University of Basel

Quantum quenches in the Tavis-Cummings model from the quasiclassical perspective

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Department of Physics, Bruder Group, February 6, 2021

Outline

1. Tavis-Cummings model and quantum criticality
   Excited-state quantum phase transitions
2. Quenching protocols
3. Quantum viewpoint
4. Quasiclassical viewpoint

Excited-state quantum phase transitions

Singularities in the level density of excited states as a generalization of QPTs

M. A. Caprio, P. Cejnar, and F. Iachello, Ann. Phys. **323**, 1106 (2008).

review: P. Cejnar, P. Stránský, M. Macek, and M. Kloc, arXiv:2011.01662

Tavis-Cummings model

$$H(\lambda) = \omega b^\dagger b + \omega_0 J_z + \frac{\lambda}{\sqrt{2j}} \left(b J_+ + b^\dagger J_- \right)$$

- collective atom-field interaction
- $b,\ b^\dagger$: photon operators
- J_z, J_+, J_-: collective quasispin operators (atoms)

- $N = 2j$: number of the atoms
- λ: tunable parameter
- Dicke model with RWA (integrable)

Conserves the total number of photons $n_b = b^\dagger b$ and atomic excitations $n_{\mathrm{ex}} = J_z + j$, i.e. $[H, M] = 0$ for $M = n_b + n_{\mathrm{ex}}$

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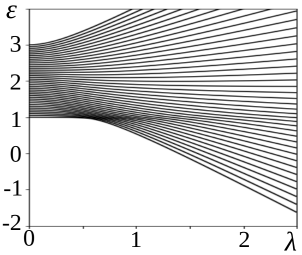
From now on:
$M = N = 2j$

Scaled energy
$\varepsilon = \frac{E}{\omega_0 j}$



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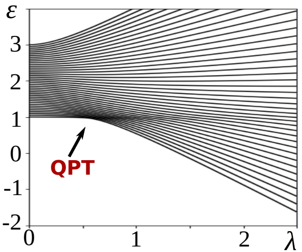

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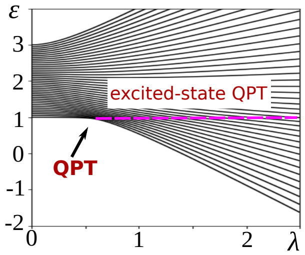

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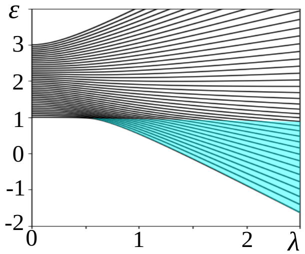

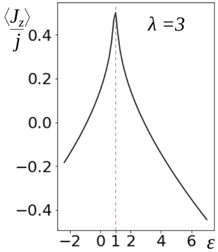

Quantum quench

- An abrupt change of the Hamiltonian $H_\mathrm{i} \to H_\mathrm{f}$.

  'Quantum tablecloth experiment'

Quantum quench

- An abrupt change of the Hamiltonian $H_{\rm i} \to H_{\rm f}$.

'Quantum tablecloth experiment'

Quench dynamics: evolution of the initial state $|\psi_{\rm i}\rangle$ under $H_{\rm f}$.

Survival probability: $$P(t) = |\langle\psi_{\rm i}|\psi(t)\rangle|^2 = |\langle\psi_{\rm i}|e^{-iH_{\rm f}t}|\psi_{\rm i}\rangle|^2$$

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- An abrupt change of the Hamiltonian $H_{\mathrm{i}} \to H_{\mathrm{f}}$.

  'Quantum tablecloth experiment'

Quench dynamics: evolution of the initial state $|\psi_{\mathrm{i}}\rangle$ under H_{f}.

