Unstable periodic orbits and scars in the Dicke model

Saúl Pilatowsky Cameo [1]
David Villaseñor [1]
Miguel A Bastarrachea-Magnani [2,3]
Sergio Lerma-Hernández [4]
Lea F Santos [5]
Jorge G Hirsch [1]

Virtual Gathering 2021
Ergodicity and chaos in many-body systems

1 Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Mexico City, Mexico.
2 Department of Physics and Astronomy, Aarhus University, Ny Munkegade, Aarhus C, Denmark.
3 Departamento de Física, Universidad Autónoma Metropolitana-Iztapalapa, Mexico City, Mexico.
4 Facultad de Física, Universidad Veracruzana, Circuito Aguirre Beltrán s/n, Xalapa, Veracruz, Mexic
5 Department of Physics, Yeshiva University, New York, NY, USA.

Bohr-Sommerfeld (Regular systems)

Periodic orbits in classical limit

↓

Quantum spectrum (ϵ_i)

$$\oint p \, \mathrm{d}q = n(2\pi\hbar) \quad \Longleftrightarrow \quad \int_{\epsilon_i}^{\epsilon_{i+1}} T \mathrm{d}\epsilon = 2\pi\hbar$$

Quantizatio
n

Example: The harmonic oscillator $\hat{H} = \hbar\omega\left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right)$

Classical Hamiltonian

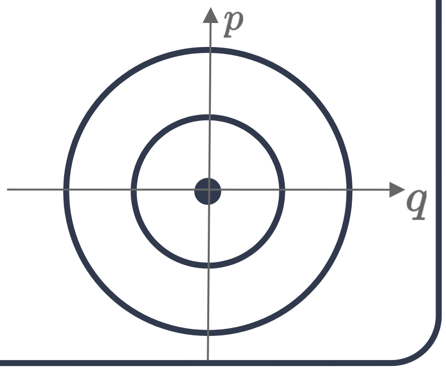
$$H = \frac{\omega}{2}\left(q^2 + p^2 + \frac{1}{2}\right)$$

Periodic orbits of energy ϵ

$$\mathcal{O}_\epsilon(t) = \Big(\underbrace{A(\epsilon)\sin(\omega t)}_{q(t)}, \underbrace{A(\epsilon)\cos(\omega t)}_{p(t)}\Big)$$

