Overview
Ergodicity and Chaos

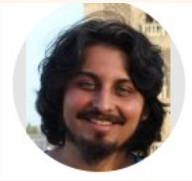
Sergio Lerma[1] & Lea F. Santos[2]

[1]Facultad de Física, Universidad Veracruzana, Veracruz, Mexico
[2]Department of Physics, Yeshiva University, New York, NY, USA

Welcoming group
Open to collaborations

Purpose of this overview is not only to promote some final discussions, but to motivate future collaborations with Lerma-Santos-Hirsch's group and among the participants.

Time line



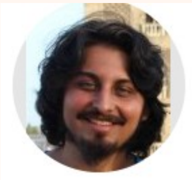
2011

Researchers and postgraduate students

Jorge Hirsch

M. Bastarrachea

Time line



2014

2011

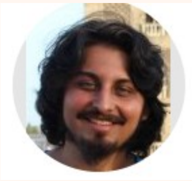
Researchers and postgraduate students

Jorge Hirsch

M. Bastarrachea

S. Lerma

Time line



2011 **2014** **2015** **2016**

Researchers and postgraduate students

Jorge Hirsch

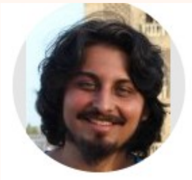
M. Bastarrachea

S. Lerma

J Chávez-Carlos

B. López-del-Carpio

Time line



2011

2014

2015

2016

2017

Researchers and postgraduate students

Jorge Hirsch

M. Bastarrachea

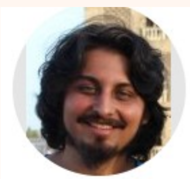
S. Lerma

J Chávez-Carlos

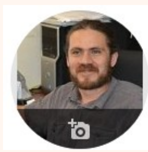
B. López-del-Carpio

A. Relaño

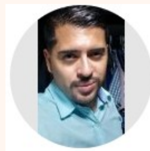
Time line

2011



2014



2015

2016

2017

2018

Researchers and postgraduate students

Jorge Hirsch

M. Bastarrachea

S. Lerma

J Chávez-Carlos

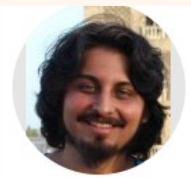
Lea Santos

B. López-del-Carpio

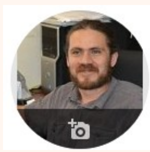
A. Relaño

2018 First Workshop

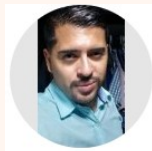
Time line



2011



2014

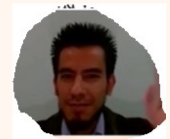


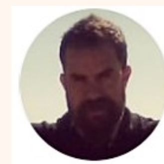
2015

2016

2017

2018





2019

Researchers and postgraduate students

Jorge Hirsch

M. Bastarrachea

S. Lerma

J Chávez-Carlos

Lea Santos

Pavel Stránský

D.Villaseñor

E.J. Torres

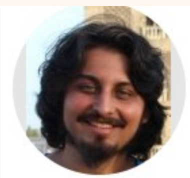
2018 First Workshop

2019 2nd Workshop

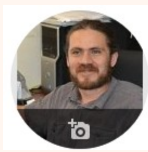
B. López-del-Carpio

A. Relaño

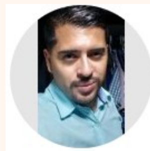
Time line

2011



2014

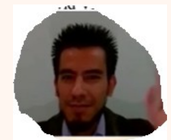
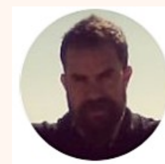


2015

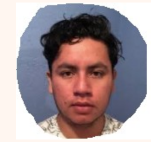
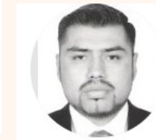
2016