Survival probability: $P(t) = |\langle\psi_{\mathrm{i}}|\psi(t)\rangle|^2 = |\langle\psi_{\mathrm{i}}|e^{-iH_{\mathrm{f}}t}|\psi_{\mathrm{i}}\rangle|^2$

either | or

$P(t) \approx 1$

$P(t) < 1$

Quench dynamics

$\{|\phi_{\mathrm{f}l}\rangle\}_{l=1}^{d}$: basis of H_{f}

$$P(t) = |\langle\psi_{\mathrm{i}}|e^{-iH_{\mathrm{f}}t}|\psi_{\mathrm{i}}\rangle|^2 = \Big|\sum_{l=1}^{d} \underbrace{|\langle\psi_{\mathrm{i}}|\phi_{\mathrm{f}l}\rangle|^2}_{|s_l|^2} \mathrm{e}^{-iE_{\mathrm{f}l}t}\Big|^2 = \left|\int \mathrm{d}E\ S(E)\mathrm{e}^{-iEt}\right|^2$$

Strength function $S(E) = \sum_{l=1}^{d} |s_l|^2 \delta(E - E_{\mathrm{f}l})$

$P(t) \leftrightarrow S(E)$
Fourier transform

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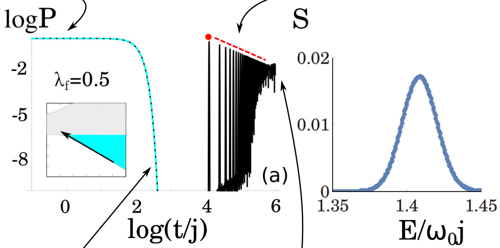
Fourier transform

Backward protocol: $\lambda_{\mathrm{i}} > \lambda_{\mathrm{f}}$
'Regular' dynamics for $|\psi_{\mathrm{i}}\rangle$ being the ground state of H_{i}:

initial decay given by the dispersion of *S(E)*

power-law decay $1/t$, both the shape and the discrete structure of *S(E)* play a role



Gaussian decay given by the shape of *S(E)*

equilibrated regime
P(t) oscillates around $\mathcal{N}^{-1} = 1/\sum_l |s_l|^4$

Quench dynamics with ESQPTs

P. Pérez-Fernández *et al.*, Phys. Rev. A **83**, 033802 (2011).
L.F. Santos, M. Távora and F. Pérez-Bernal, Phys. Rev. A **94**, 012113 (2016).

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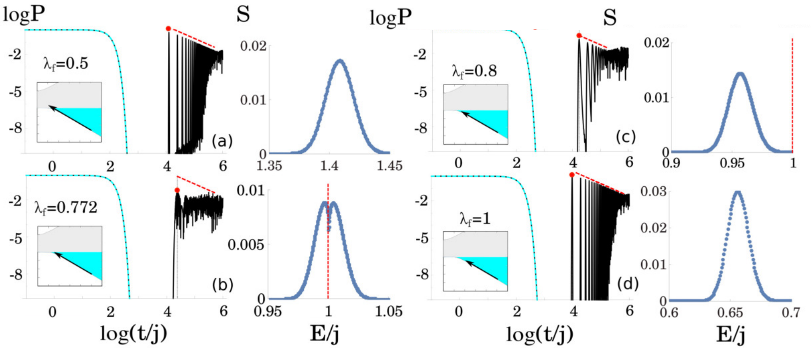

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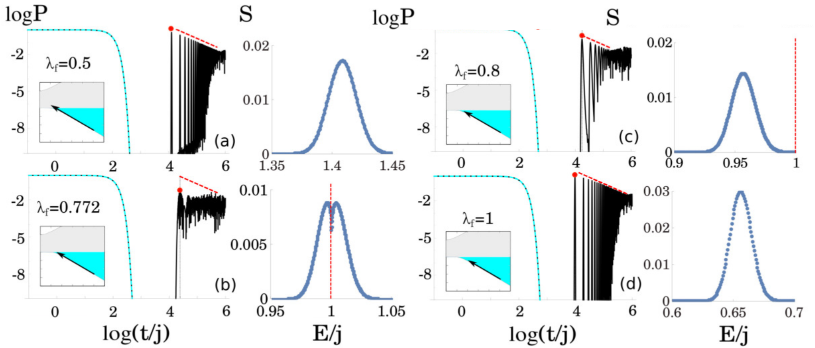

$1/t$ power law decay not present, **faster transition to the equilibrated regime**.