with period

$$T = \frac{2\pi}{\omega}$$

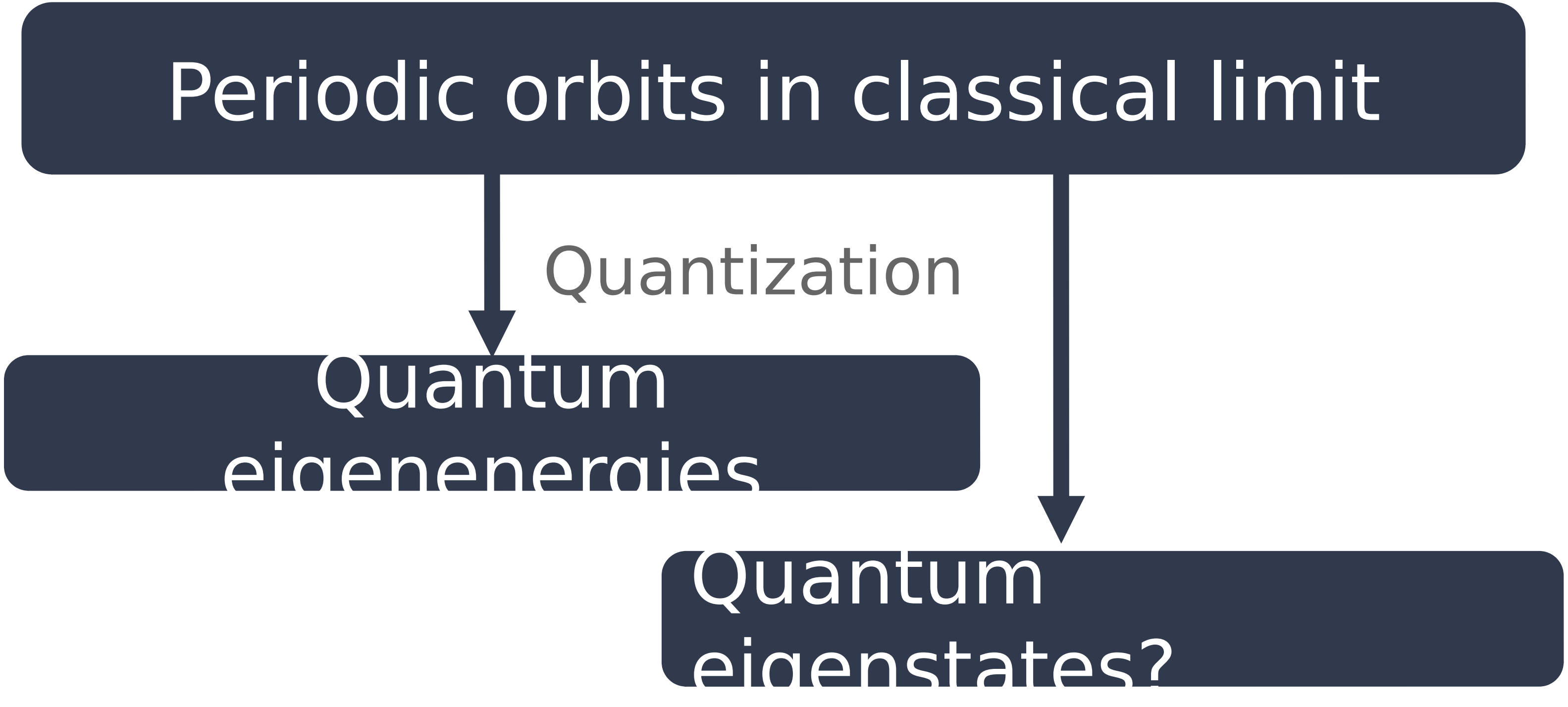


Eigenenergies ϵ_n

$$\int_{\epsilon_{n+1}}^{\epsilon_n} T \mathrm{d}\epsilon = 2\pi\hbar$$

$$\Rightarrow \epsilon_{n+1} - \epsilon_n = \boxed{\hbar\omega}$$

Regular Systems

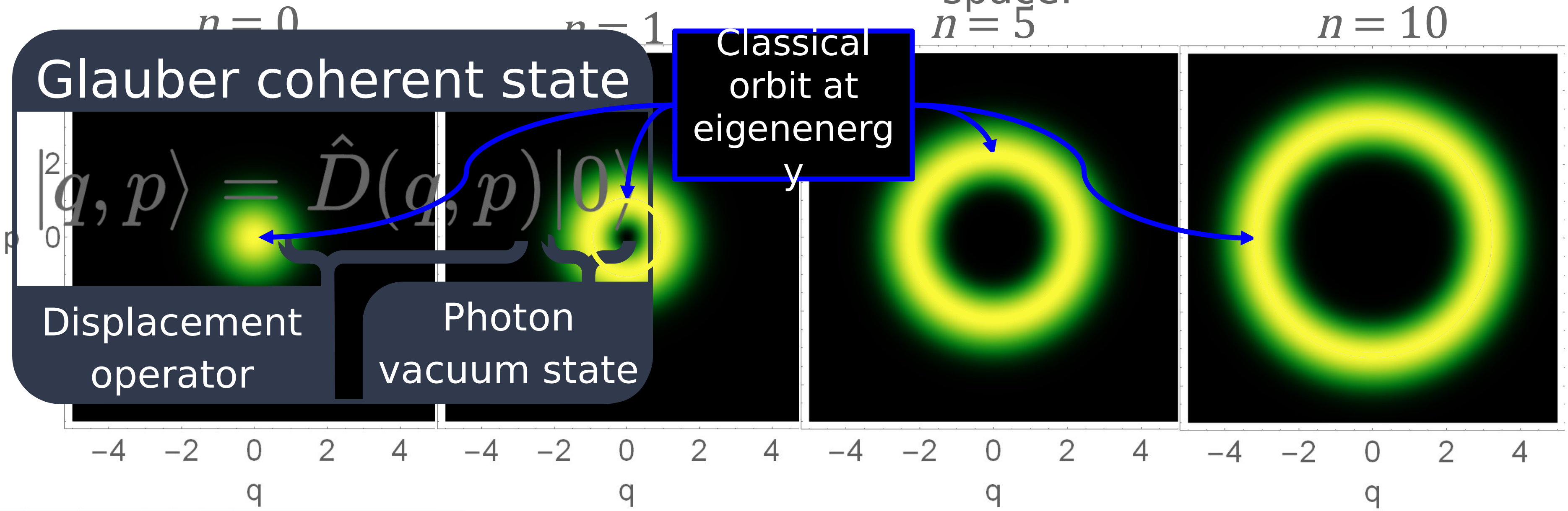


Example: The harmonic oscillator $\hat{H} = \omega\left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right)$

