Chaos in the Bose Hubbard model.

Ergodicity and chaos in many-body systems

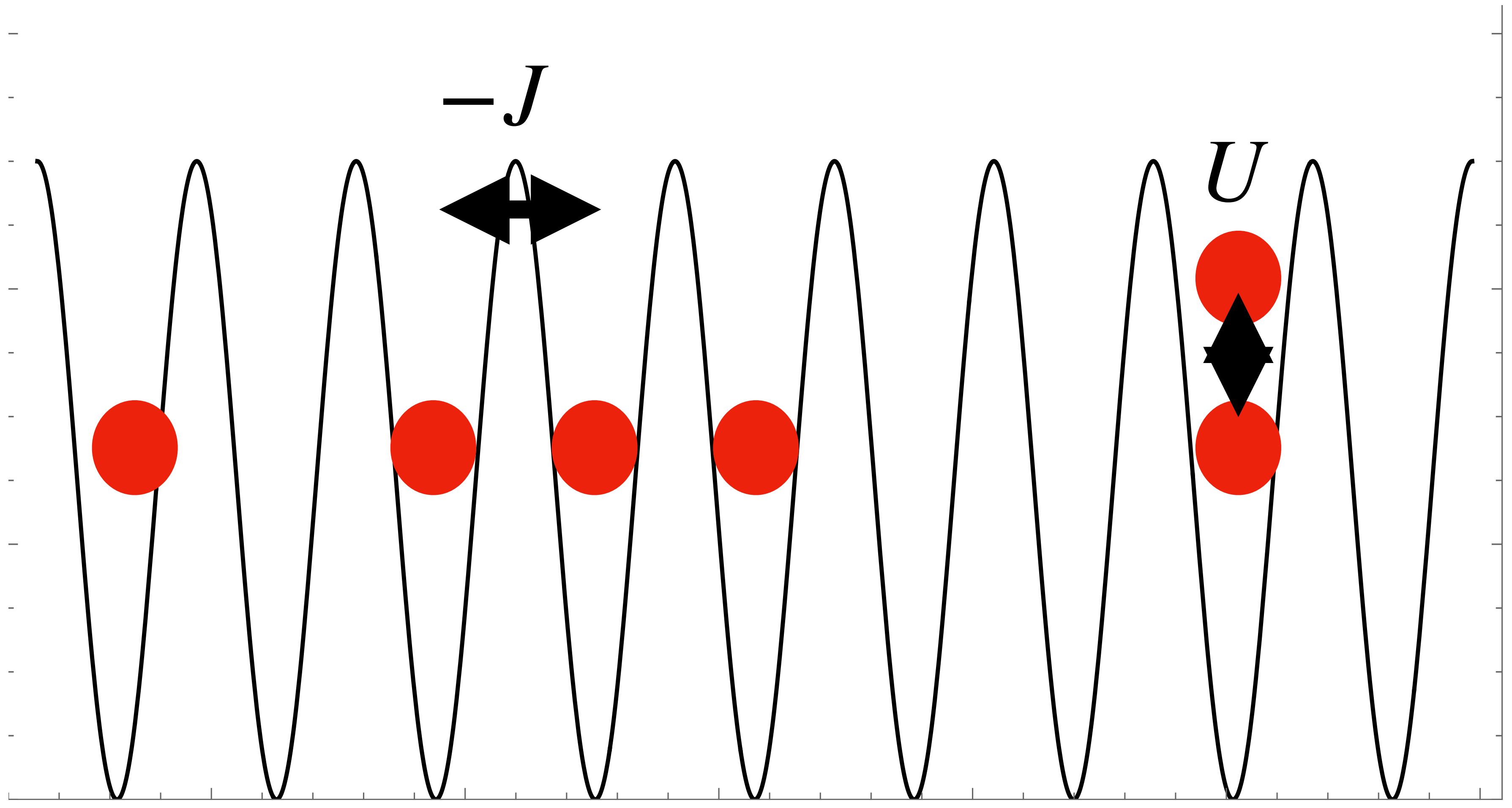
Javier de la Cruz, Carlos Díaz, Sergio Lerma and Jorge G. Hirsch

