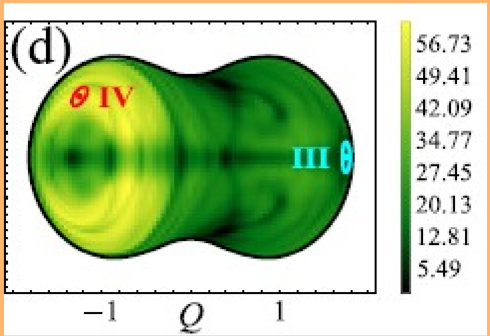




Poincaré sections

Quantum results





Participation ratio
of coherent states

E/J=-1.1

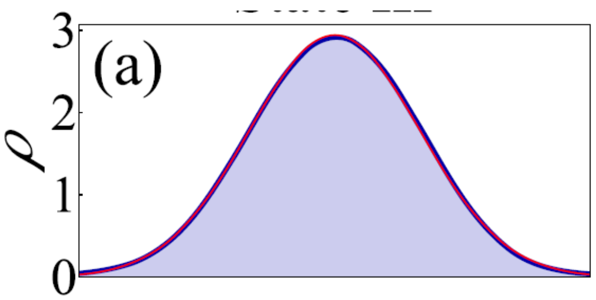
Where do the observed structures in the chaotic regions come from?

