Quantum phase transitions of bosons trapped in three wells.

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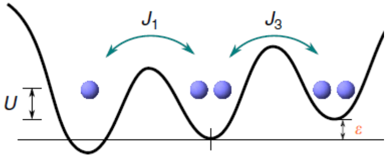
February 07, 2021

Table of contents

1. The model of the three well boson system
   - The quantum Hamiltonian and their semi-classical limit
2. The Integrable cases
   - The case $U = 0$
   - The case $J = 0$
   - The case $\epsilon = 0$
3. The non-integrable cases
   - The non-integrable cases
   - Perspectives

I- The model of the three well boson system



The quantum Hamiltonian and their semi-classical limit

- The model is a variation of the known bosonic Hubbard model with N bosons (K. W. Wilsmann, L. H. Ymai, A. P. Tonel, J. Links, and A. Foerster, Comm. Phys. 1 (2018).)

$$H = \underbrace{\frac{U}{N}\left(\hat{N}_1 - \hat{N}_2 + \hat{N}_3\right)^2}_{\text{Interaction boson term}} \qquad \overbrace{+\epsilon\left(\hat{N}_3 - \hat{N}_1\right)}^{\text{External field interaction term}}$$
$$\underbrace{+\frac{J}{\sqrt{2}}\left(a_1^\dagger a_2 + a_2^\dagger a_1\right) + \frac{J}{\sqrt{2}}\left(a_2^\dagger a_3 + a_3^\dagger a_2\right)}_{\text{Tunneling boson term}} \qquad (1)$$

- In the base $\{|N_1, N_2, N_3\rangle\}$ the matrix representation is diagonalizable, obtaining the eigen-energies and their eigenvectors.

The quantum Hamiltonian and their semi-classical limit

- The classical limit is obtained taking $N \to \infty$ from the association

$$a_j \to \sqrt{N_j}\exp(i\phi_j), \qquad a_j^\dagger \to \sqrt{N_j}\exp(-i\phi_j) \tag{2}$$

- In the classical variables (N_i, ϕ_i), we obtain the classical Hamiltonian

$$\begin{aligned}\mathcal{H} =& \frac{U}{N}\left(N_1 - N_2 + N_3\right)^2 + \epsilon\left(N_3 - N_1\right) \\ &+ J\left(\sqrt{2N_1N_2}\cos(\phi_2 - \phi_1) + \sqrt{2N_3N_2}\cos(\phi_2 - \phi_3)\right) \\ &+ L(N_1 + N_2 + N_3 - N)\end{aligned} \tag{3}$$

- The search for quantum phase transition in the parameter space (U, J, ϵ) is realized from the properties of the classical minimal energy configurations.

The quantum Hamiltonian and their semi-classical limit

- Classical minimal energy configurations satisfy the equilibrium conditions

$$\frac{\partial \mathcal{H}}{\partial N_i} = \frac{\partial \mathcal{H}}{\partial \phi_i} = 0 \tag{4}$$

- Quantum phase transitions are associated in this model with bifurcations of the classical minimal energy equilibrium points.
- The minimal energy coordinates $N_i(U, J, \epsilon)$ behaves as a typical order parameters.
- We compare the classical minimal energy variable, with the respective quantum ground state energy and quantum ground state mean value $\langle \hat{N}_i \rangle$

II- The integrable cases

The case $U = 0$

- We have three possible stationary points in this case, considering $\epsilon > 0$ we have (For the quantum case we use $N = 20$)

