Universal correlations in many-body systems: the Random Wave Model in Fock space

An ongoing project by

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(Regensburg-Northumbria-Regensburg)

MB systems Mexico 2021

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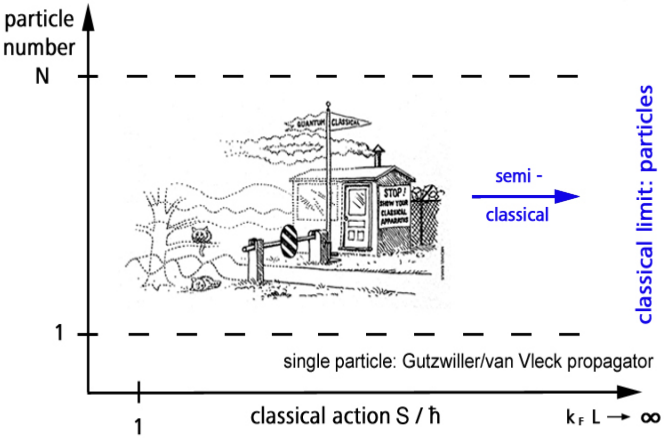
- They are applicable within a regime, the semiclassical regime, where typical actions are larger compared with $\hbar$, but semiclassical methods are asymptotic instead of non-perturbative in $\hbar$

Life at the border...



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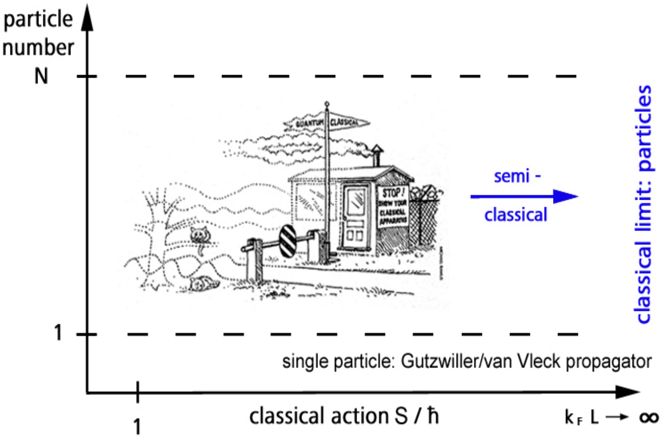

$N = 1, \hbar \to 0$ and decoherence $\to 0$: Classical Particle

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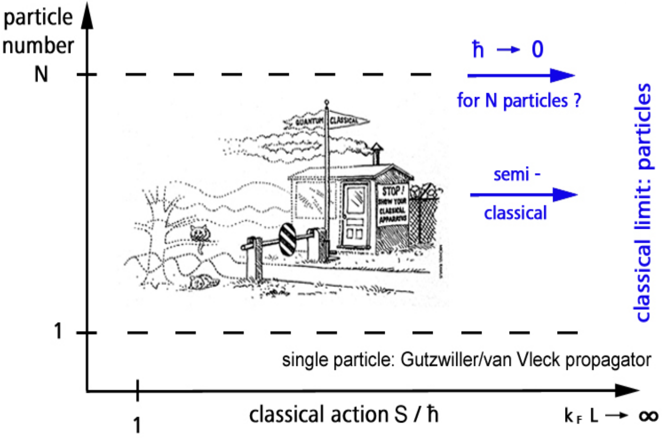

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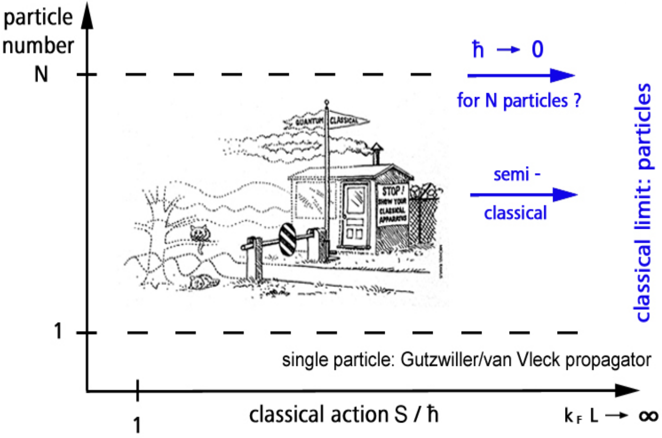