E/J=-0.5

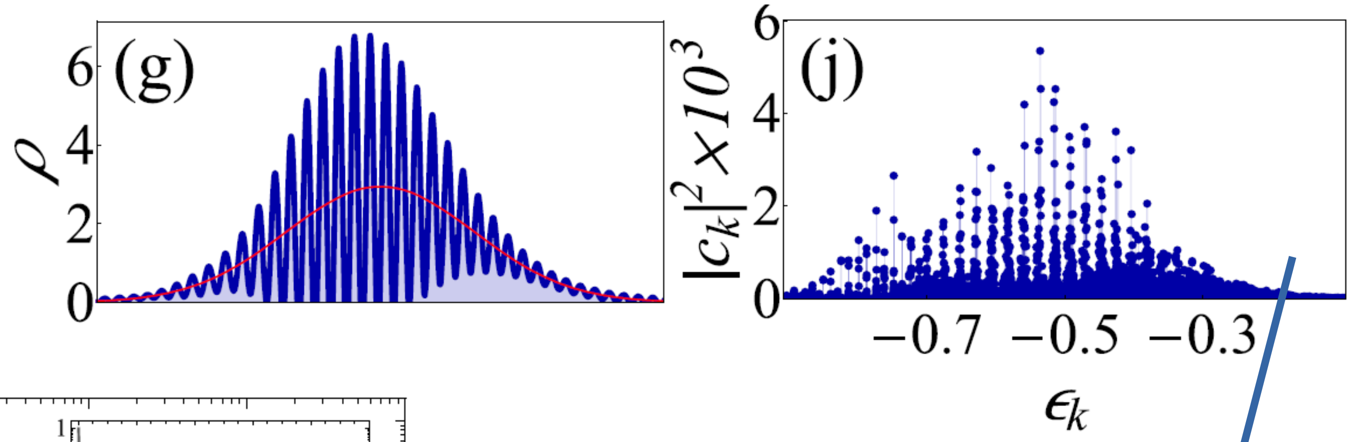
Hard chaotic

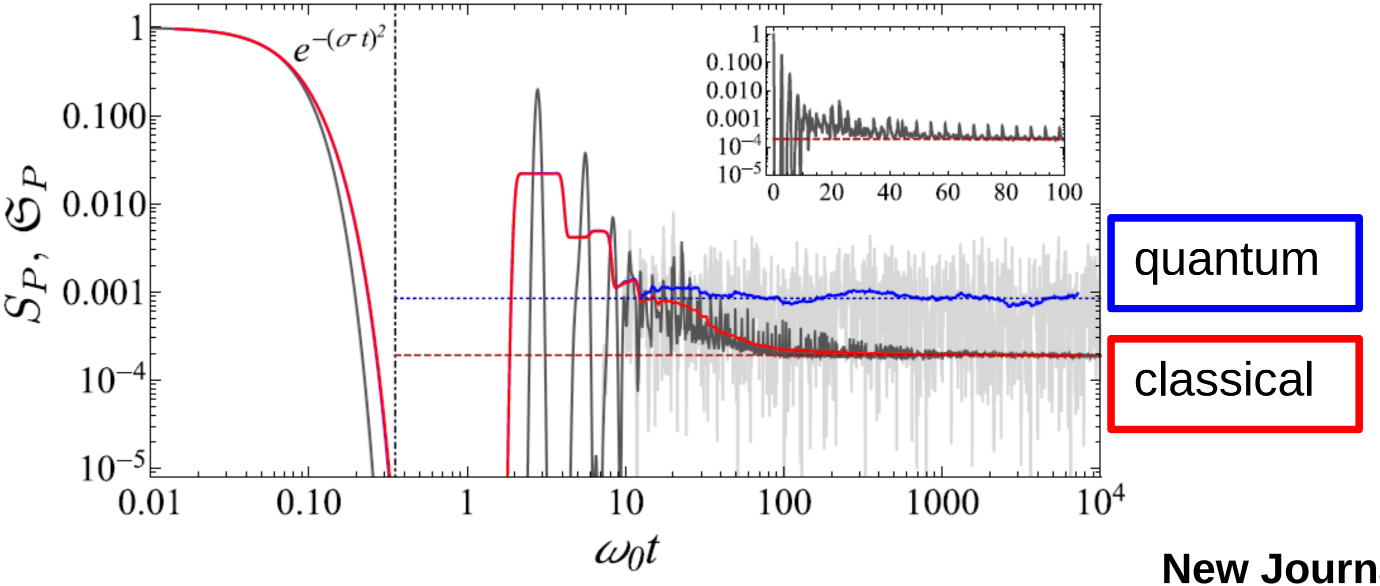
Initial state with a (relative) **low** participation ratio shows periodic revivals

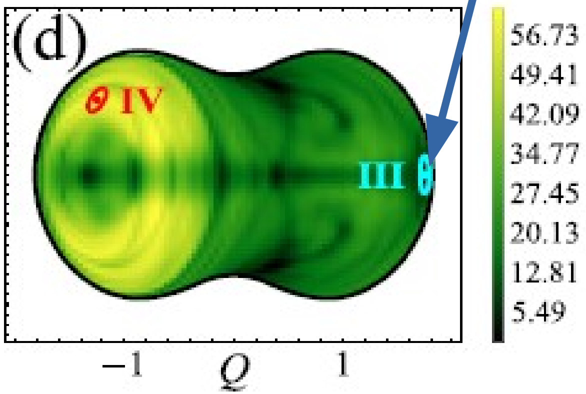
LDOS at different resolutions



Comb-like structure (Heller scarring detected)