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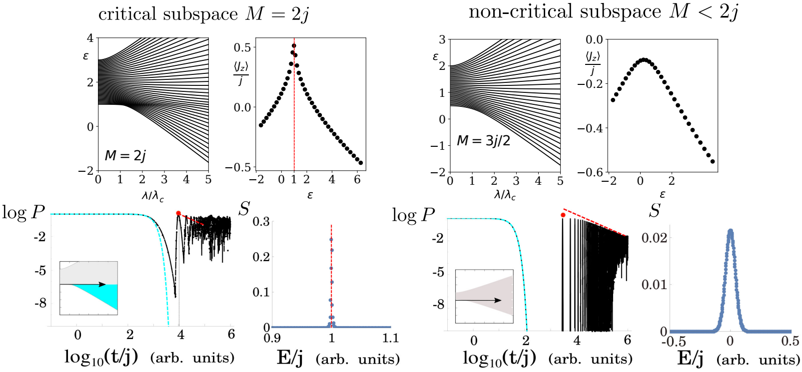

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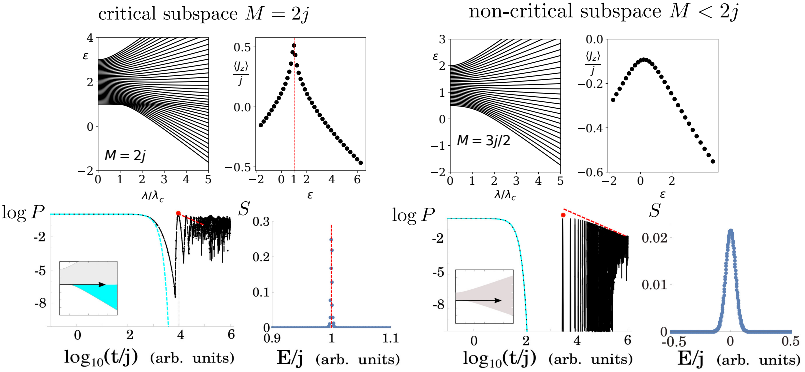

$1/t$ power law decay not present, **slower initial decay, stabilization of the initial state**.

Truncated Wigner Approximation

Wigner function of the initial state:

$$W_\mathrm{i}(x, p) = \frac{1}{\pi} \int_{-\infty}^{\infty} \psi_\mathrm{i}^*(x + y)\psi_\mathrm{i}(x - y)\mathrm{e}^{2\mathrm{i}py}\, \mathrm{d}y$$

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Classical version of the Hamiltonian and classical dynamics:

$$H_{\mathrm{cl}}(x,p) = \frac{\Delta\omega}{2}(x^2+p^2) + \omega_0(M-j) + \lambda x\sqrt{j}\sqrt{1-\frac{\left[M-j-\frac{1}{2}(x^2+p^2)\right]^2}{j^2}}$$

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Quasiclassical survival probability:

$$P_{\mathrm{cl}}(t)=2\pi\int W_{\mathrm{i}}(x,p)\;W(x,p,t)\,\mathrm{d}x\,\mathrm{d}p$$

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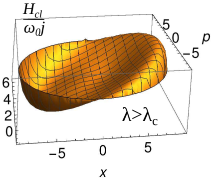

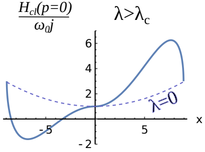

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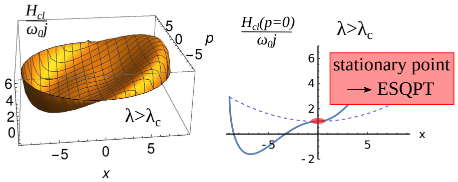