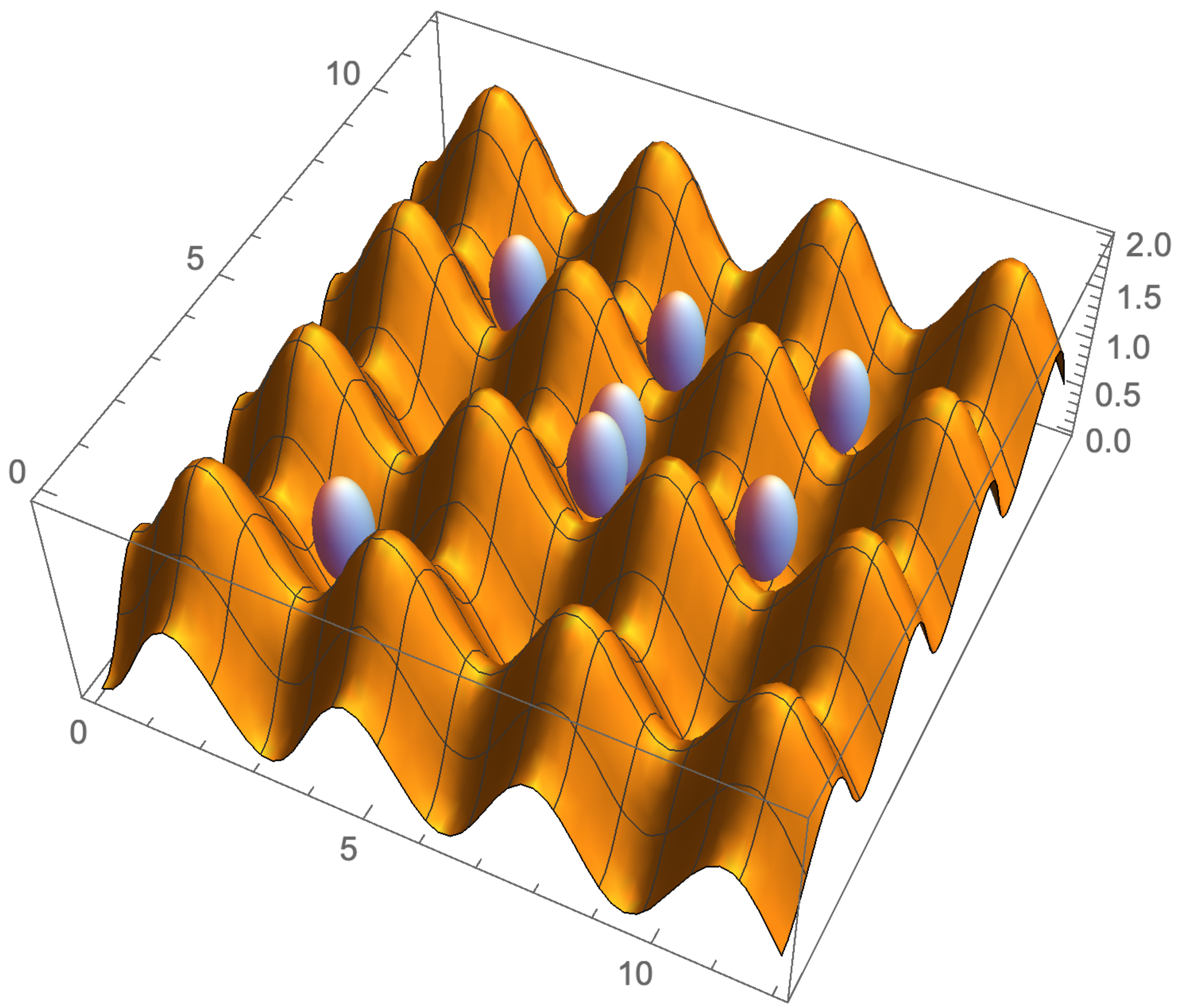
Bose-Hubbard Model (BHM)

The model which we consider is the one dimensional Bose Hubbard model with periodic boundary conditions. The Hamiltonian of the system with N bosons in a lattice of L sites is

$$\hat{H}_{BH} = -J \sum_{j=1}^{N} \left(\hat{a}_{j+1}^{\dagger} \hat{a}_j + \hat{a}_j^{\dagger} \hat{a}_{j+1} \right) + \frac{U}{2} \sum_{j=1}^{N} \hat{n}_j \left(\hat{n}_j - 1 \right)$$