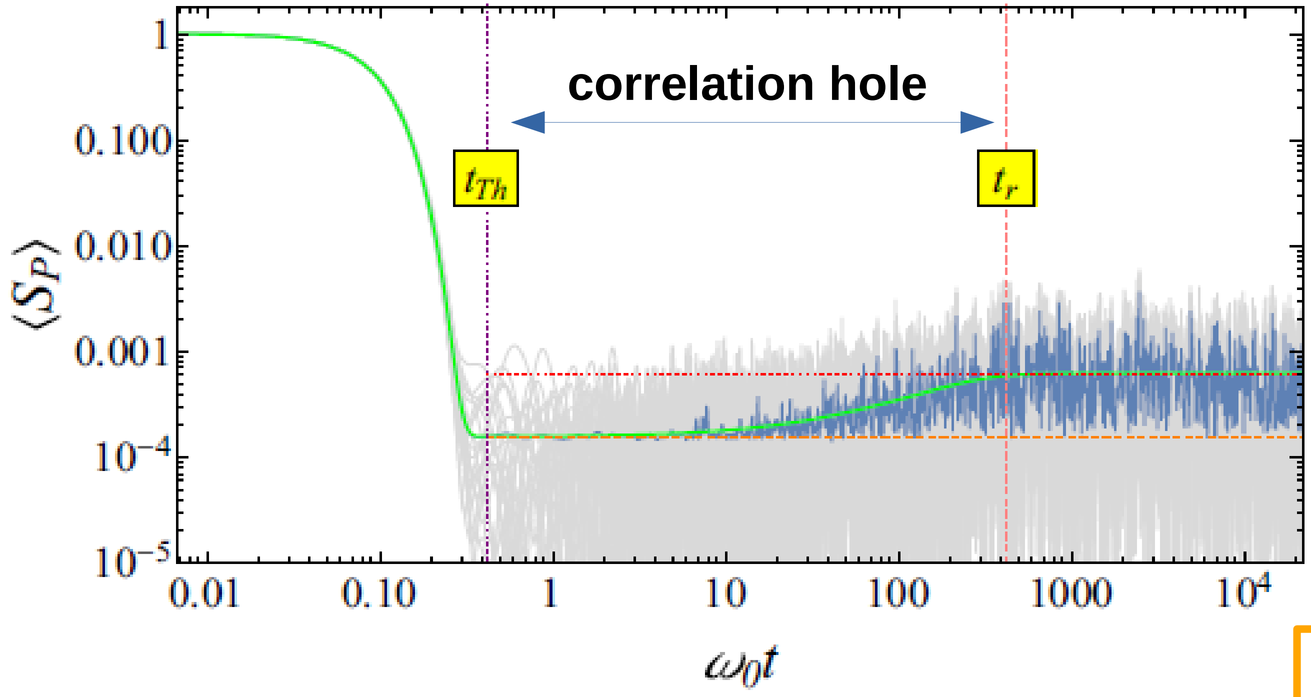




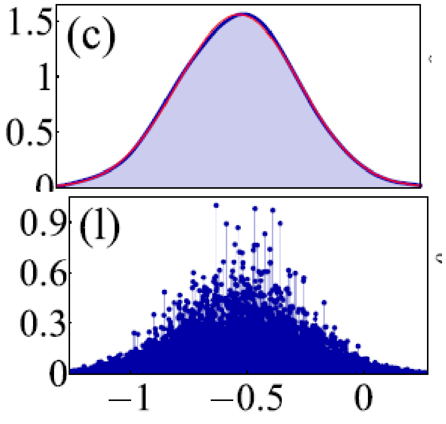


New Journal of Physics 22 (6), 063036(2020)

Random states show a dip below the saturation value, which is governed by GOE correlations: correlation hole