Finite N, $\hbar \to 0$ and decoherence $\to 0$: Classical Particles

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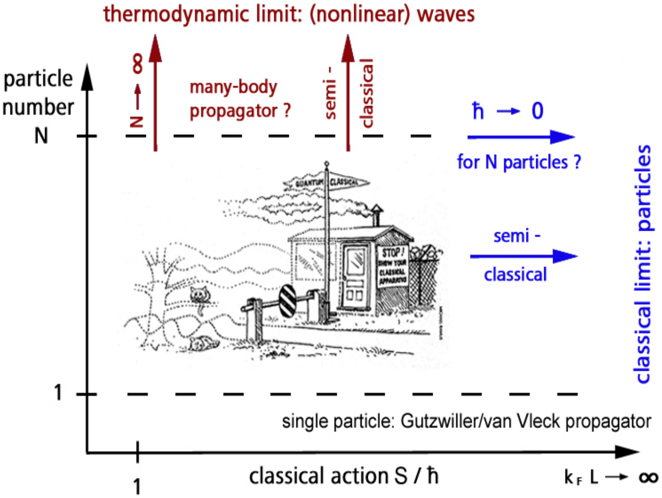

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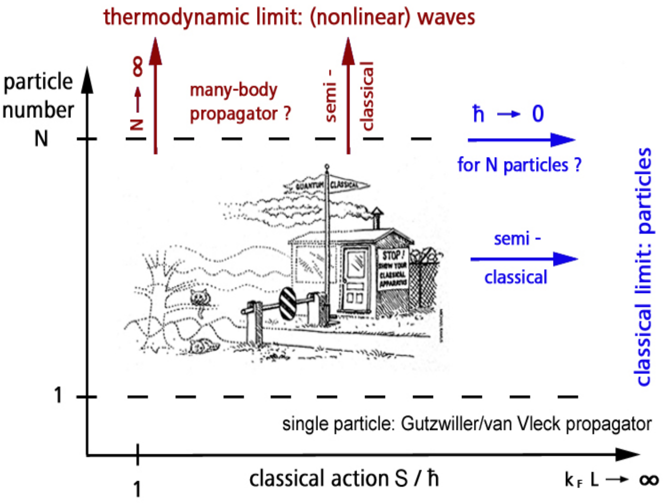

$N \to \infty$ and decoherence $\to 0$: Classical Fields

Life at the border... can be quite singular!

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quantum(S)

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$$\text{quantum}(S) \neq \text{classical}(S) + \text{corrections}(\hbar/S)$$

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Interference is missing

$$e^{iS/\hbar}, e^{iNR}$$

Non-perturbative!

The transition probability

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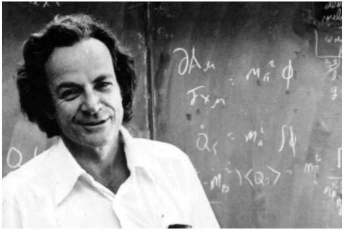

Everything starts with
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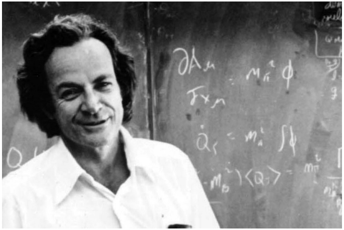