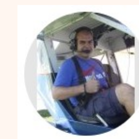
2017

2018

2019



2020

Researchers and postgraduate students

Jorge Hirsch

M. Bastarrachea

S. Lerma

J Chávez-Carlos

Lea Santos

Pavel Stránský

D.Villaseñor

E.J. Torres

Javier de la Cruz

Saúl-Pilatowsky

B. López-del-Carpio

A. Relaño

Mario Quiroz

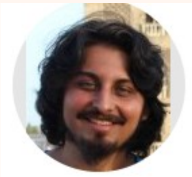
Roberto Montiel

2018 First Workshop

2019 2nd Workshop

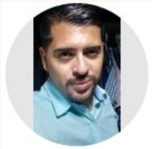
2020 3d Workshop

Time line



2011

2014



2015

2016

2017

2018

2019

2020

2021

Researchers and postgraduate students

Jorge Hirsch

M. Bastarrachea

S. Lerma

J Chávez-Carlos

Lea Santos

Pavel Stránský

D.Villaseñor

E.J. Torres

Javier de la

Saúl-Pilatowsky

A. Relano

Mario Quiroz

Roberto Montiel

Ergodicity and chaos in many-body systems

Around 50 participants!

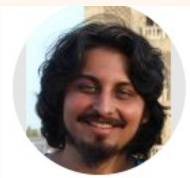
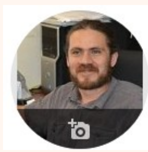
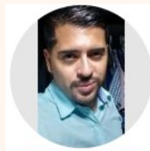
2018 First Workshop

2019 2nd Workshop

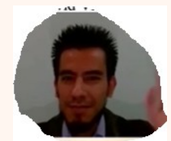
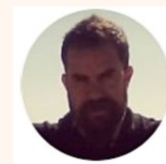
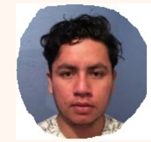
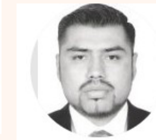
2020 3d Workshop

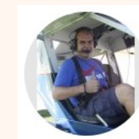
2021 4th Workshop

Time line





2011 **2014** **2015** **2016** **2017** **2018** **2019** **2020** **2021**

Researchers and postgraduate students

Jorge Hirsch
M. Bastarrachea
S. Lerma
J Chávez-Carlos
Lea Santos

Pavel Stránský
D.Villaseñor
E.J. Torres
Javier de la Cruz
Saúl-Pilatowsky

We hope many new collaborations to come

B. López-del-Carpio
A. Relaño
Mario Quiroz
Roberto Montiel

2018 First Workshop
2019 2nd Workshop
2020 3d Workshop
2021 4th Workshop

Topics for discussion

1) Definitions of **ergodicity**
- classical ergodicity
- mixing

quantum ergodicity
- ETH - chaos
- self-averaging
- phase space

2) **Quantum scarring**

Why is it ubiquitous
in the Dicke model?

Measures of scarring:
- scar function
- P from NJP
- ETH vs scarring

3) **Localization**

relationship with scarring
and lack of ergodicity?
- L (phase space)
- Robnik’s group
- IPR, R (Hilbert space)

phase space vs Hilbert space

Eigenvalues vs eigenstates:
- **beta vs PR**
- dependence on basis


nature COMMUNICATIONS

ARTICLE

https://doi.org/10.1038/s41467-021-21123-5 OPEN

Ubiquitous quantum scarring does not prevent ergodicity

Saúl Pilatowsky-Cameo[1], David Villaseñor[1], Miguel A. Bastarrachea-Magnani[2,3], Sergio Lerma-Hernández[4], Lea F. Santos[5] & Jorge G. Hirsch[1]

Topics for discussion

5) **Wigner-Dyson distribution: What is many-body quantum chaos?**

WD needed for thermalization?

WD = chaos in many-body?

WD, but ballistic transport in the SINGLE defect model

6) **Chaos detectors vs symmetries**

- correlation hole
- purity
- off-diagonal elements of observables (SPECK of CHAOS)

Useful also for identifying

- symmetries
- integrable models

7) **Open systems**

- by adding a little bit of **dissipation**, find ***stable regions***
- what PR tells us that classical phase space does not

Topics for discussion

8) **Time scales**
quantum-classical correspondence
- Ehrenfest time
- Thouless time (correlation hole)

- Diffusion time tD
vs
- Heisenberg time tH
- when tD>tH and tH>tD

Relaxation in
real space
momentum space
dependence on observable:
chaos, integrability, initial state
multifractality

9) **Multifractality**
- phase space fragmentation
vs
- Hilbert space fragmentation

10) **Correlations in realistic systems**
Is there any physical quantity that could detect the correlations of the eigenstates of realistic systems?