Random state LDOS



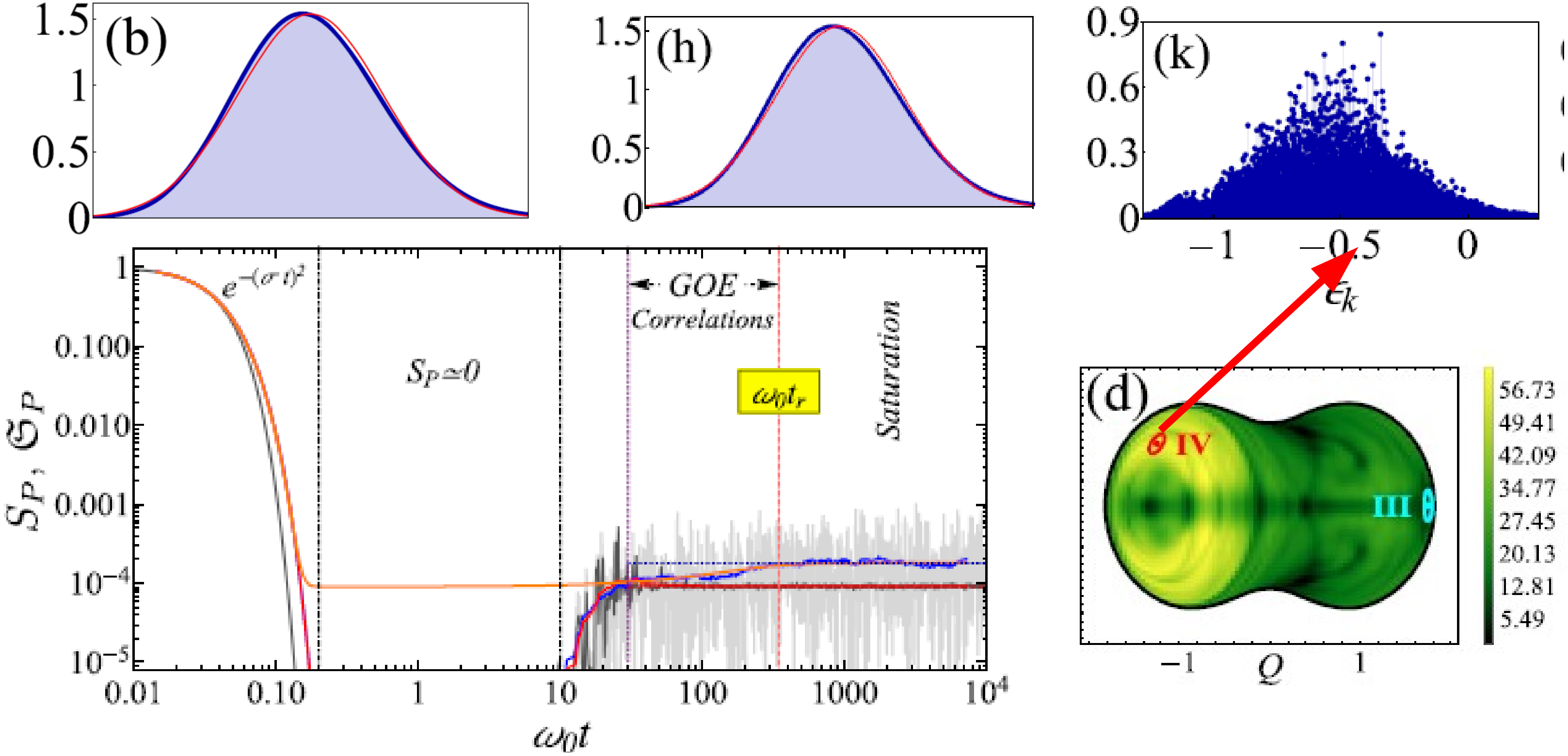
GOE Two level form factor

$$\langle S_P(t) \rangle = \frac{1 - \langle I_{PR} \rangle}{\eta - 1} \left[\eta S_P^{bc}(t) - b_2\left(\frac{t}{2\pi \nu_c}\right) \right] + \langle I_{PR} \rangle.$$

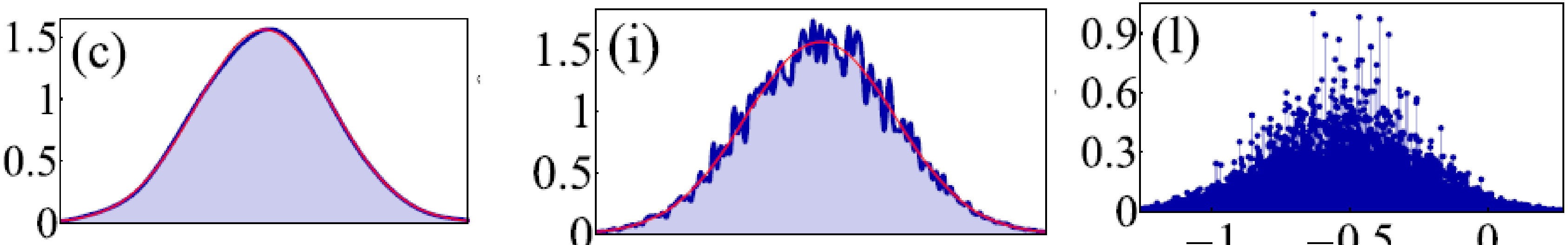
Phys. Rev. E 100 (1), 012218(2020)