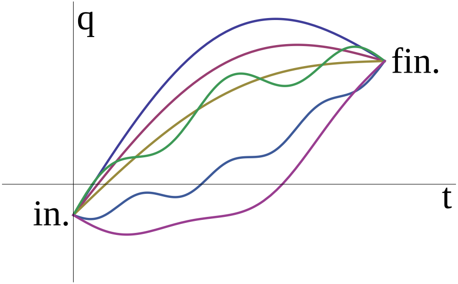

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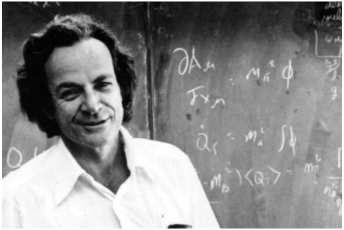

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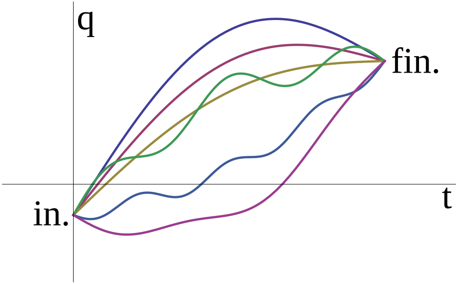
Feynman path integral



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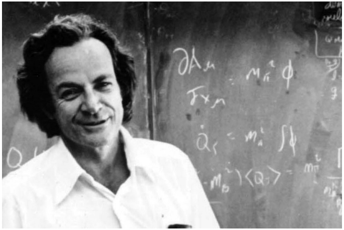

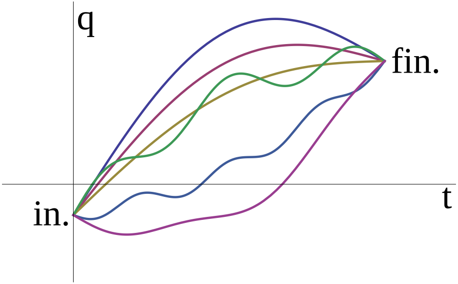

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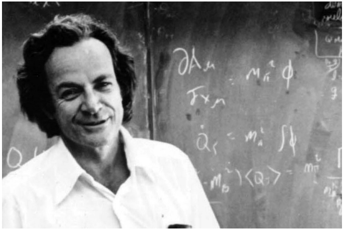

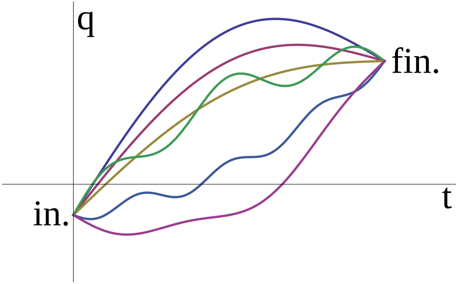

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Where are the classical paths?, can we use them?

The semiclassical approximation $(R[q(t)] \gg \hbar)$

$$\int \mathcal{D}[q(t)] \mathrm{e}^{\frac{i}{\hbar} R[q(t)]} \simeq \sum_\gamma \sqrt{W_\gamma} \mathrm{e}^{\frac{i}{\hbar} R_\gamma + i \frac{\pi}{4} \mu_\gamma}$$

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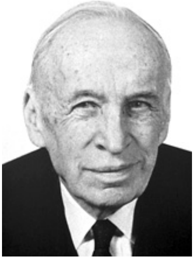
- 1930's
- Starts from WKB
- Only short times



John H. van Vleck

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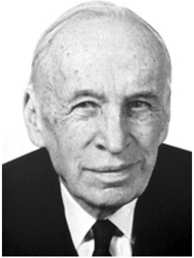

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Martin Gutzwiller

- 1970's
- Starts from Feynman
- Short and large times μ

Crash course on semiclassics (a bit technical)

Start with an action $R[q(t)]$ and the exact path integral

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If we are talking about statistical properties
we need an ensemble!!