Observables:
few-body observable, entropies,
quantum Fisher, FOTOC, OTOC
short vs long times

Entropy growth & Kolmogorov-Sinai

Topics for discussion

8) **Time scales**

quantum-classical correspondence

- Ehrenfest time
- Thouless time (correlation hole)

- Diffusion time tD

vs

- Heisenberg time tH
- when tD>tH and tH>tD

Relaxation in

real space

momentum space

dependence on observable:

chaos, integrability, initial state

multifractality

9) **Multifractality**

- phase space fragmentation

vs

- Hilbert space fragmentation

10) **Correlations in realistic systems**

Is there any physical quantity that could detect the correlations of the eigenstates of realistic systems?

Observables:

few-body observable, entropies,

quantum Fisher, FOTOC, OTOC

short vs long times

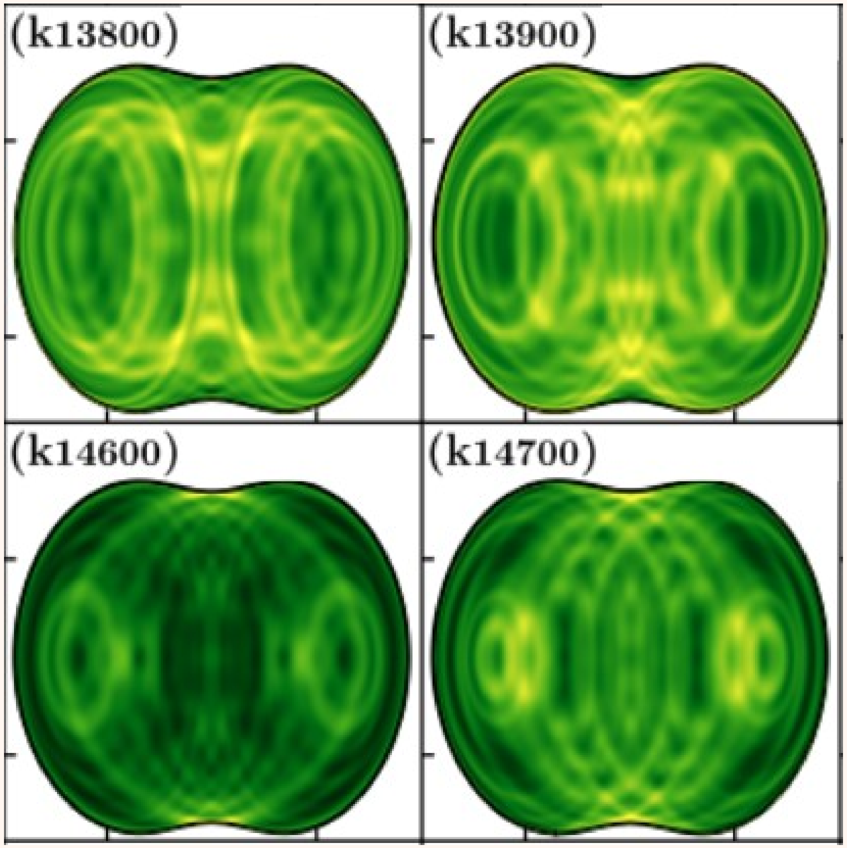
Entropy growth & Kolmogorov-Sinai

ESQPTs from Wigner and Hus funct

Topics for discussion

OTHERS????