Initial state with a **high** participation ratio

NO revivals, NO comb-like structure

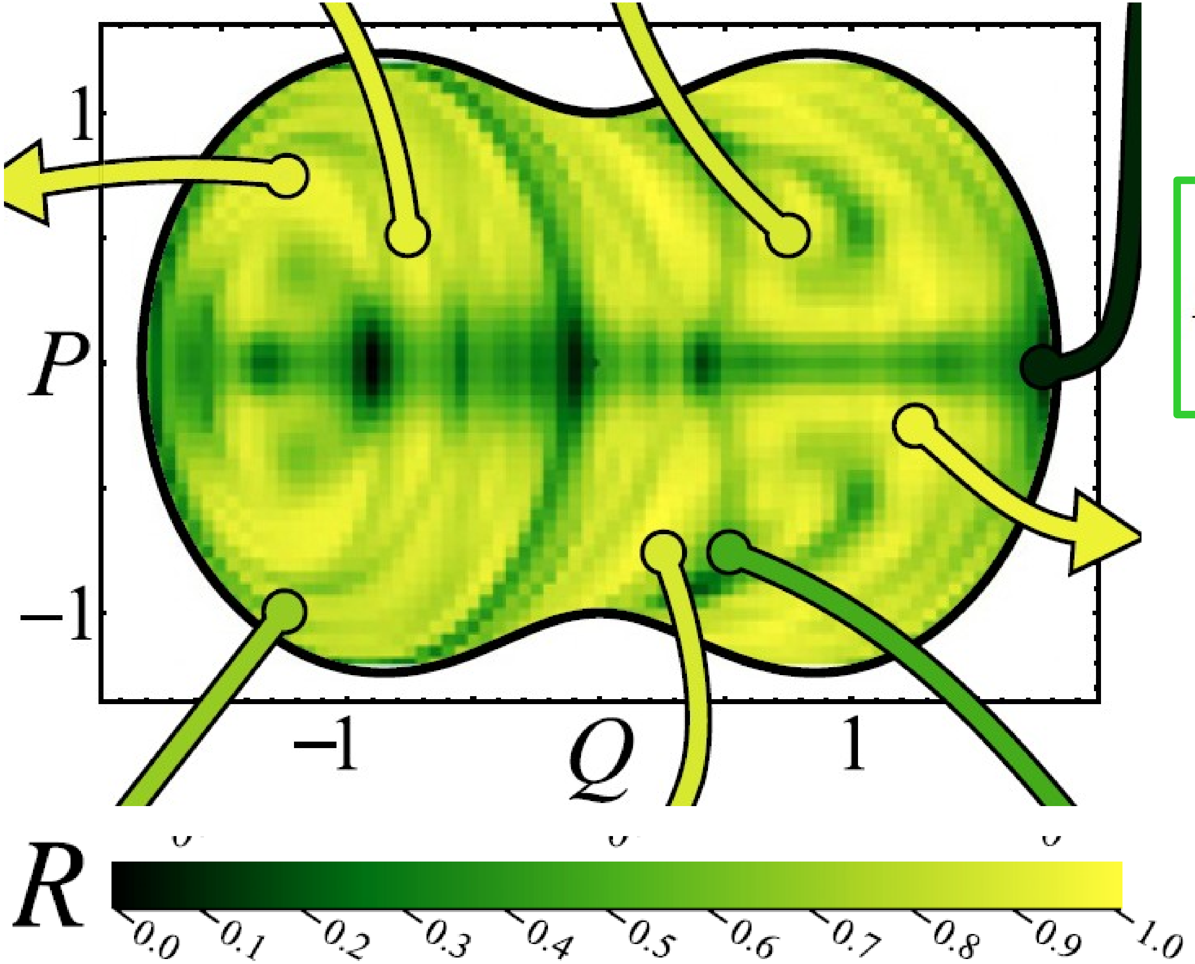


All the energy components participate. Ramp to saturation value described by GOE correlations