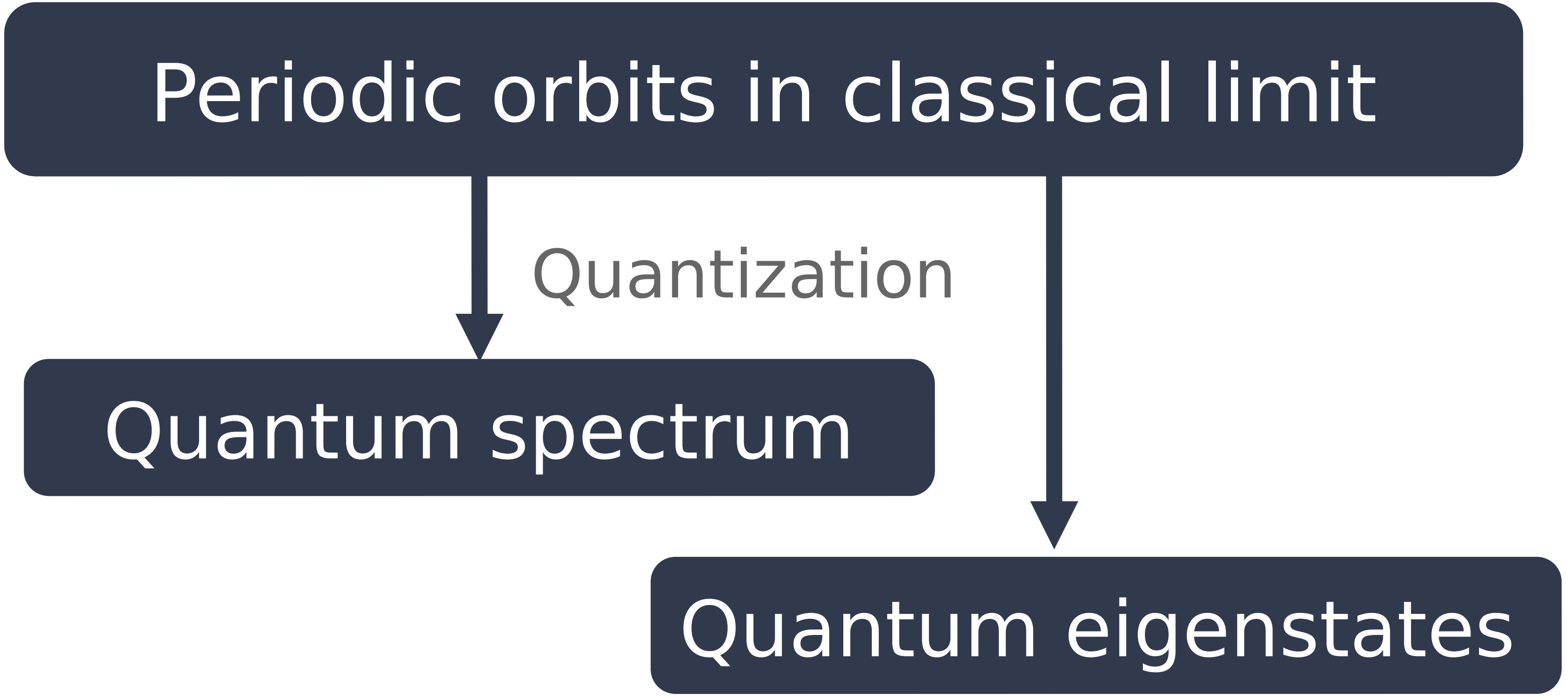
Husimi function

$$\mathcal{Q}_n(q,p) = \left|\langle q,p|n\rangle\right|^2$$

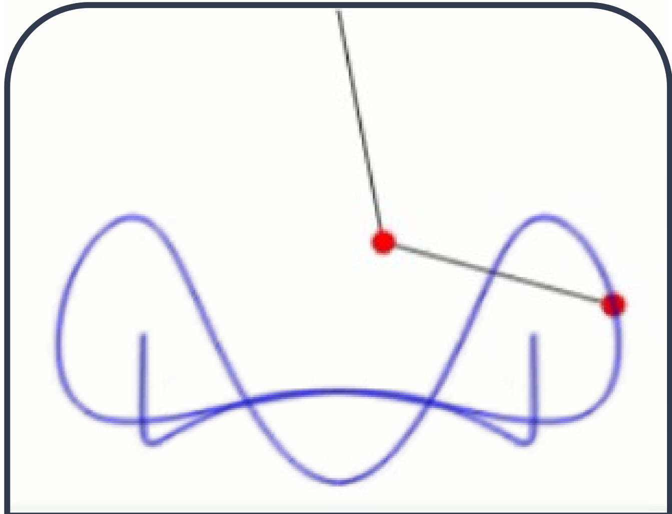
Allows to visualize how state $|n\rangle$ is distributed in phase space.



Regular Systems

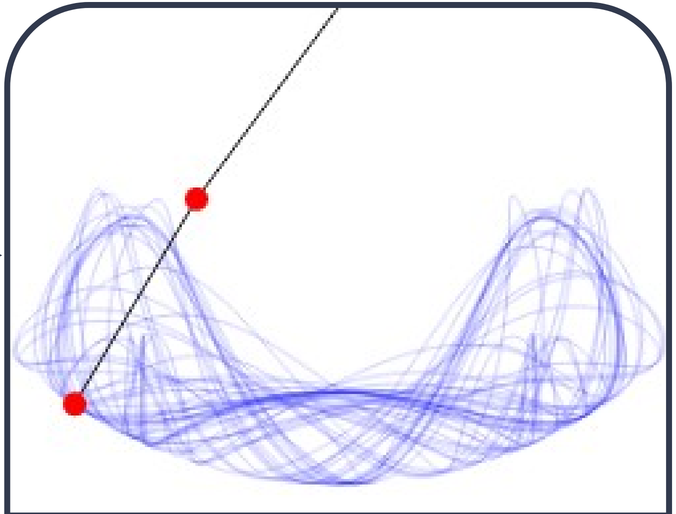


Chaotic systems also have periodic orbits



Periodic orbit of the chaotic double pendulum

Perturbation in random direction



Chaotic trajectory

Animation from: Stricker, H-P Mathematics Stack Exchange, https://math.stackexchange.com/q/2971394 (201

Gutzwiller theory (chaotic systems)

Gutzwiller, M. C. J. Math. Phys. **12**, 343–358 (1971)