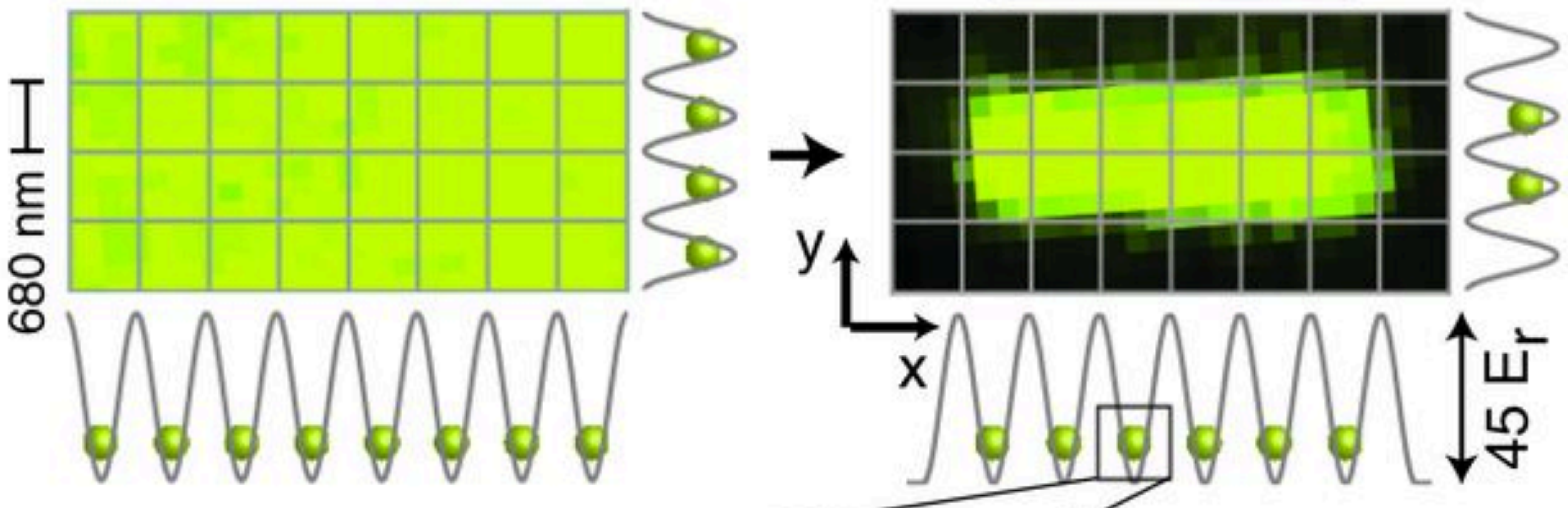
$$D = \frac{(N+L-1)!}{N!(L-1)!}$$





Bose-Hubbard Model

- There are multiple experiments where the model has been done[1].
- The experimenter has high control over the system parameters (particles, sites, repulsion energy U, tunneling rate J) and also over the time evolution.

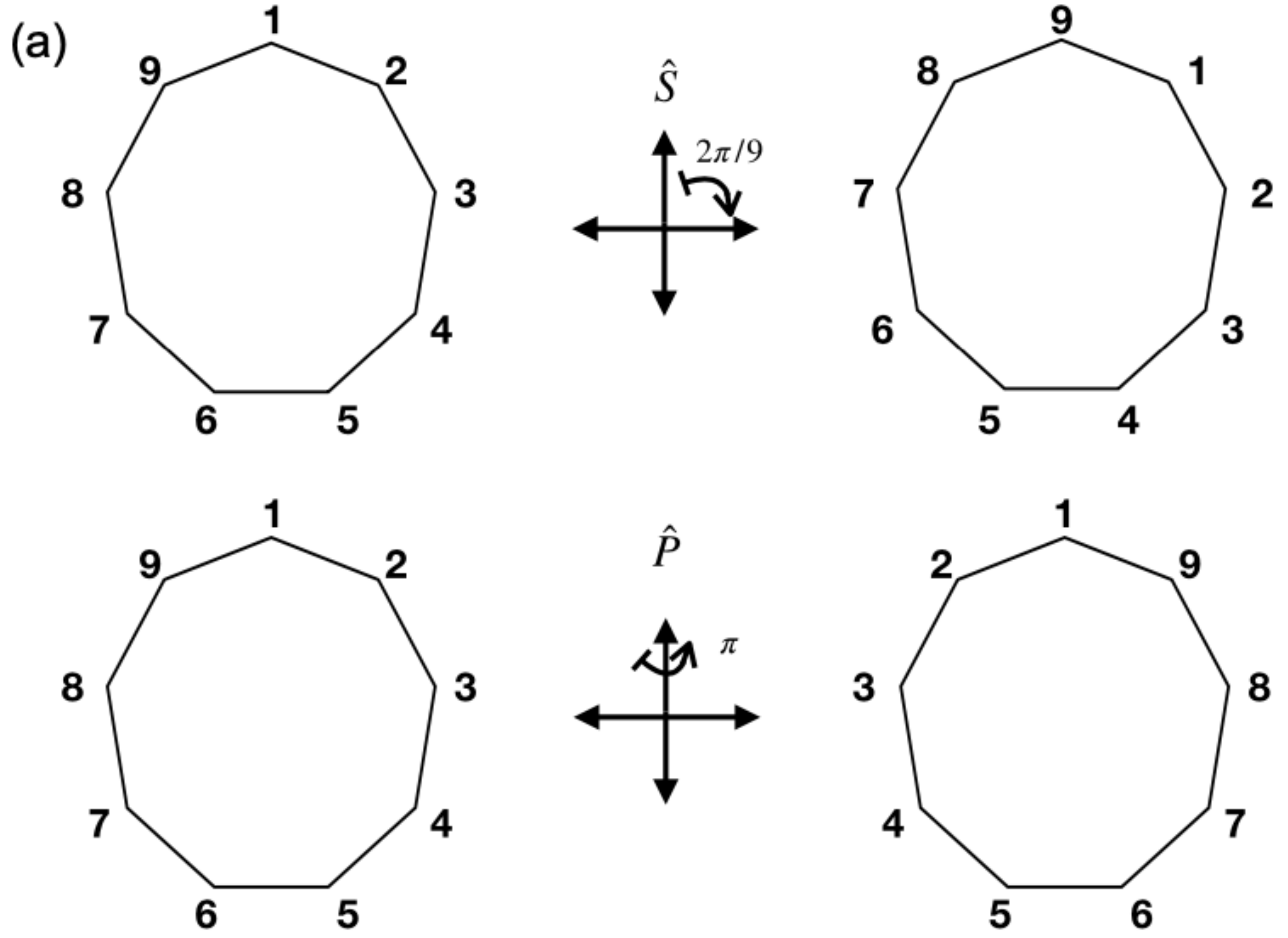


Microscopy of the experimental BHM with N = 6 and L=6.
The flourecence squares are sites in the lattice with a boson.

Greiner, M., Mandel, O., Esslinger, T. *et al.* Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms. *Nature* **415,** 39–44 (2002).