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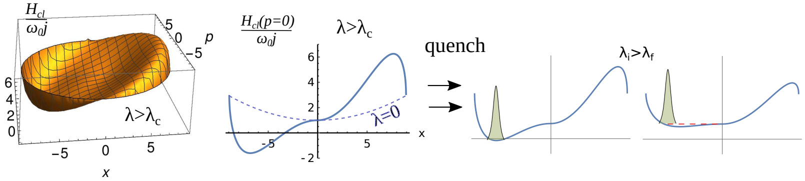
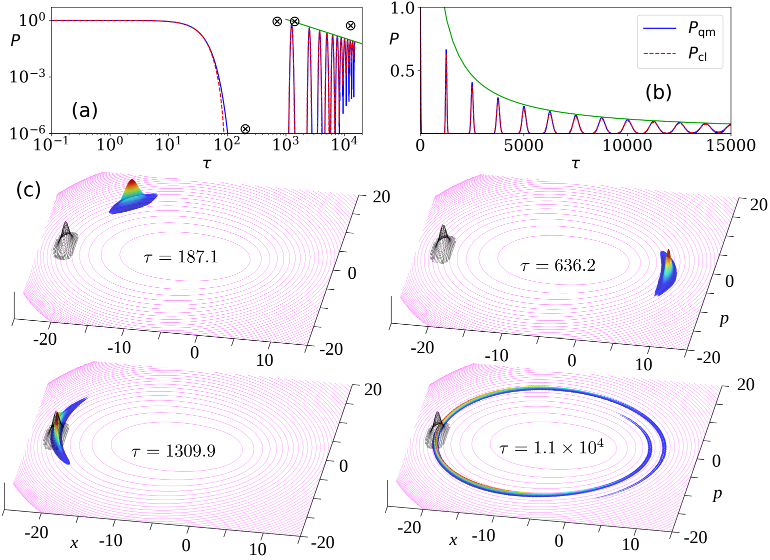


Quasiclassical simulation

'Regular' dynamics, backward quench $\lambda_{\rm i} = 2.5$, $\lambda_{\rm f} = 0.5$, $\tau = t\omega_0 j$,

M. Kloc & D. Šimsa *et al.*, arXiv:2010.07750



Quasiclassical simulation

'Critical' dynamics, backward quench $\lambda_{\rm i} = 2.5$, $\lambda_{\rm f} = 0.772$, $\tau = t\omega_0 j$,

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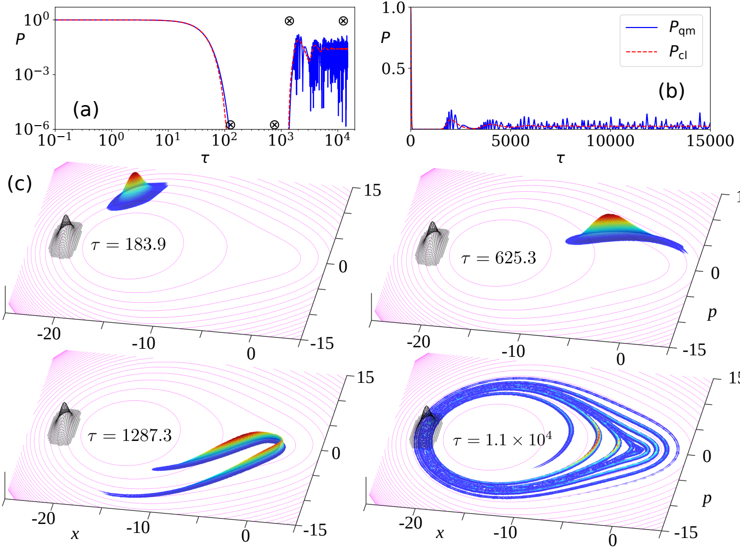

Quasiclassical simulation

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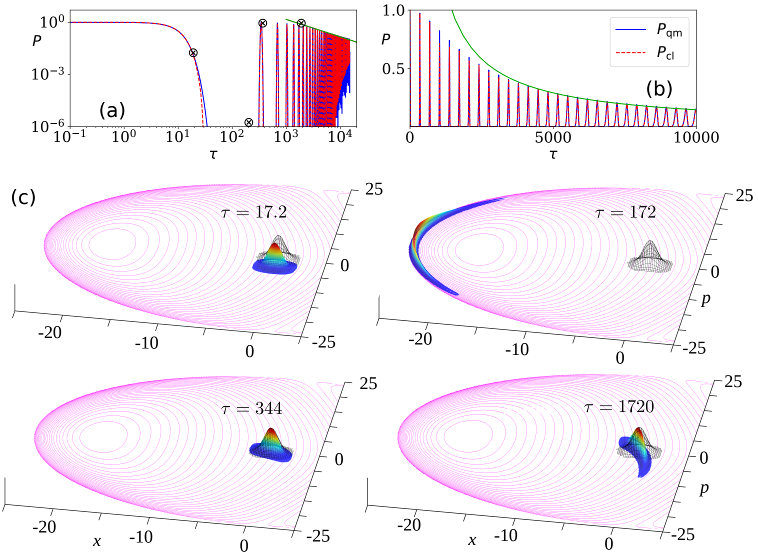