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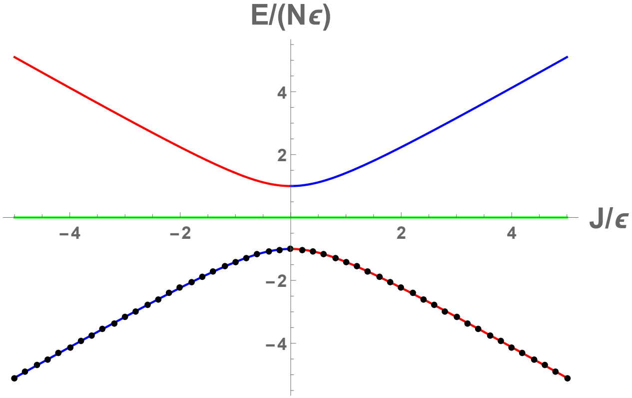
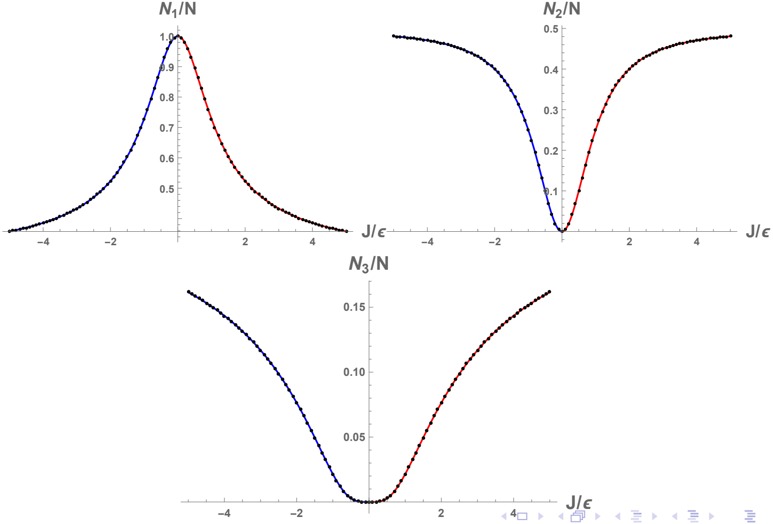
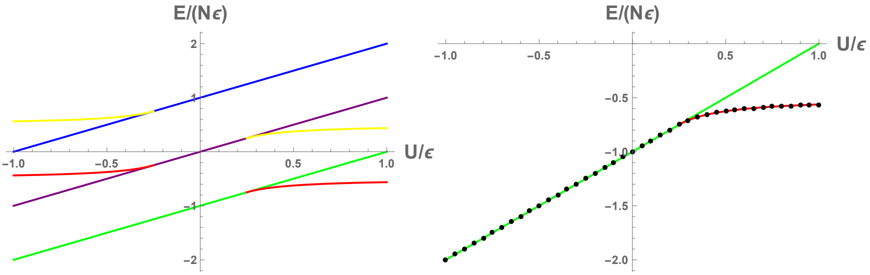


- The coordinates N_i the minimal energy point, and the quantum mean value $\langle N_i \rangle$ are $(i = 1, 2, 3)$



The case $J = 0$

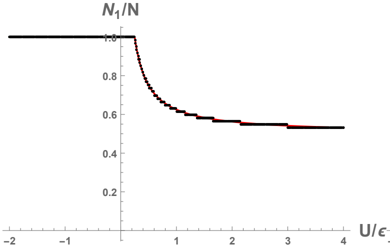
- We have five possible stationary points in this case, considering $\epsilon > 0$ we have (For the quantum case we use $N = 60$)

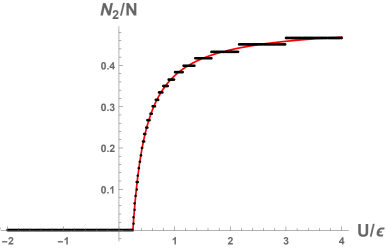


- Note as in $U/\epsilon = 1/4$, we have explicitly a bifurcation.
- In $U/\epsilon = 1/4$ we have a second order phase transition with $U = \epsilon/4$ the critical line.