Symmetries

- The symmetries of the one dimensional BHM with periodic boundary conditions are the same that those of a regular polygon with L sides.
- The translational symmetry
- The reflection symmetry
- The Hamiltonian can be written in Jordan blocks associated to each symmetrie.



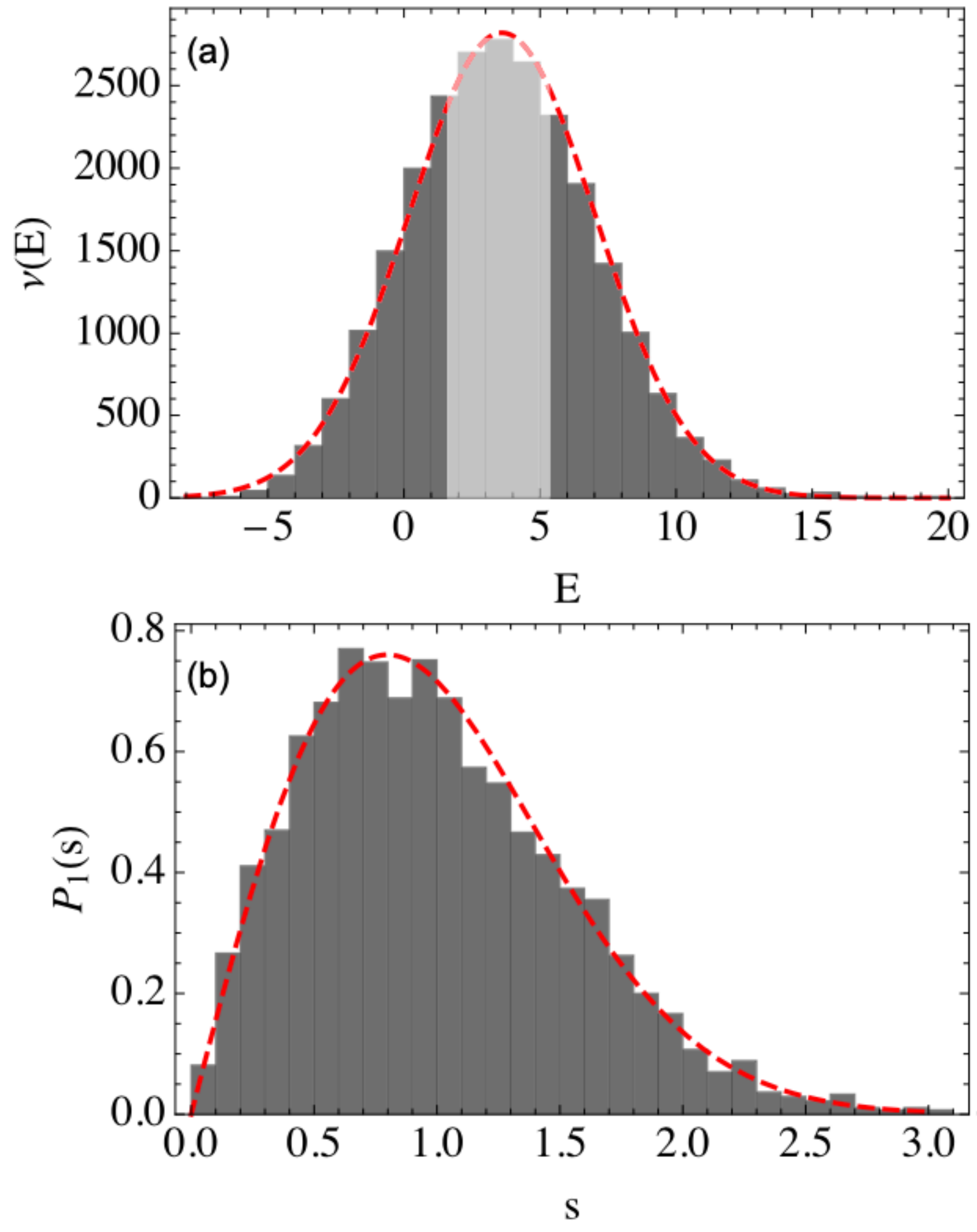
$$[\hat{S}, \hat{P}] \neq 0$$

Chaos in the BHM

The spectral properties associated with quantum chaos in the BHM were studied by Kolovsky and Buchleitner [1]. We will use a parameter introduced by Kolovsky. $u = U$ and $1 - J = u$. with this parametrization the system is integrable $u = 0$ and $u = 1$.