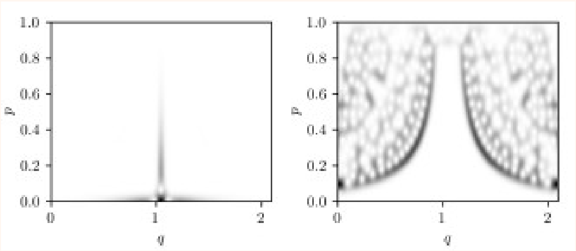
Quantum scarring



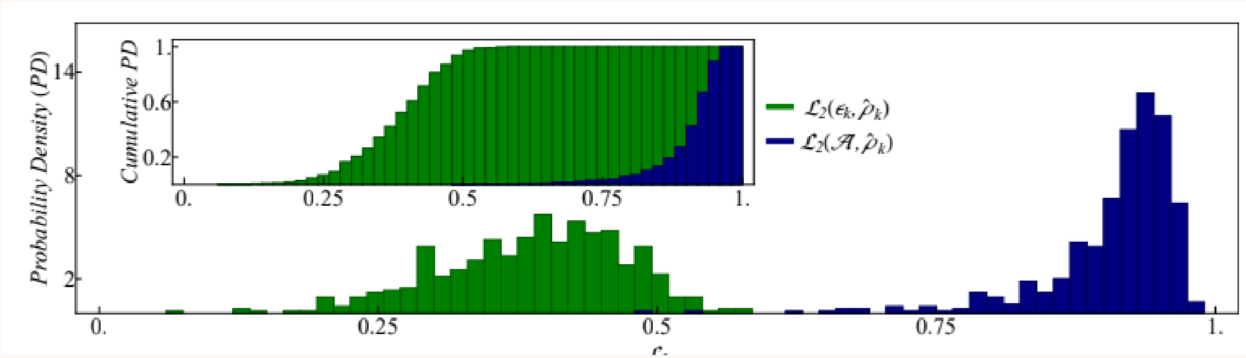
May the semiclassical random wave model presented by Juan Diego help to understand the ubiquitous quantum scarring in the eigenstates of the Dicke model?

States that are NON-ergodic (localized in phase-space L<1) in phase space lead to a violation of ETH?

Are we missing small regularity island in the large energy region of the Dicke model?



Localization measures



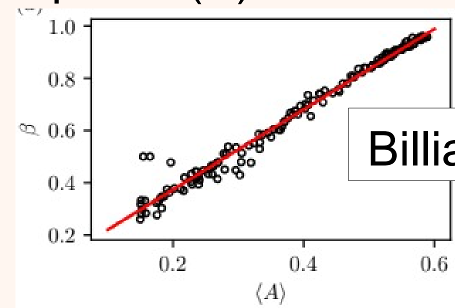
David Villaseñor
L

Crt Lozej
Qian Wang
A

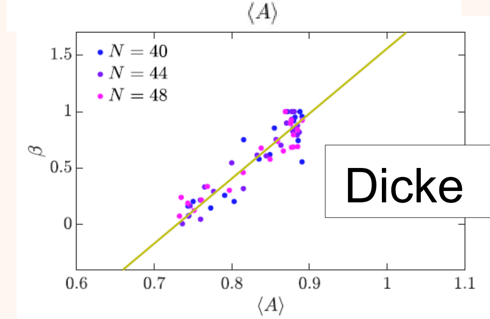
Localization measures in phase space using

-Projection over energy surfaces (L)

-Projection over Bloch sphere (A)



Billiards



Dicke

How does the tD/tH ratio depends on energy and coupling in the Dicke model?

Does the beta distribution describe P(L) as it does for P(A)?

Is the relation between Brody parameter and localization L also linear?

Ergodicity

Wikipedia:
The origins of ergodicity lie in statistical physics, where Ludwig Boltzmann formulated the ergodic hypothesis.

In physics and thermodynamics, the **ergodic hypothesis** says that, over long periods of time, the time spent by a system in some region of the phase space of microstates with the same energy is proportional to the volume of this region, i.e., that all accessible microstates are equiprobable over a long period of time.

The ergodic hypothesis is often assumed in the statistical analysis of computational physics. The analyst would assume that the

average of a process parameter over time and the average over the statistical ensemble are the same.

This assumption - that it is as good to simulate a system over a long time as it is to make many independent realizations of the same system – is not always correct. (FPUT)

Quantum Ergodicity vs ETH

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta-E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha^{ini} |\alpha\rangle$$

$$O_{\beta\alpha} = \langle\beta|O|\alpha\rangle$$

$$O_{\alpha\alpha} = \langle\alpha|O|\alpha\rangle$$

Infinite time average   Thermodynamic average

$$\overline{\langle O(t)\rangle} \equiv \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha} \overset{=?}{\longleftrightarrow} O_{micro} \equiv \frac{1}{N_{E_0,\Delta E}} \sum_{\substack{\alpha \\ |E_0-E_\alpha|<\Delta E}} O_{\alpha\alpha}$$

depends on the initial conditions   depends only on the energy

ETH: the expectation values $O_{\alpha\alpha}$ of few-body observables do not fluctuate for eigenstates close in energy