The case $J = 0$

- The minimal energy coordinates N_i and the ground state quantum mean values $\langle N_i \rangle$ are

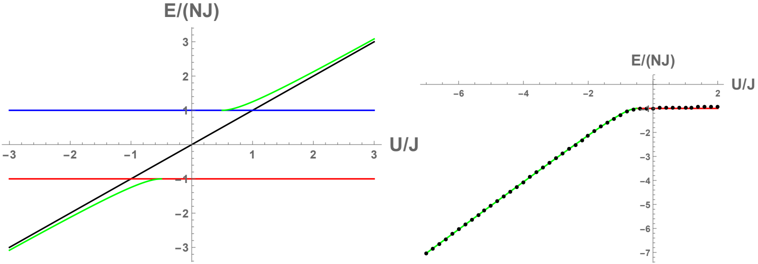




- In particular, always we have that $N_3 = \langle \hat{N}_3 \rangle = 0$
- As the states $|N_1, N_2, N_3\rangle$ are the eigenstates of the system, the values of $\langle \hat{N}_i \rangle$ are non-negative integers.
- In $U/\epsilon = 1/4$ we have a discontinuity in the the derivative of N_i.

The case $\epsilon = 0$

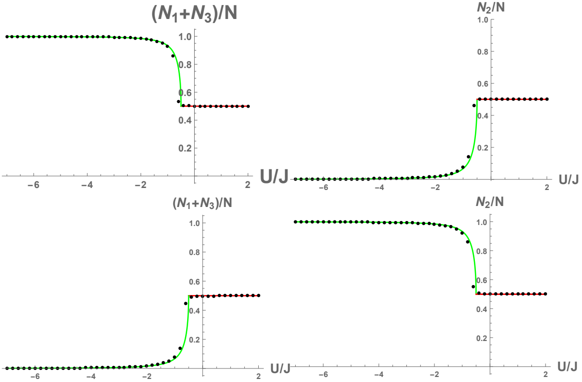
- In this case, we use $N = 20$. There are a symmetry between the wells 1 and 3.
- We have 5 equilibrium points



- In $U/J = -0.5$, we have a second order phase transition.
- We have for $U/J < -0.5$ two minimal energy points with the same energy.
- This is an expression of a doubly degenerated ground state for $U/J < -0.5$.

The case $\epsilon = 0$

- For the ground state, the system behaves effectively as a two well system, where the wells 1 and 3 behaves as an unique well $\{13\}$.
- The degeneration is caused by the two well symmetry $2 \leftrightarrow \{13\}$
- We only have coincidence with the semi classical limit considering a tiny non-zero value of ϵ.



The case $\epsilon = 0$

- With this tiny external field the well 1 and 3 still behaves as an unique well $\{13\}$.
- But the symmetry between the wells 1 and 3 is explicitly broken.

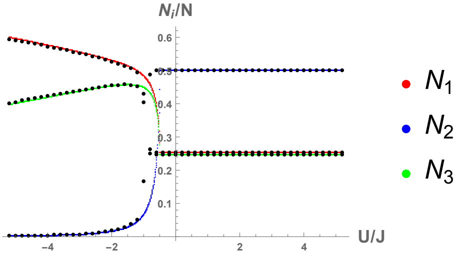


Figure: $\epsilon = 0.005$

- This suggests that the system is extremely sensitive to the presence of tiny external field ϵ in this regime.

II- The non-integrable cases

- In the general problem, the classical equilibrium points are associated with the roots of a seventh degree polynomial.
- Therefore, there are at most seven equilibrium points.