Density of States with $u = 0.5$

Nearest-neighbour level spacing distribution for the unfolded spectrum of one subspace, u=0.5

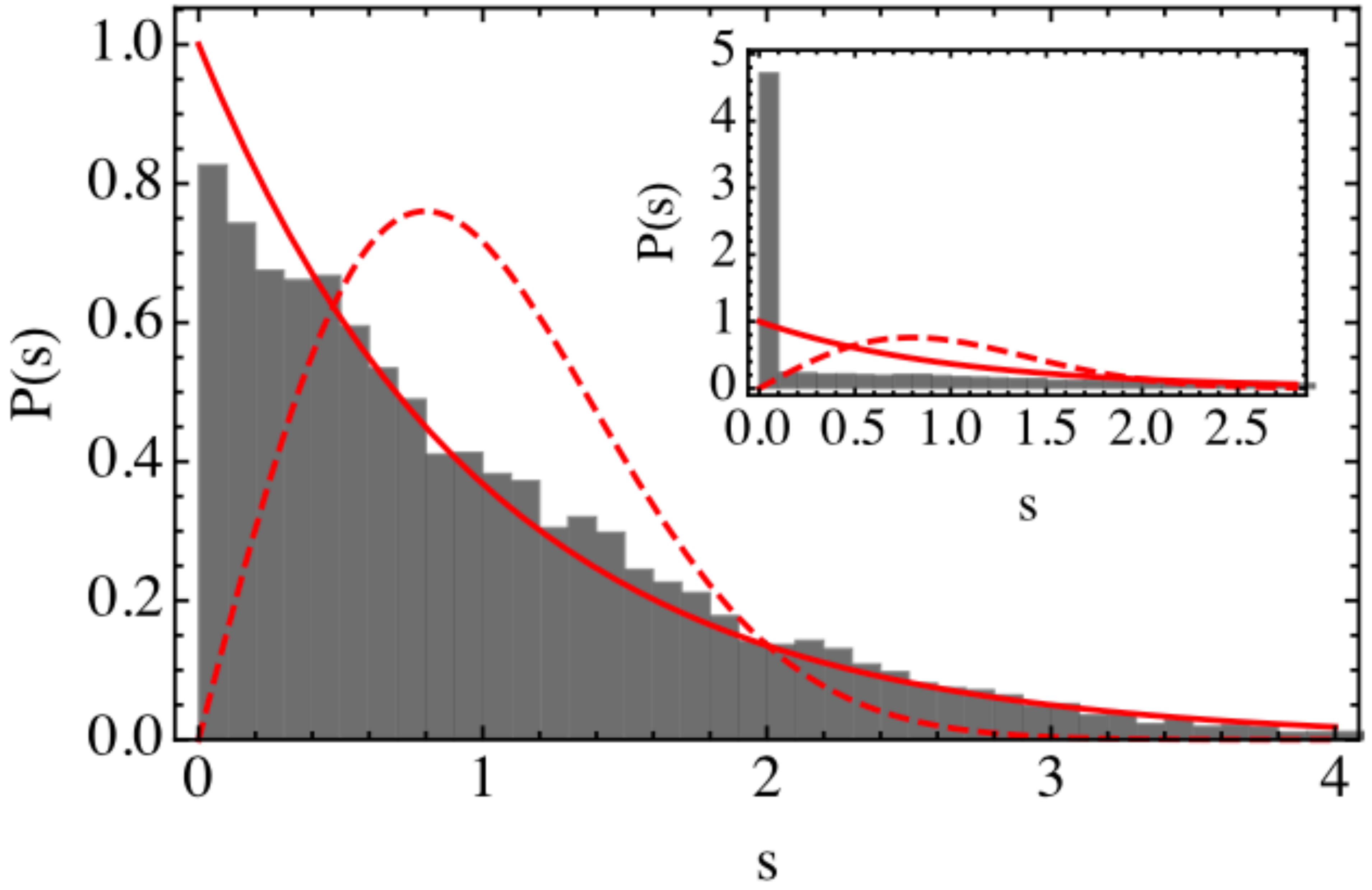


- [1] A. R. Kolovsky and A. Buchleitner, Quantum chaos in the Bose-Hubbard model, Europhys. Lett. **68**, 632 (2004).

Chaos in the BHM

Could we detect chaos without separating by symmetries?

Main panel shows the nearest-neighbor spacing distribution of energy levels in the light gray region of Fig 1 considering only 4 subspaces. Inset shows the same distribution for the whole set of symmetry subspaces.