Unstable periodic orbits in classical limit

Quantization

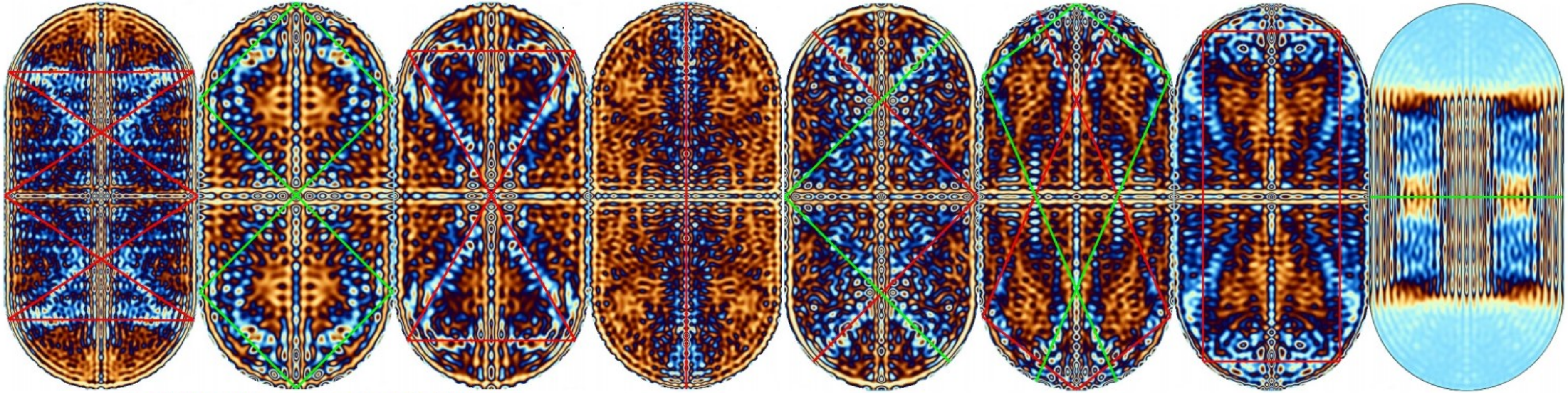
Quantum spectrum

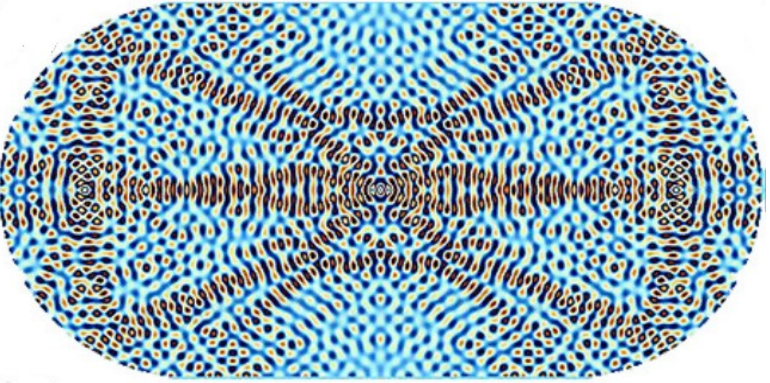
Quantum eigenstates?

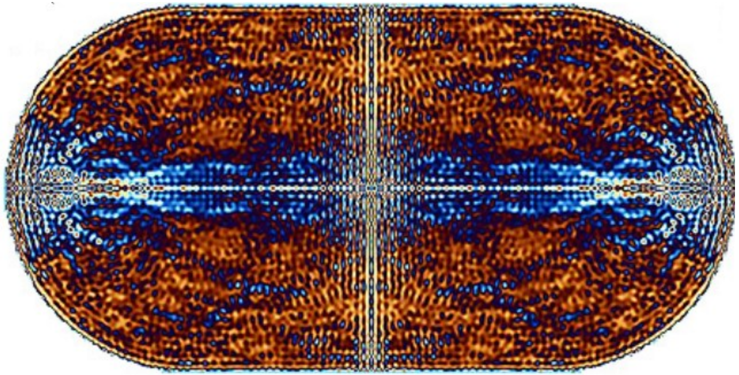
Yes! (some of them)
-Heller

Quantum scars

Heller, E. J. Phys. Rev. Lett. **53**, 1515–1518 (1984)







King, C. C. Quanta **3**, 16-31 (2014)

The Dicke model

Dicke, R. H. Phys. Rev. **93**, 99 (1954).

A **single-mode cavity** of frequency ω interacting with $\mathcal{N}$ **two-level atoms** with level spacing ω_0.

$$\hat{H}_D = \underbrace{\omega \hat{a}^\dagger \hat{a}}_{\text{Energy of photons}} + \underbrace{\omega_0 \hat{J}_z}_{\text{Energy of atoms}} + \underbrace{\frac{\gamma}{\sqrt{\mathcal{N}}} \hat{J}_x (\hat{a}^\dagger + \hat{a})}_{\text{Coupling}}$$

Energy of photons

Energy of atoms

$$\hat{J}_z = \sum_{i=1}^{\mathcal{N}} \hat{\sigma}_z^{(i)} \quad \hat{J}_y = \sum_{i=1}^{\mathcal{N}} \hat{\sigma}_y^{(i)} \quad \hat{J}_x = \sum_{i=1}^{\mathcal{N}} \hat{\sigma}_x^{(i)}$$

Collective spin operators (all atoms are identical)

Coupling

γ

Coupling strength

The Dicke model: classical limit

- de Aguiar, M. A. M. et al. EPL **15**, 125 (1991),
- Bastarrachea-Magnani, M. A. et al. PRA **89**, 032101 (2014)