Regular and irregular wavefunctions

A quantum system is a generator of energies and states

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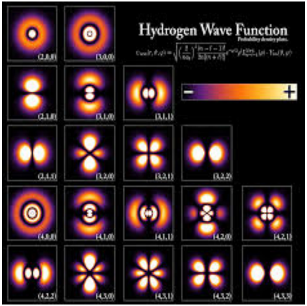
Regular



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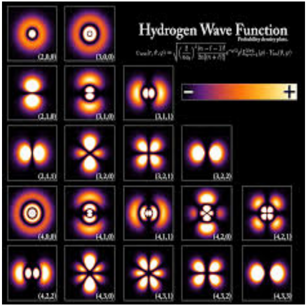
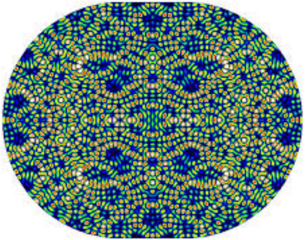


Irregular, Quasi-Random



Defining ensembles of states

Quantum systems that generate ensembles of states

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Defining ensembles of states THIS TALK

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Classical limit with complex dynamics

$\downarrow$

Universality of spatial correlations

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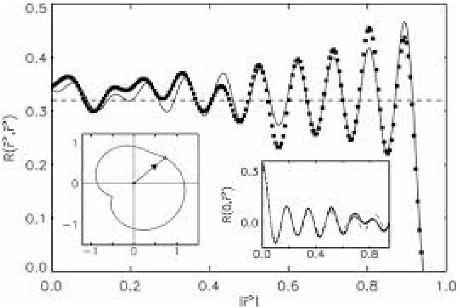

Irregular eigenstates as gaussian random fields

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Random Wave Model
Berry, 1977

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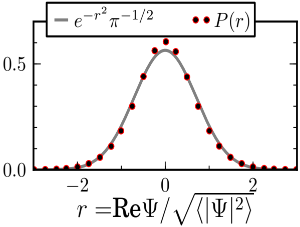

Random Wave Model
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Universal Gaussian fluctuations,

$$P[\psi(\vec{r})] = \mathcal{Z}^{-1} \mathrm{e}^{-\frac{1}{2}\int d\vec{r}d\vec{r}'\psi^*(\vec{r})R^{-1}(\vec{r},\vec{r}')\psi(\vec{r}')}$$

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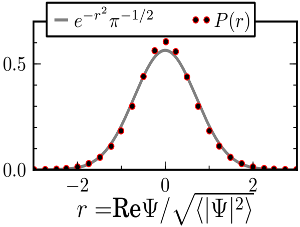

Random Wave Model
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Universal Gaussian fluctuations, microscopic covariance

$$P[\psi(\vec{r})] = \mathcal{Z}^{-1} e^{-\frac{1}{2}\int d\vec{r} d\vec{r}' \psi^*(\vec{r}) R^{-1}(\vec{r},\vec{r}')\psi(\vec{r}')}$$
$$R(\vec{r},\vec{r}') = \overline{\psi^*(\vec{r})\psi(\vec{r}')}$$

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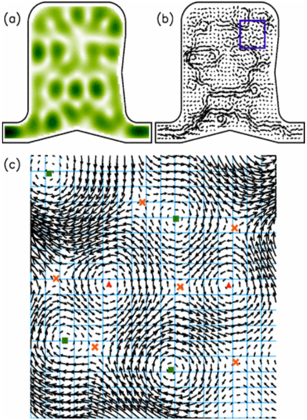

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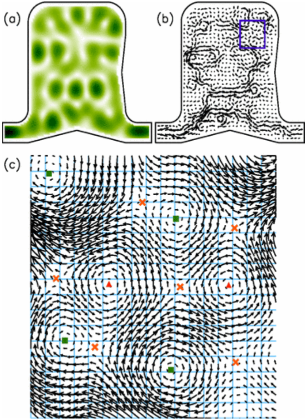