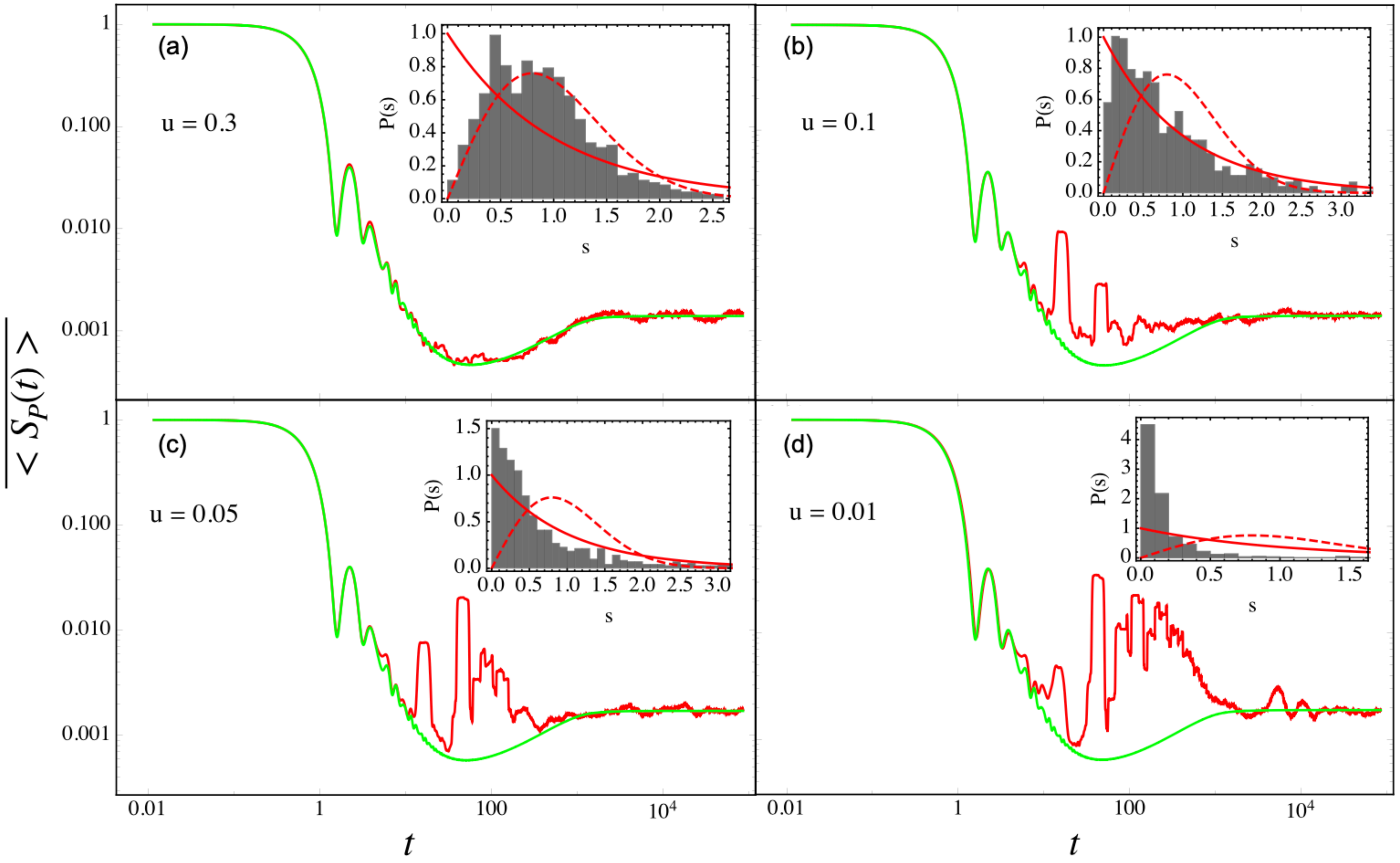
With level spacing distribution we don't detect chaos if the energy levels are not classified according to their symmetries

Chaos in the BHM

The survival probability is a dynamical observable defined as the probability to find a given initial state $|\Psi(0)>$ at time t. *The initial state has random components following a rectangular distribution centered in the center of the espectrum*

An analytical expression for the ensemble average of the survival probability was derived, which applies for the case of one sequence of nondegenerate energy levels with energy density ν and correlations similar to those of random matrices of a Gaussian orthogonal ensemble (GOE)

$$\langle S_P(t)\rangle = \frac{1-\langle S_P^{\infty}\rangle}{\eta - 1}\left[\eta S_P^{bc}(t) - b_2\left(\frac{t}{2\pi\bar{\nu}}\right)\right] + \langle S_P^{\infty}\rangle,$$