Glauber coherent

$$|q,p\rangle = \hat{D}(q,p)|0\rangle$$

$\hat{D}(q,p)$: Displacement operator

$|0\rangle$: Photon vacuum state

Bloch coherent states $(j=\mathcal{N}/2)$

$$|Q,P\rangle = \hat{R}(Q,P)|j,-j\rangle$$

$\hat{R}(Q,P)$: Rotation operator

$|j,-j\rangle$: All atoms in ground state

Classical Hamiltonian

$$|\mathbf{x}\rangle = |q,p\rangle \otimes |Q,P\rangle$$

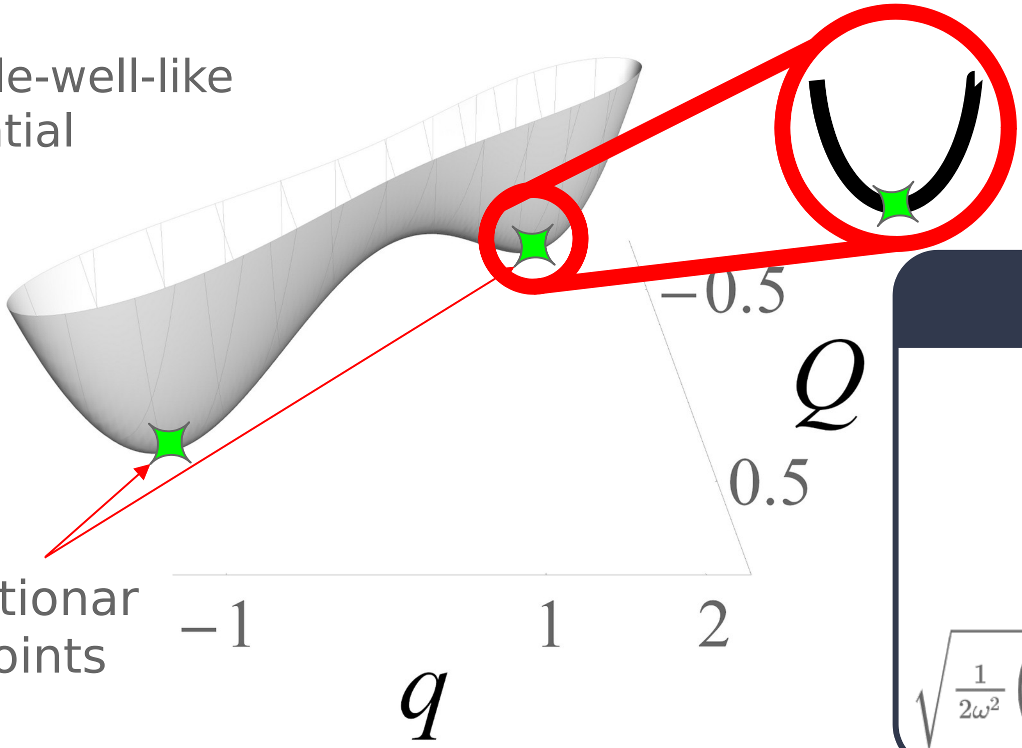
$$\mathbf{x} = (q,p,Q,P)$$

$$h_{\text{cl}}(\mathbf{x}) = \frac{\langle \mathbf{x}|\hat{H}_D|\mathbf{x}\rangle}{j} =$$

$$\frac{\omega}{2}\left(q^2+p^2\right) + \frac{\omega_0}{2}\left(Q^2+P^2\right) + 2\gamma q Q\sqrt{1-\frac{Q^2+P^2}{4}} - \omega_0$$

Periodic orbits in the Dicke model

Double-well-like potential



Stationary Points

≈ Harmonic oscillator

Normal frequencies

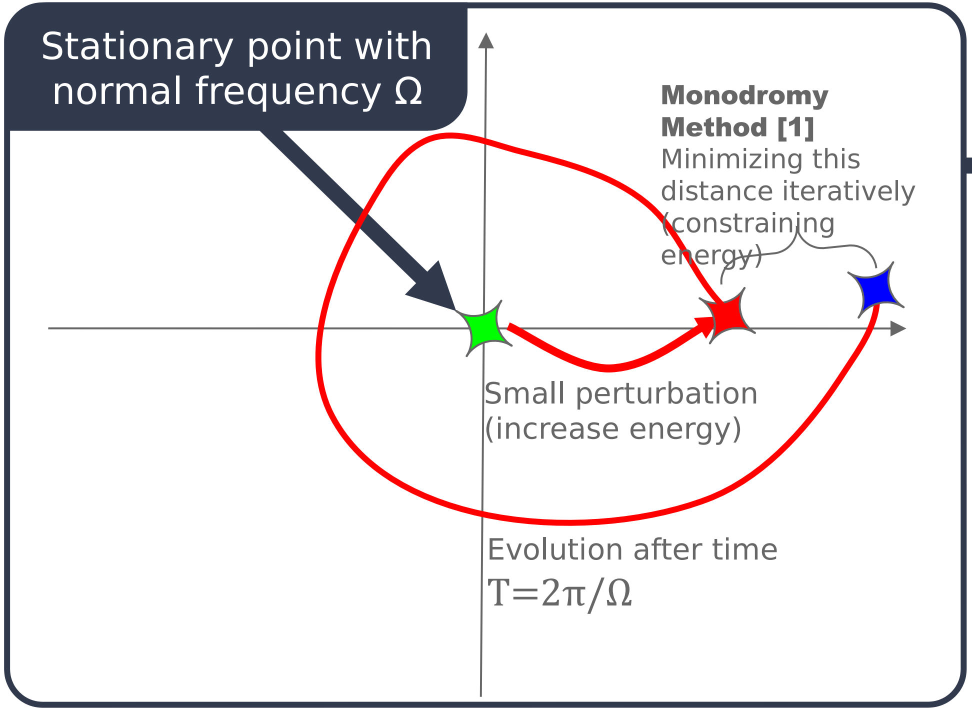
$$\Omega^{A,B} = \sqrt{\frac{1}{2\omega^2}\left((16\gamma^4+\omega^4) \pm \sqrt{(\omega^4-16\gamma^4)^2+4\omega^6\omega_0^2}\right)}$$

Theorem 2.1 of [Alan Weinstein, Inv. Math. **20**, 47–57 (1973)]

“A family of periodic orbits emanates from for **each** normal frequency.”