Phase singularities

$$\psi(\vec{r}) = 0$$

Critical points

$$\nabla \psi(\vec{r}) = 0$$

Morphology of eigenfunctions: a complicated observable

Universal spatial correlations of critical points

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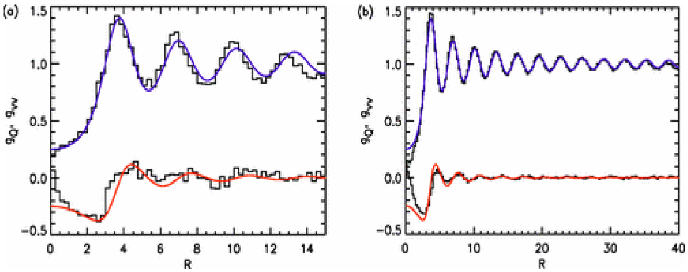
Experiment: Stöckmann group, Marburg

Theory: JDU and M. Dennis



R. Höhmann, U. Kuhl, H.-J. Stöckmann, JDU and M. Dennis PRE09

Intermezzo

The power of the RWM comes from combining universal Gaussian statistics (due to classical chaos) with system-dependent covariance matrix

This is a general recipe to construct a RWM!

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Let's go Many-Body!!

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Let's start from the begining...

Checking gaussian fluctuations

Does a single expansion coefficient look gaussian?

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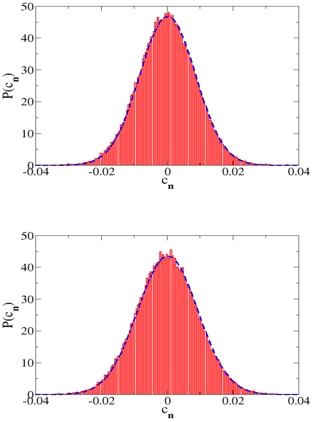

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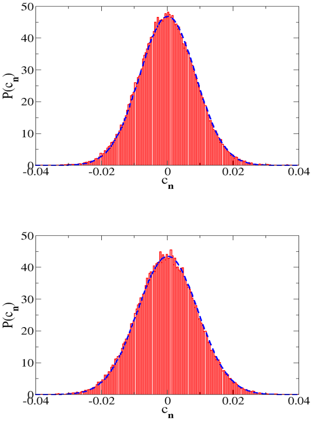

The dirt old Gaussian we all know and love....
NOT ENOUGH!

Checking gaussian fluctuations for real

Do the expansion coefficients look as a multivariate Gaussian?

$$\langle|\psi_E^{(\mathbf{m})}|^2|\psi_E^{(\mathbf{n})}|^2\rangle = \langle|\psi_E^{(\mathbf{m})}|^2\rangle\langle|\psi_E^{(\mathbf{n})}|^2\rangle + 2|\langle\psi_E^{(\mathbf{m})}\psi_E^{(\mathbf{n})}\rangle|^2 \quad ???$$

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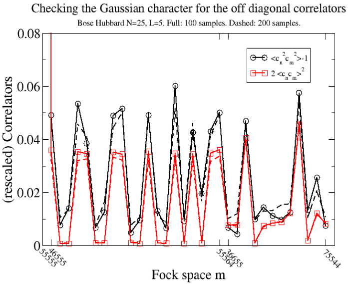

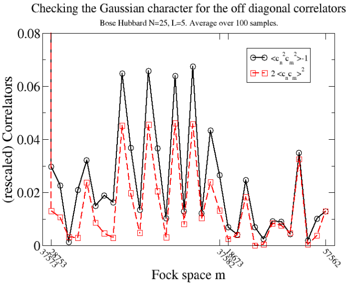

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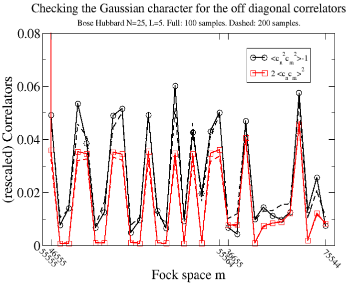

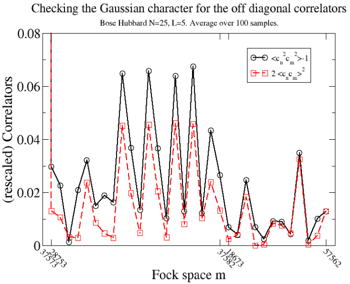

The dirt old Gaussian we all know and love....
but in all directions!!