Quantum chaos in a system with high degree of symmetries

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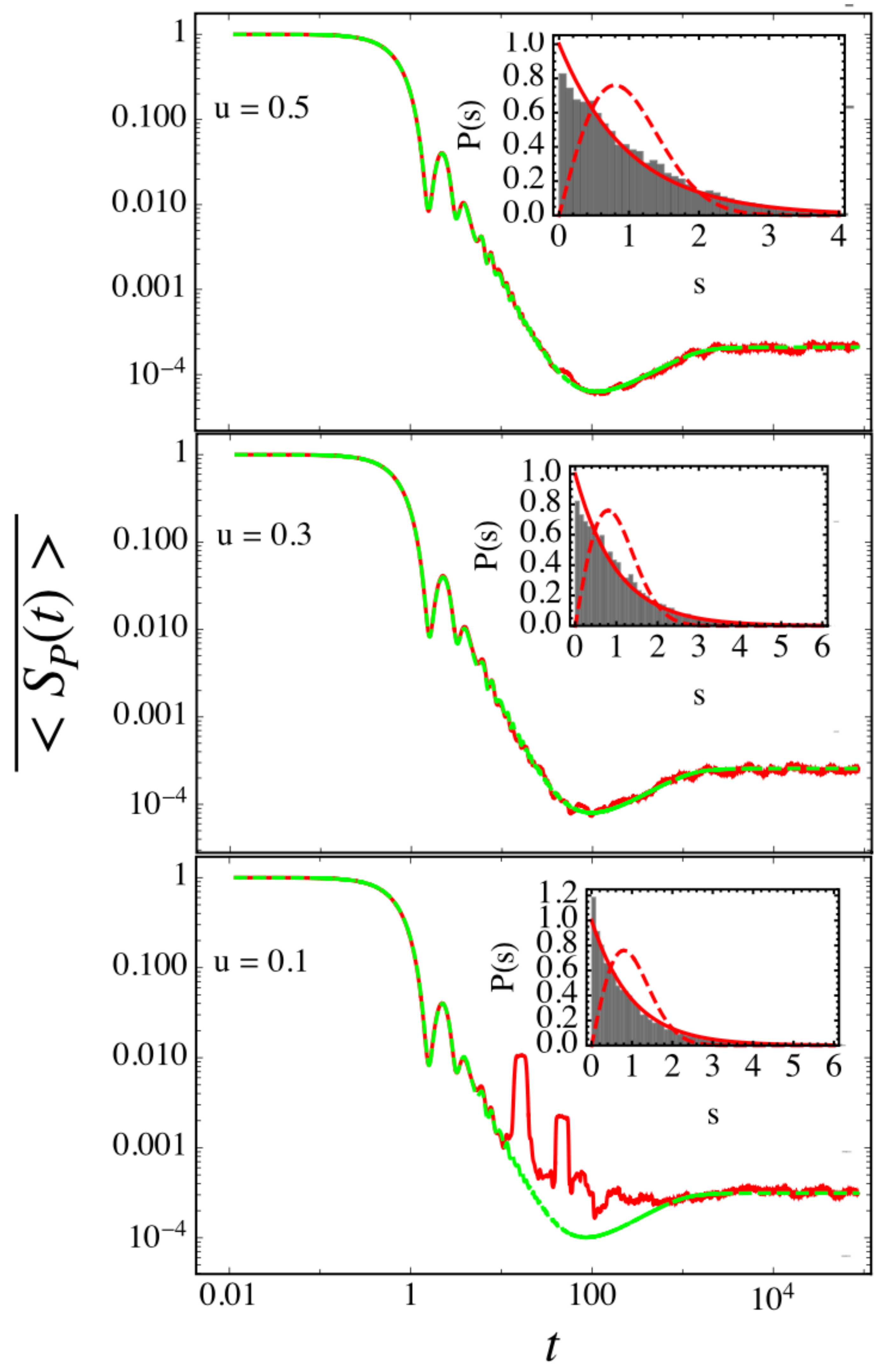
Phys. Rev. E **102**, 032208 – Published 8 September 2020

Chaos in the BHM

Three different couplings u, indicated in each panel, were employed. Light green lines are the saem rolling temporal averaged of the analytical expression in Eq. (21). Insets show the corresponding nearest-neighbor spacing distribution of energy levels considering all subspaces of the system.

$$\langle S_P(t)\rangle_a = \frac{1-\frac{4}{3\eta}}{\eta-1}\left[\eta S_P^{bc}(t) - \frac{16 b_2\left(\frac{9t}{2\pi\bar{\nu}}\right) + b_2\left(\frac{18t}{2\pi\bar{\nu}}\right)}{9}\right] + \langle S_P^{\infty}\rangle_a, \qquad (21)$$



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The Bose-Hubbard model yields a large number of invariant subspaces and degenerate energy levels. The standard procedure to reveal signatures of quantum chaos requires classifying the energy levels according to their symmetries. We show that this classification is not necessary to observe manifestations of spectral correlations in the temporal evolution of the survival probability, which makes this quantity a powerful tool in the identification of chaotic many-body quantum systems.

Introducing disorder in the BHM

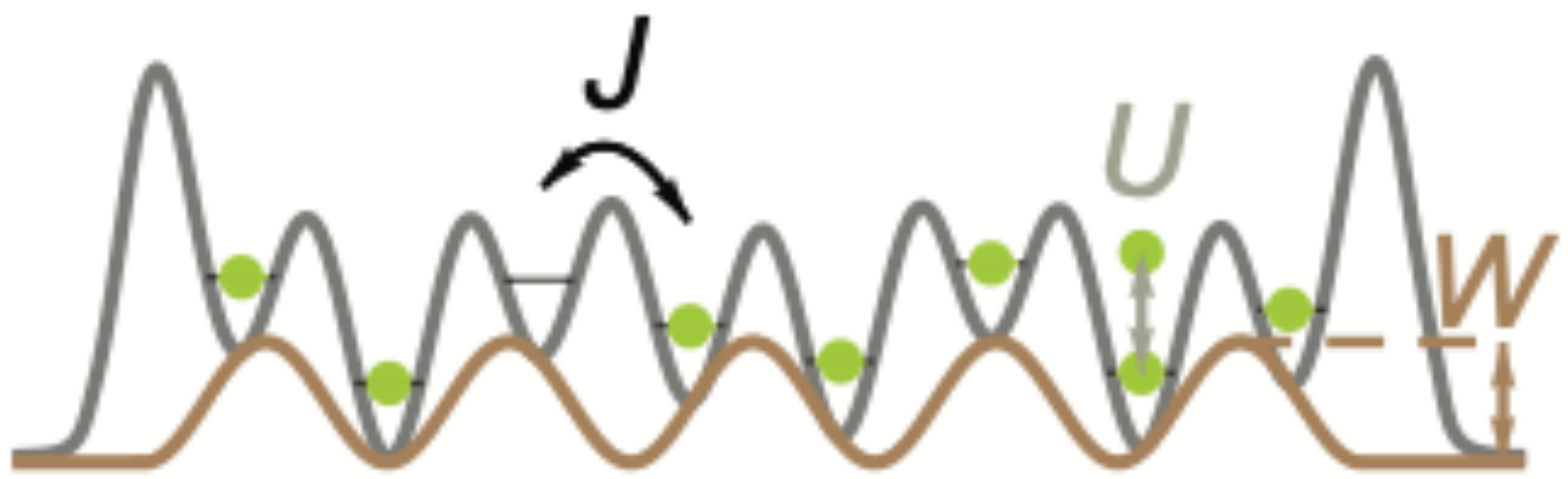
The Aubry-Andre model is similar to the Bose Hubbard model, but where disorder is introduced by a periodic modulation of the on-site energies, with a spatial period incommensurate with the lattice period. In this case we are working with open boundary conditions.