Chaos guarantees thermalization, ETH

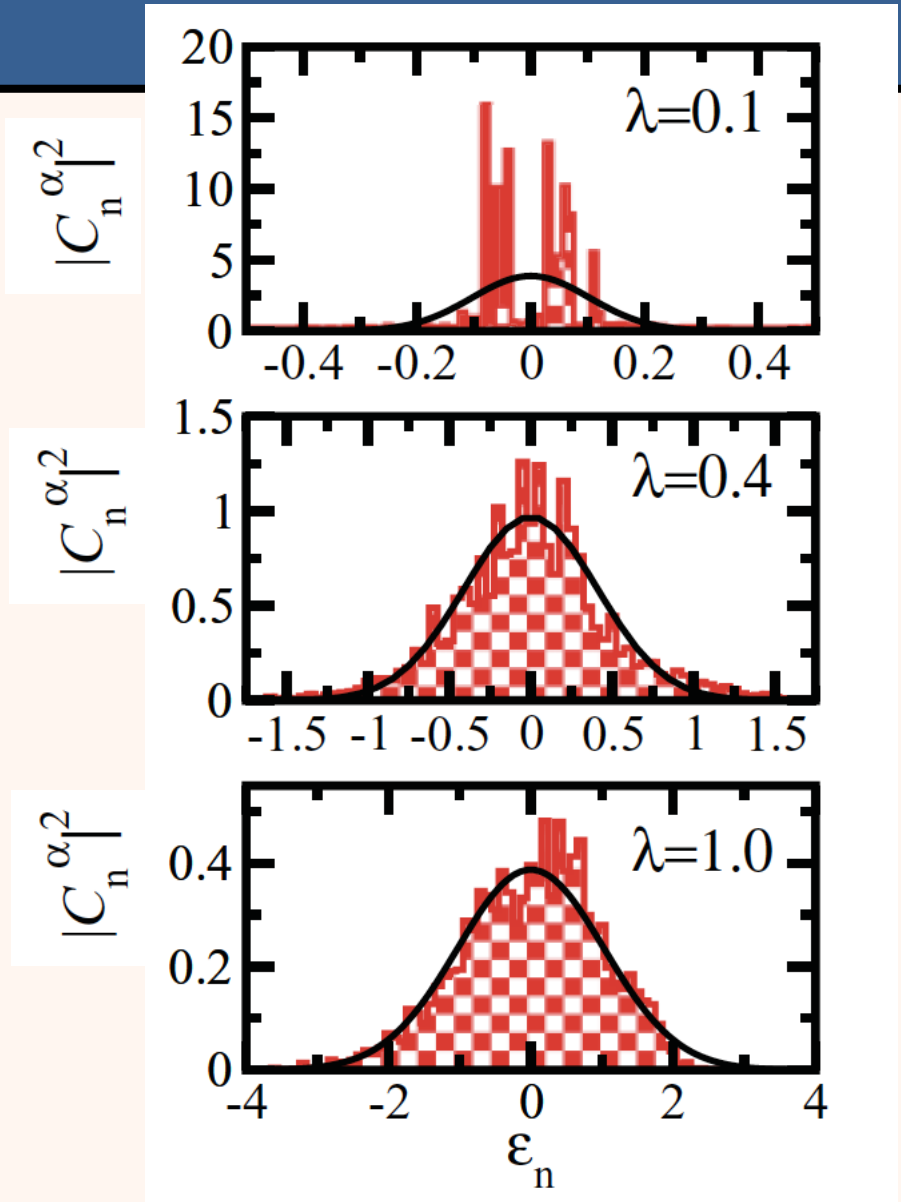
- ***CHAOTIC STATES***
- ***FEW-BODY O***
- ***SELF-AVERAGING***

Chaotic states

Chaotic Eigenstates

$$H = H_0 + \lambda V$$

- ***CHAOTIC STATES***
  Up to which point?
- ***FEW-BODY Observables***



Chaos guarantees thermalization, ETH

Quantum Ergodicity vs ETH

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha^{ini} |\alpha\rangle$$

$$O_{\beta\alpha} = \langle\beta|O|\alpha\rangle$$

$$O_{\alpha\alpha} = \langle\alpha|O|\alpha\rangle$$

Infinite time average | Thermodynamic average

$$\overline{\langle O(t)\rangle} \equiv \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha} \overset{=?}{\longleftrightarrow} O_{micro} \equiv \frac{1}{\mathcal{N}_{E_0,\Delta E}} \sum_{\substack{\alpha \\ |E_0 - E_\alpha| < \Delta E}} O_{\alpha\alpha}$$

depends on the initial conditions | depends only on the energy

Literature of many-body systems:

~~Chaotic regime, chaotic states~~

Thermal phase or ergodic phase, ergodic states

Why this preference?