The two-point correlation

Enter semiclassics: Two-point correlators in Fock space

$$R(\mathbf{m}-\mathbf{n},\mathbf{m}+\mathbf{n},\tilde{E})$$
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$$\int_0^{2\pi} d\theta_1 \ldots d\theta_L \delta\left[\tilde{E} - H(\hat{b}_i \to \sqrt{m_i}\mathrm{e}^{i\theta_i})\right] \mathrm{e}^{2i\mathbf{n}\cdot\theta}$$

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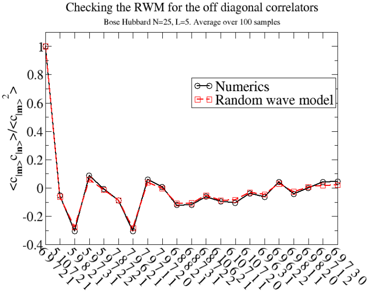

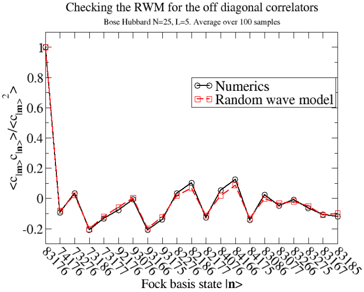

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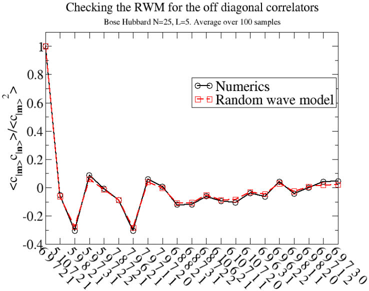

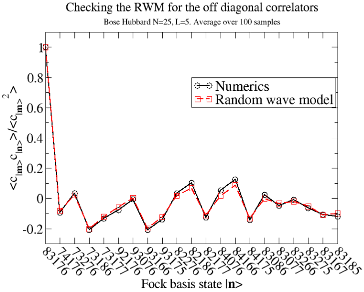

Quantum eigenstates of many-body chaotic systems

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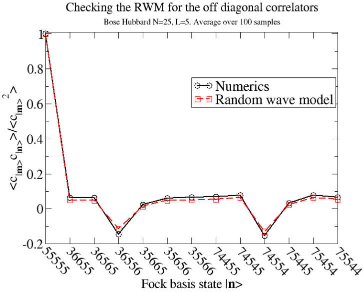

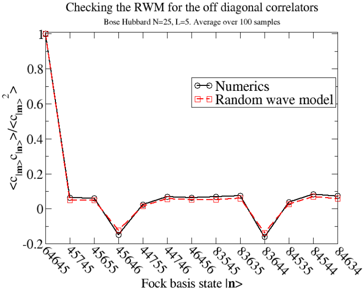

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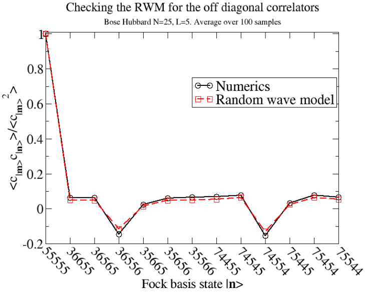