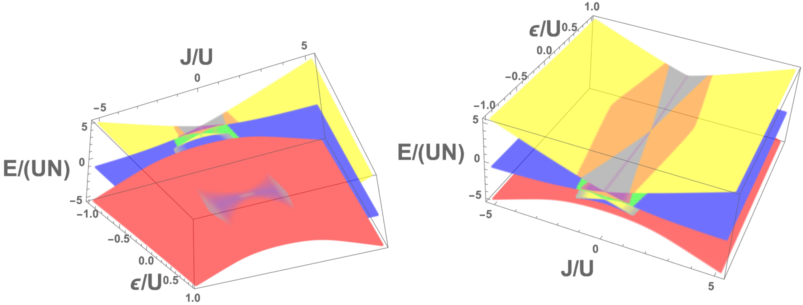
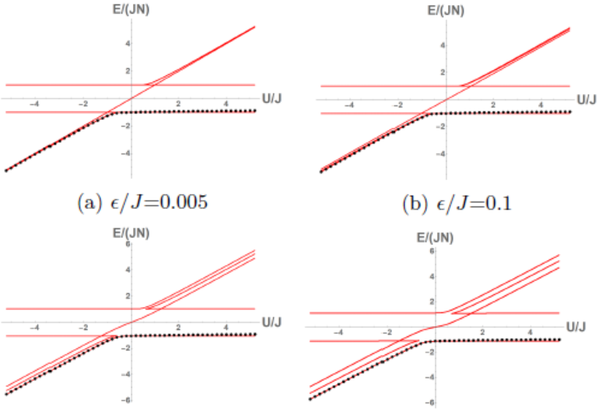


Figure: Here $U < 0$

- In the left figure, the red and purple regions are different phases!

- In particular, we can analyze the behavior close the integrable cases (close of $\epsilon = 0$).



(a) ϵ/J=0.005

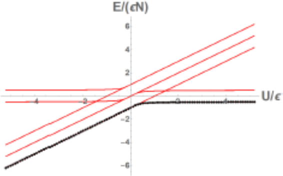
(b) ϵ/J=0.1

(c) ϵ/J=0.3

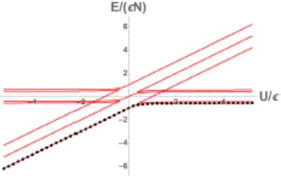
(d) ϵ/J=0.5

The non-integrable cases

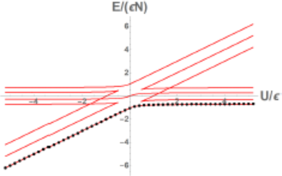
- Close of $J = 0$.



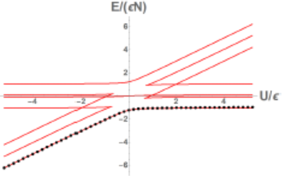
(a) J/ϵ=0.001



(b) J/ϵ=0.1

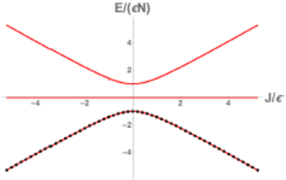


(c) J/ϵ=0.3

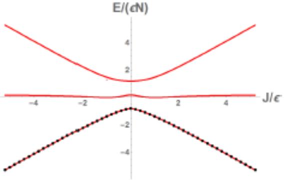


(d) J/ϵ=0.7

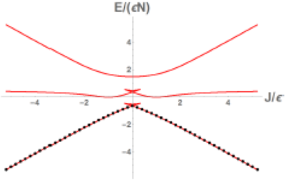
- Close of $U = 0$.



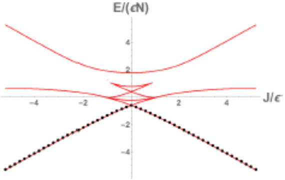
(a) U/ϵ=0.001



(b) U/ϵ=0.2



(c) U/ϵ=0.5



(d) U/ϵ=0.8

Perspectives

- We characterize the stationary properties of the semi classical system, finding legitimate quantum phase transitions points in the limit $N \to \infty$.
- The semi classical approximations explain very well many quantum ground state properties.
- We intend to study the classical and quantum dynamics using a quantum chaos perspective.

Collaborators

- Jorge G. Hirsch (UNAM).
- Jorge Chávez-Carlos (UNAM).
- Lea F. Santos (Yeshiva University).
- Itzhak Roditi (CBPF)
- In the future dynamical study: Angela Foerster and Karin Wilsmann.