What is
many-body quantum chaos?
(lack of semiclassical)
--WD? -- Diffusive transport?
<u>Eigenstates</u>

Quantum Ergodicity vs ETH

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha^{ini} |\alpha\rangle$$

$$O_{\beta\alpha} = \langle\beta|O|\alpha\rangle$$

$$O_{\alpha\alpha} = \langle\alpha|O|\alpha\rangle$$

Infinite time average | Thermodynamic average

$$\overline{\langle O(t)\rangle} \equiv \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha} \overset{=?}{\longleftrightarrow} O_{micro} \equiv \frac{1}{N_{E_0,\Delta E}} \sum_{\substack{\alpha \\ |E_0 - E_\alpha| < \Delta E}} O_{\alpha\alpha}$$

depends on the initial conditions | depends only on the energy

Literature of many-body systems:

~~Chaotic regime, chaotic states~~

Thermal phase or ergodic phase, ergodic states

QUESTIONS:

- Dependence on initial state?
- Ergodic state! Too strong?
- Hilbert space
- Dependence on observables

Quantum Ergodicity vs Phase Space

In the classical limit, a system is ergodic if the trajectories cover the energy shell homogeneously. We then adopt the same definition for quantum ergodicity.
To quantify how much of the energy shell is visited on average by the evolved state

$$\hat{\rho}(t) = e^{-i\hat{H}_D t}\hat{\rho}\, e^{i\hat{H}_D t}$$

we consider the infinite-time average

$$\overline{\rho} = \lim_{T\to\infty}\frac{1}{T}\int_0^T \mathrm{d}t\, \hat{\rho}(t)$$

and compute

$$\overline{\mathfrak{L}}(\epsilon,\hat{\rho}) \equiv \mathfrak{L}(\epsilon,\overline{\rho})$$

Measure of localization in phase space

$$\mathfrak{L}(\epsilon,\hat{\rho})^{-1} = \frac{1}{N}\int_{\mathcal{M}_\epsilon} \mathrm{ds}\, \mathcal{Q}^2_{\hat{\rho}}(\mathbf{x})$$

If the whole energy shell is homogeneously visited by $\hat{\rho}$, then

and th $\overline{\mathfrak{L}}(\epsilon,\hat{\rho}) = 1$ e is ergodic.

Stationary states (eigenstates of Dicke, superpositions) are NON-ergodic

Non-stationary states can be: Random states and coherent states

Quantum Ergodicity vs Phase Space

In the classical limit, a system is ergodic if the trajectories cover the energy shell homogeneously. We then adopt the same definition for quantum ergodicity.
To quantify how much of the energy shell is visited on average by the evolved state

$$\hat{\rho}(t) = e^{-i\hat{H}_D t}\hat{\rho}\, e^{i\hat{H}_D t}$$

we consider the infinite-time average

$$\overline{\rho} = \lim_{T\to\infty} \frac{1}{T}\int_0^T \mathrm{d}t\, \hat{\rho}(t)$$

and compute

$$\overline{\mathfrak{L}}(\epsilon,\hat{\rho}) \equiv \mathfrak{L}(\epsilon,\overline{\rho})$$

Measure of localization in phase space

$$\mathfrak{L}(\epsilon,\hat{\rho})^{-1} = \frac{1}{N}\int_{\mathcal{M}_\epsilon} \mathrm{ds}\, \mathcal{Q}_{\hat{\rho}}^2(\mathbf{x})$$

If the whole energy shell is homogeneously visited by $\hat{\rho}$ then

$$\overline{\mathfrak{L}}(\epsilon,\hat{\rho}) = 1$$

and th… e is ergodic.

- ❖ Quantum ergodicity in phase space vs quantum ergodicity in Hilbert space (ETH)? $O_{\alpha\alpha}$
- ❖ Eigenstates that are NON-ergodic in phase space lead to a violation of ETH?
  no -- few-body observables

Ergodicity and chaos in many-body systems

Around 50 participants!

2021 4th Workshop