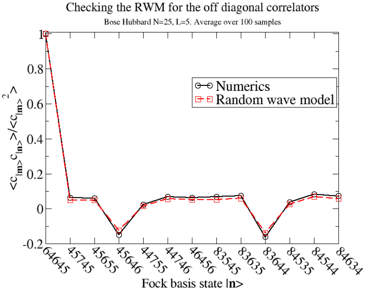

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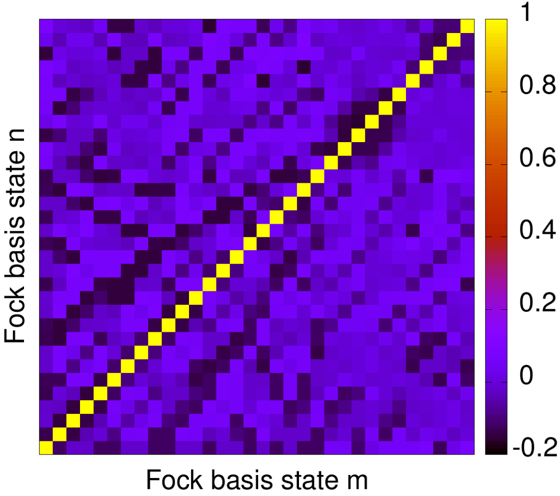
Let's crank some serious numbers



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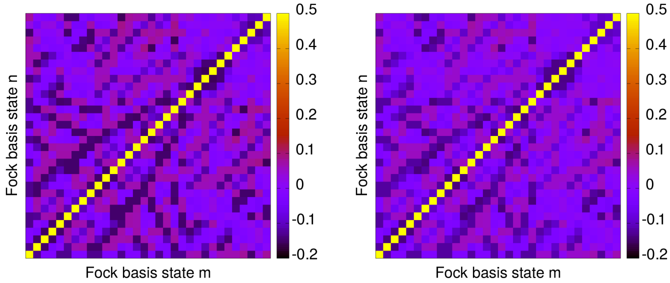

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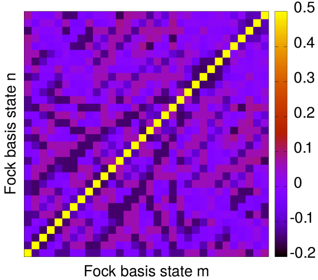

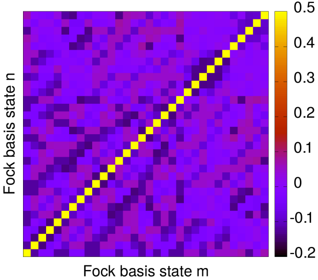

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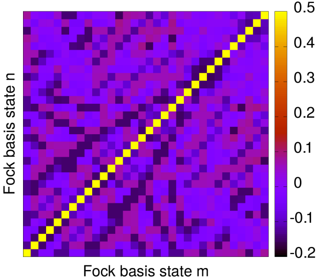

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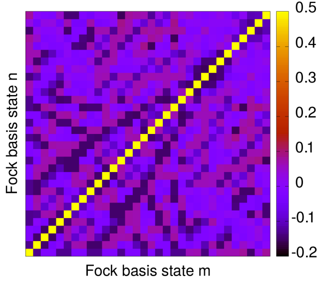

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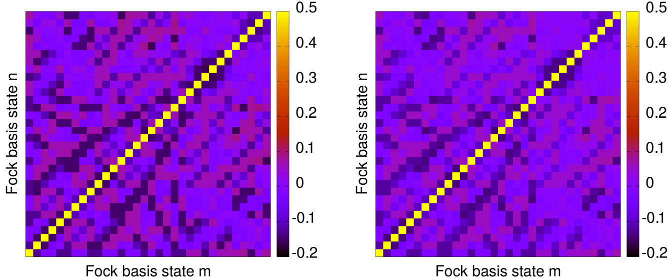

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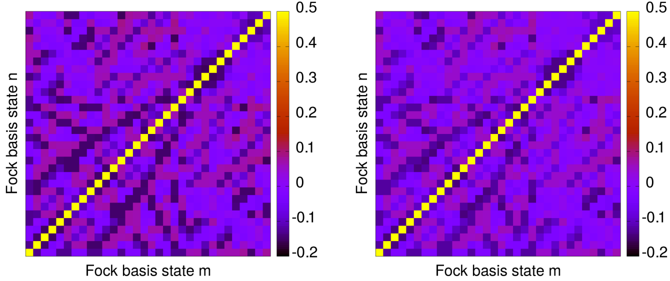

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Please do not say "expectation values"... we have ETH for that!