Random state

Most of the coherent states behave as random states at long times

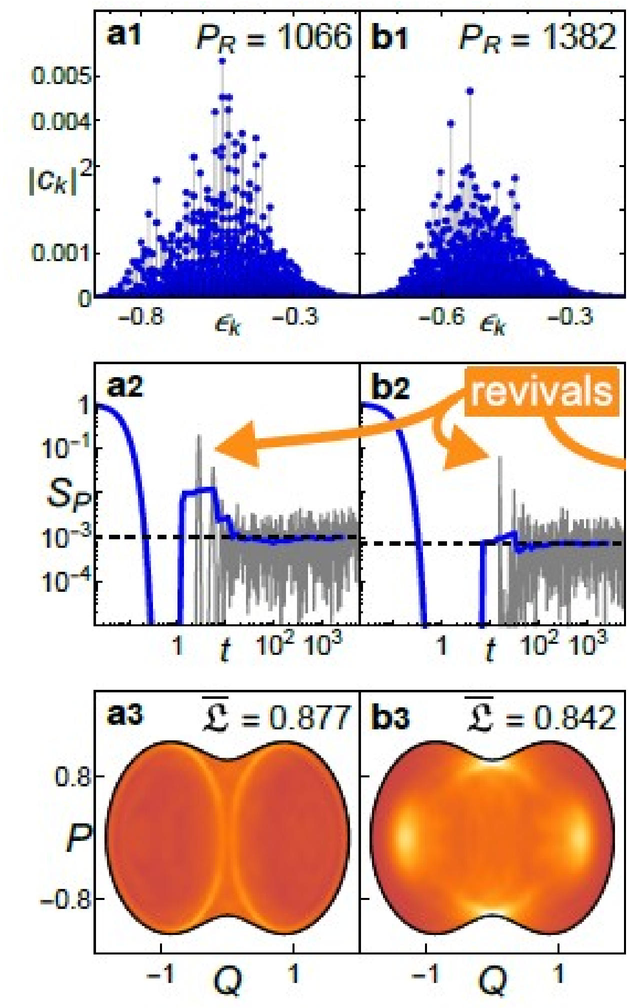


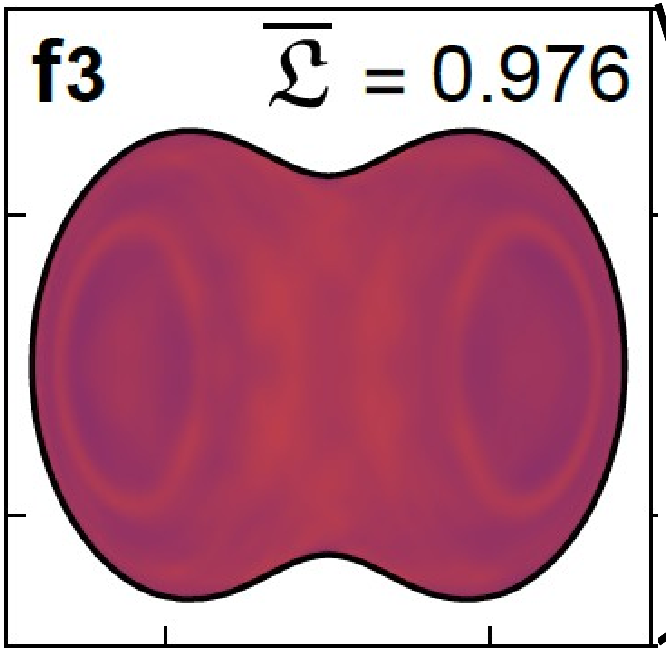
$$R = \frac{S_P^{\infty,(\mathrm{rand})}}{S_P^{\infty}} = \frac{P_R}{P_R^{(\mathrm{rand})}}$$