Finding periodic orbits by perturbing stationary points

[1] M.A.M de Aguiar et. al, Ann. Phys. **180** 167-205 (1987)
[2] S. Pilatowsky Cameo et. al, New. J. Phys. (2021) In press. 10.1088/1367-2630/abd2e6

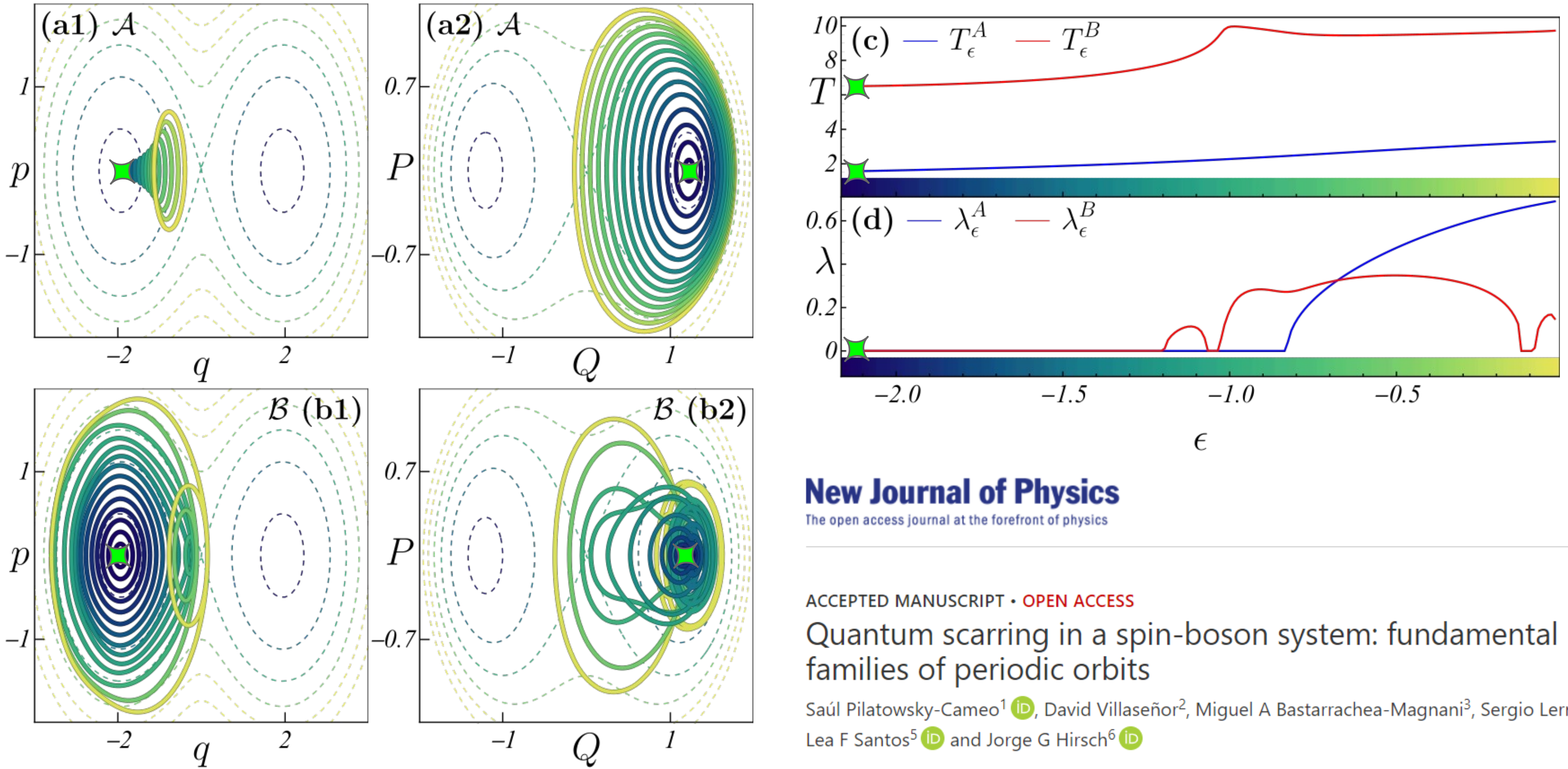


True periodic orbit with increased energy

repeat

Family of periodic orbits corresponding to fundamental frequency Ω. [2]

The Dicke model: fundamental families of PO



New Journal of Physics

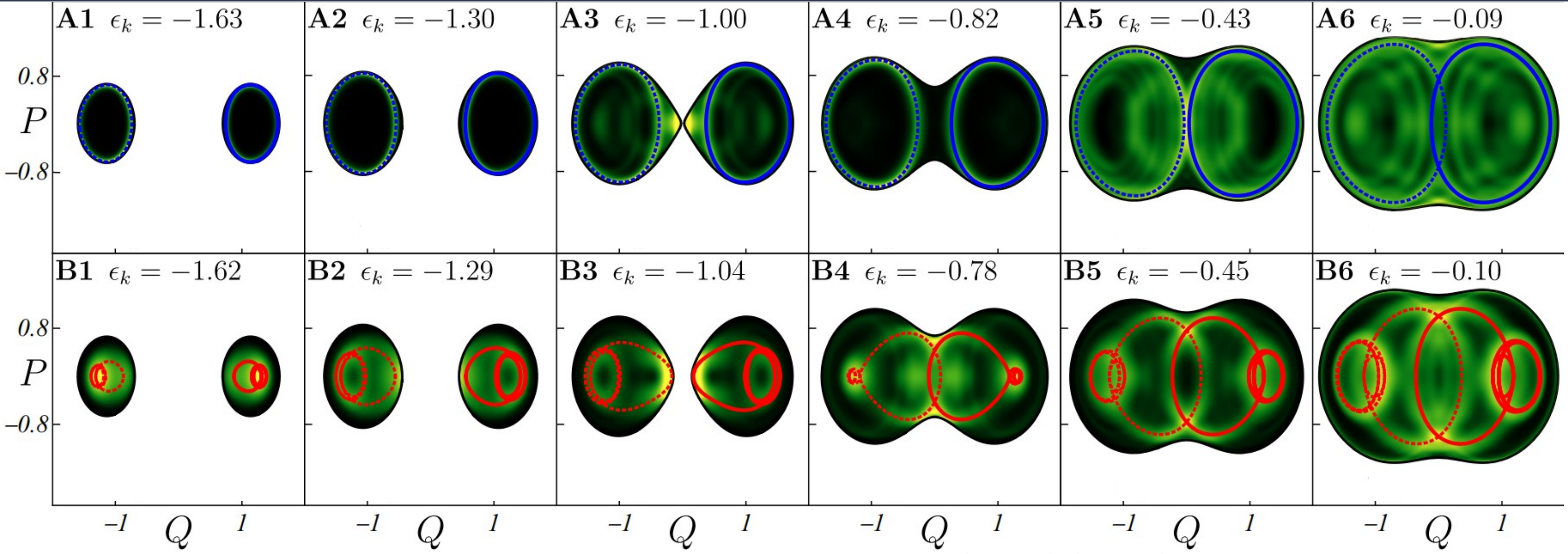
The open access journal at the forefront of physics