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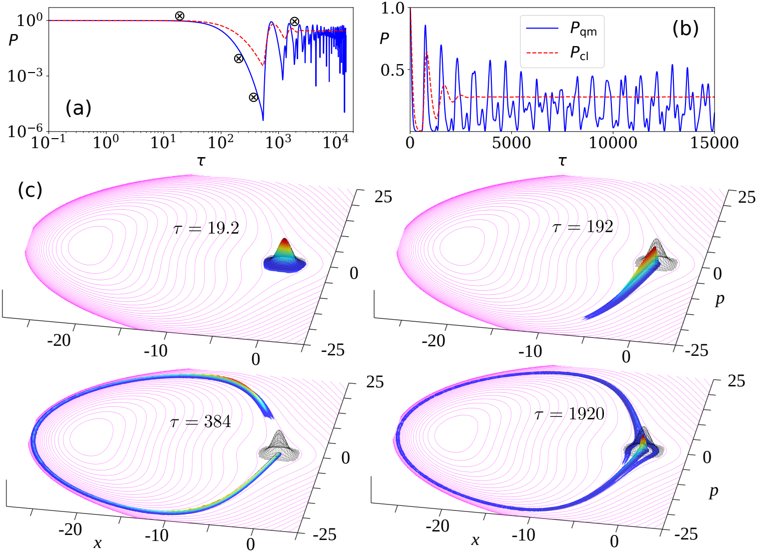
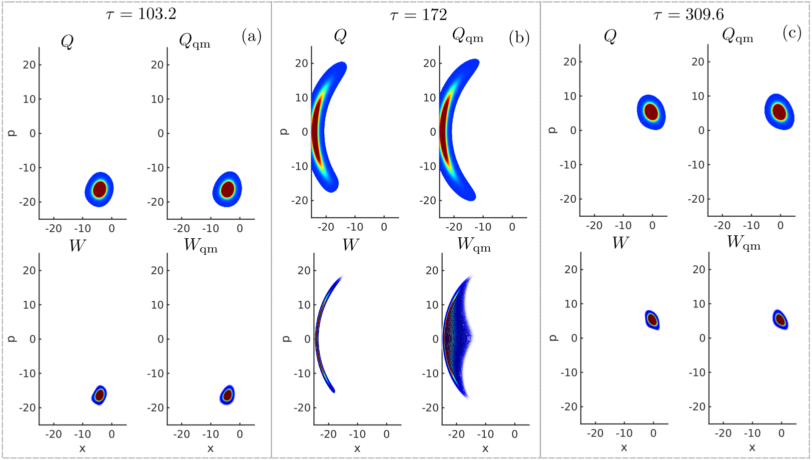


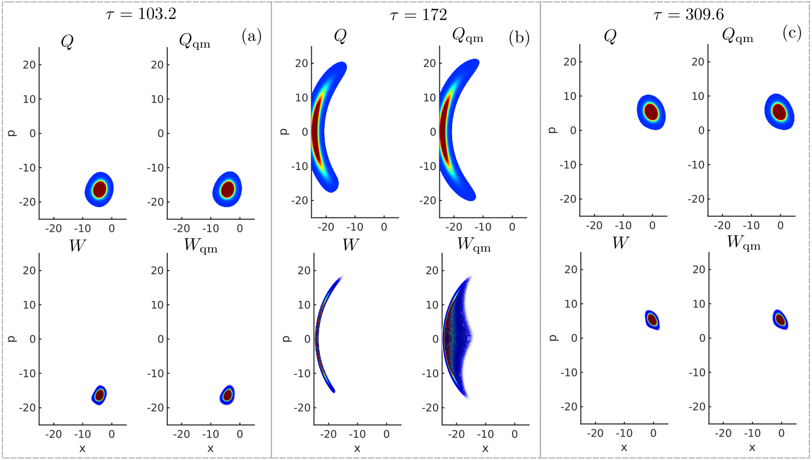
Comparison to the full quantum dynamics I