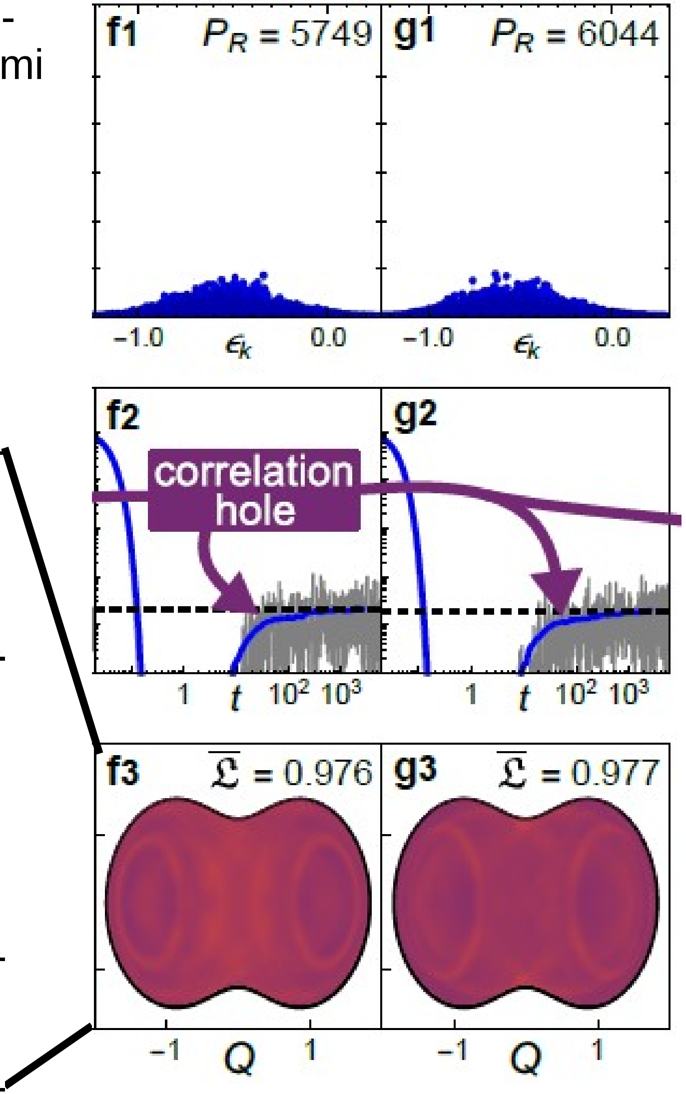
The participation ratio of most coherent states is very similar to the participation ratio of random sates

Even for states with high PR, no comb-like structure and no revivals, the Husimi function of the temporal average of its density matrix reveals a *soft* concentration around periodic orbits.

$$\overline{\rho} = \lim_{T\to\infty} \int_0^T dt\ |\Psi t\rangle\langle\Psi t|$$





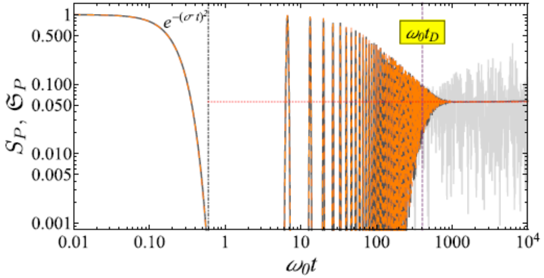


Is there a way to detect this phase-space scarring from the expectation value of an observable?

Nat. Comm. in press (2021)

Final remarks

Important insight obtained from studying energy participation ratio and survival probability of initial coherent states



At regular regions

The survival probability can be classically understood: the major difference is observed at times larger than Heisenberg time when the discrete nature of spectrum reveals in the form of fluctuations around ($1/P_R$).

At chaotic regions

-The quantum participation ratio uncovers structures associated to classical unstable periodic orbits.

- Most of initial coherent states are random-like and GOE correlations in the spectrum determine their final ramp toward the saturation value ($1/P_R$)