ACCEPTED MANUSCRIPT • OPEN ACCESS

Quantum scarring in a spin-boson system: fundamental families of periodic orbits

Saúl Pilatowsky-Cameo[1], David Villaseñor[2], Miguel A Bastarrachea-Magnani[3], Sergio Lerma[4], Lea F Santos[5] and Jorge G Hirsch[6]

The Dicke model: scars of fundamental families



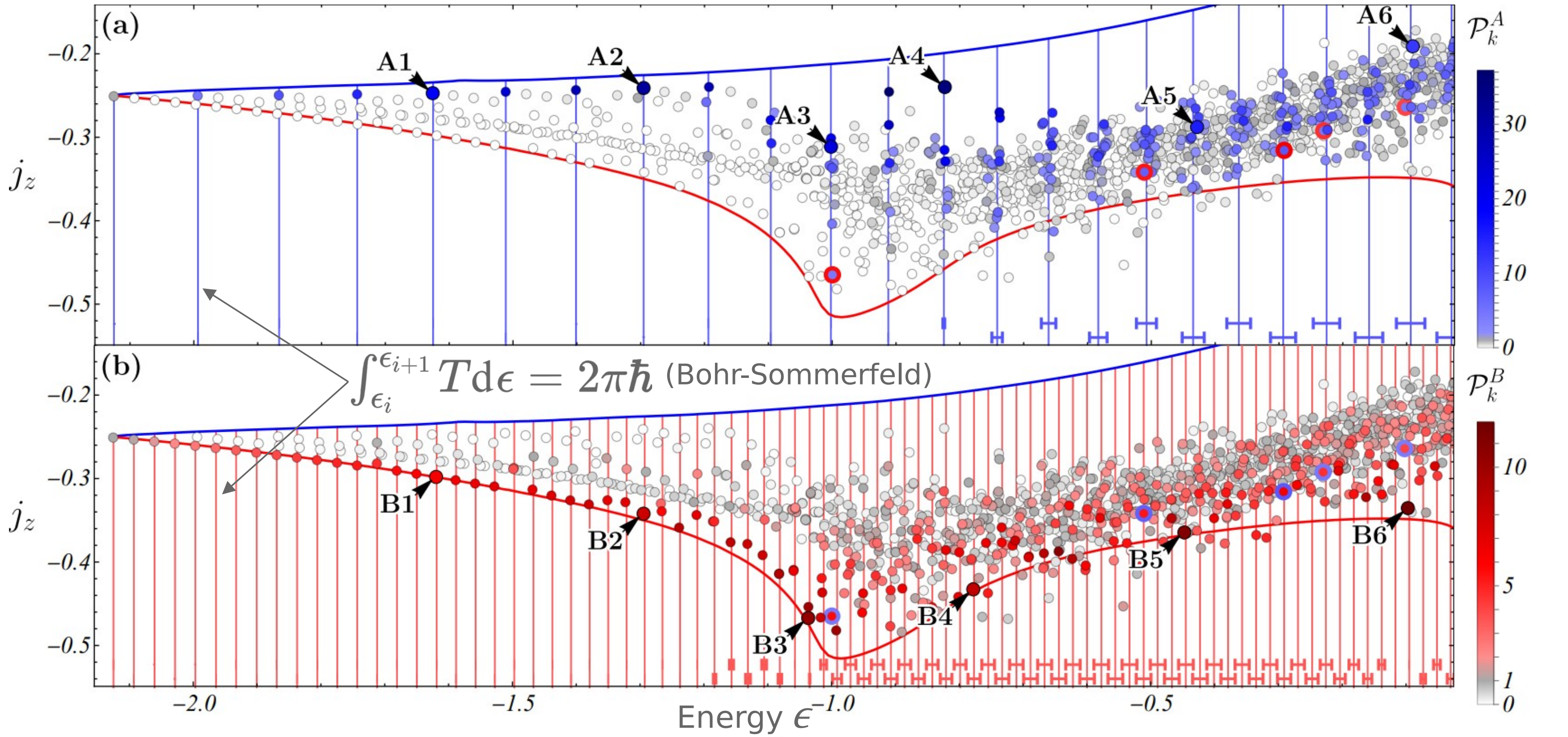
$$\mathcal{P}_k = \frac{\mathrm{tr}\left(\hat{\rho}_{\mathrm{PO}}\,\hat{\rho}_k\right)}{\mathrm{tr}\left(\hat{\rho}_{\mathrm{PO}}\,\hat{\rho}_{\mathrm{DS}}\right)}$$