Forward quench protocol, 'regular'



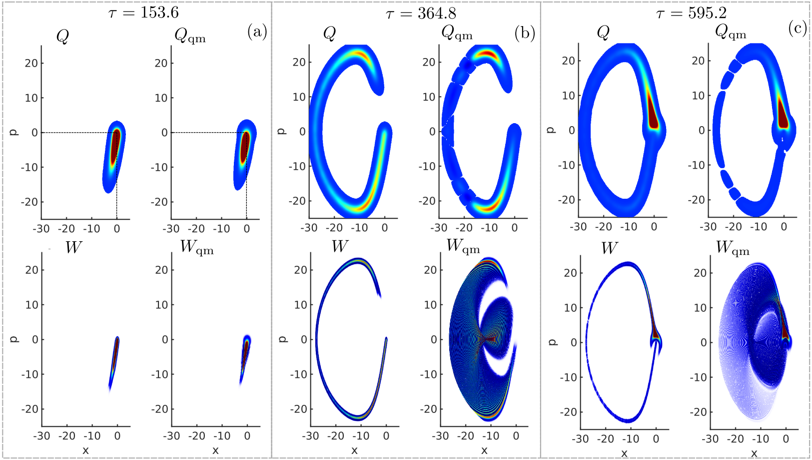
Comparison to the full quantum dynamics II

Forward quench protocol, 'regular'



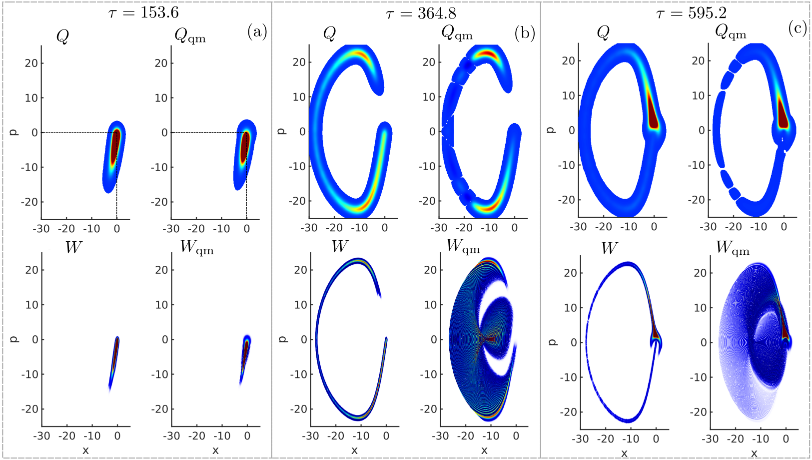
Comparison to the full quantum dynamics III

Forward quench protocol, 'critical'



Comparison to the full quantum dynamics IV

Forward quench protocol, 'critical'



Summary

- ESQPTs ('spectral singularities') *may have* a strong influence on the quench dynamics.
- Chaotic dynamics may smear out the effect but for weakly chaotic systems we get qualitatively similar results, see

  M. Kloc, P. Stránský, and P. Cejnar, Phys. Rev. A **98**, 013836 (2018).
- Quasiclassical simulations reveal intuitively the differences between the 'regular' and 'critical' dynamics for all quench protocols.
- Quasiclassical survival probability faithfully captures the main features of the equlibration.

M. Kloc, D. Šimsa, F. Hanák, P. Kaprálová-Žďánská, P. Stránský and P. Cejnar, arXiv:2010.07750

Collaborators:

Charles University, Prague, Czech Rep.
P. Cejnar, P. Stránský

Luleå university of technology, Luleå, Sweden.
D. Šimsa

Academy of Sciences, Prague, Czech Rep.
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