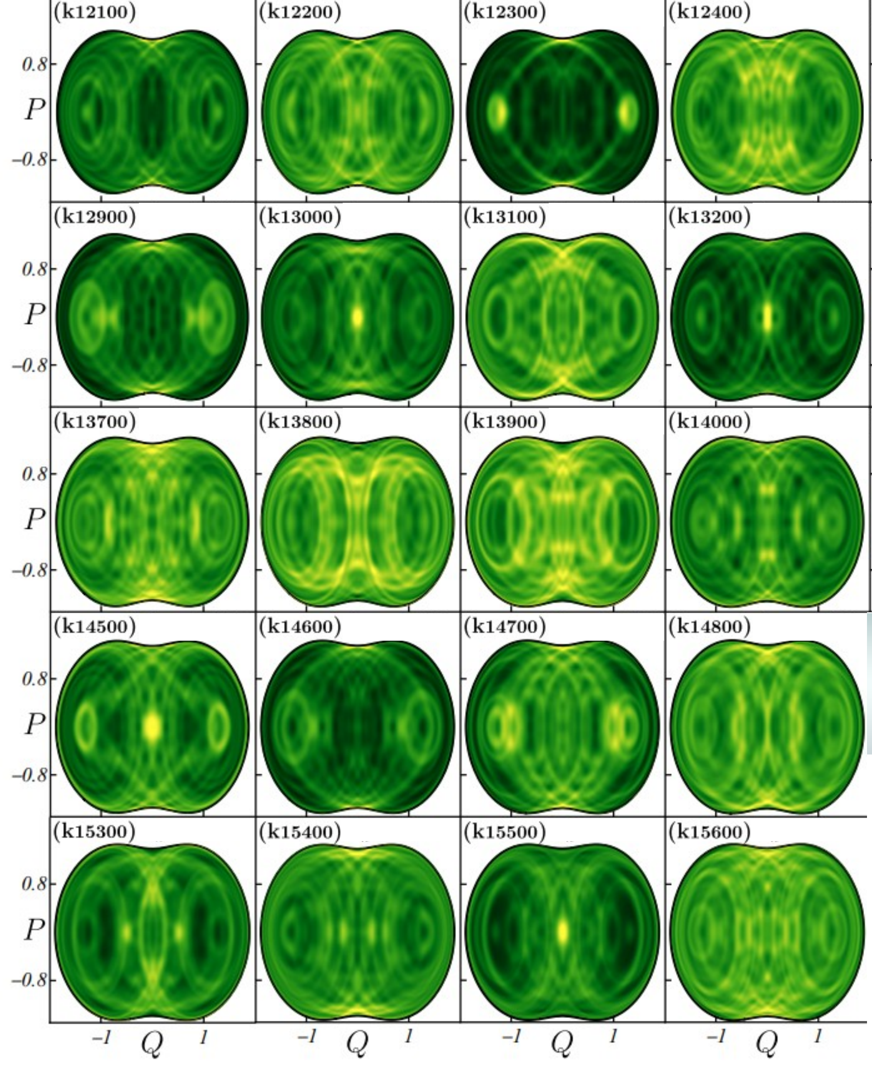
$\hat{\rho}_{\mathrm{k}} = |E_k\rangle\langle E_k|$

$\hat{\rho}_{\mathrm{PO}} =$ Tubular state around periodic orbit

$\hat{\rho}_{\mathrm{DS}} =$ Totally delocalized state

Peres lattice colored by scarring measure





nature COMMUNICATIONS

Release date: 8-feb-2021

ARTICLE

https://doi.org/10.1038/s41467-021-21123-5 OPEN

Ubiquitous quantum scarring does not prevent ergodicity

Saúl Pilatowsky-Cameo [1], David Villaseñor [1], Miguel A. Bastarrachea-Magnani [2,3], Sergio Lerma-Hernández [4], Lea F. Santos [5]✉ & Jorge G. Hirsch [1]✉

Release date: 8-feb-2021

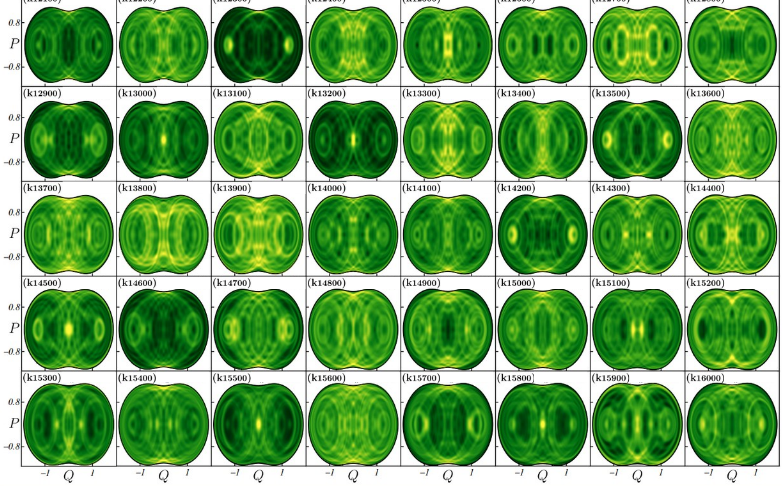
nature COMMUNICATIONS

ARTICLE

https://doi.org/10.1038/s41467-021-21123-5 OPEN

Ubiquitous quantum scarring does not prevent ergodicity

Saúl Pilatowsky-Cameo[1], David Villaseñor[1], Miguel A. Bastarrachea-Magnani[2,3], Sergio Lerma-Hernández[4], Lea F. Santos[5] & Jorge G. Hirsch[1]



- **All** eigenstates of the chaotic Dicke model are scarred
- Pure states never occupy more than half of the available phase space.
- Quantum ergodicity is achievable only as an ensemble property, after temporal averages are performed

See David Villaseñor’s talk tomorrow