- $$\hat{H}_{AA} = -J\sum_{j=1}^{N}\left(\hat{a}_{j+1}^{\dagger}\hat{a}_j + \hat{a}_j^{\dagger}\hat{a}_{j+1}\right) + \frac{U}{2}\sum_{j=1}^{N}\hat{n}_j\left(\hat{n}_j - 1\right) + W\sum_i h_i\hat{n}_i$$
  $$= \hat{H}_{BH} + W\sum_i h_i\hat{n}_i$$

$$h_i = cos(2\pi\beta i + \phi)$$

-No symmetry is preserved

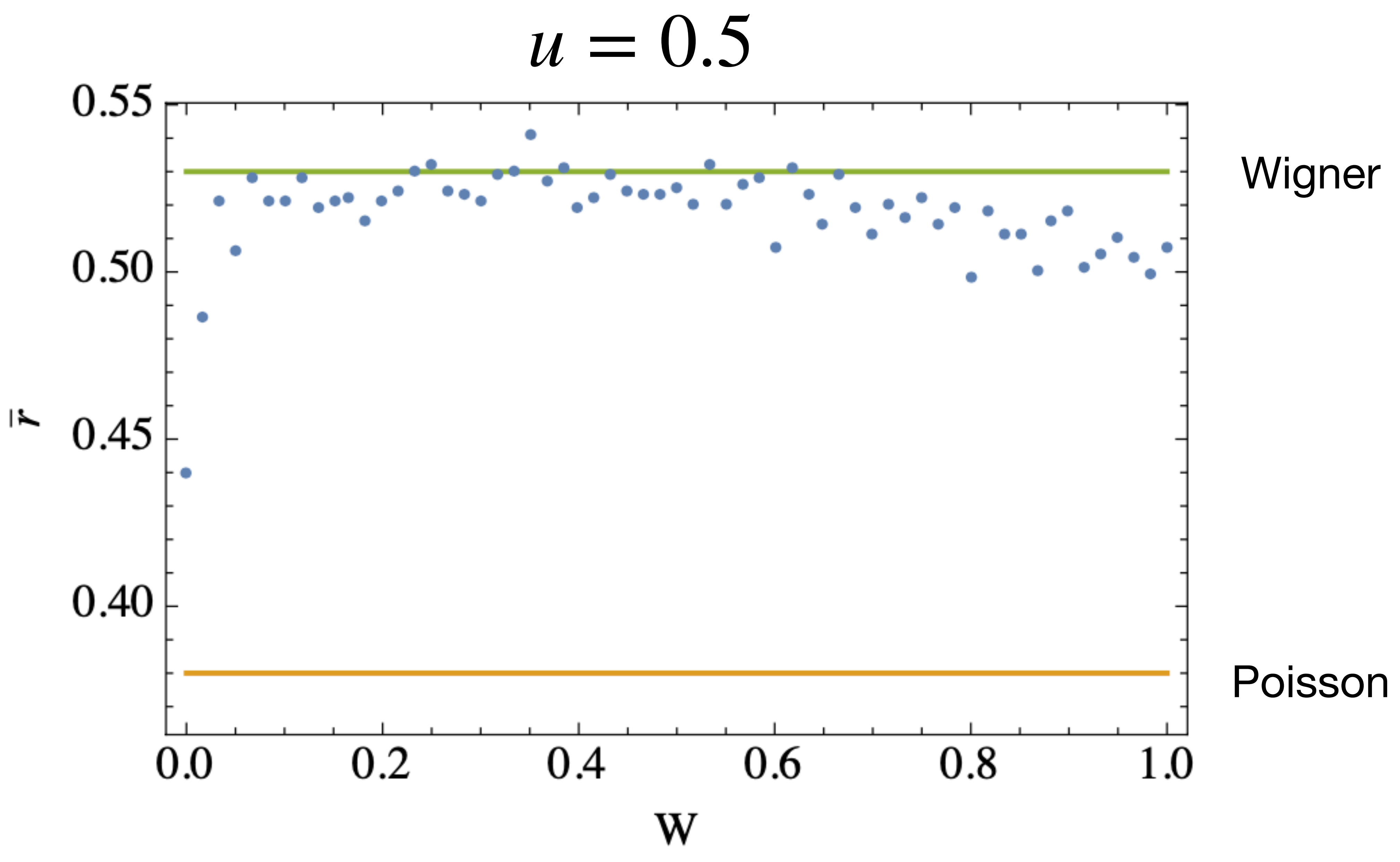
Physical model



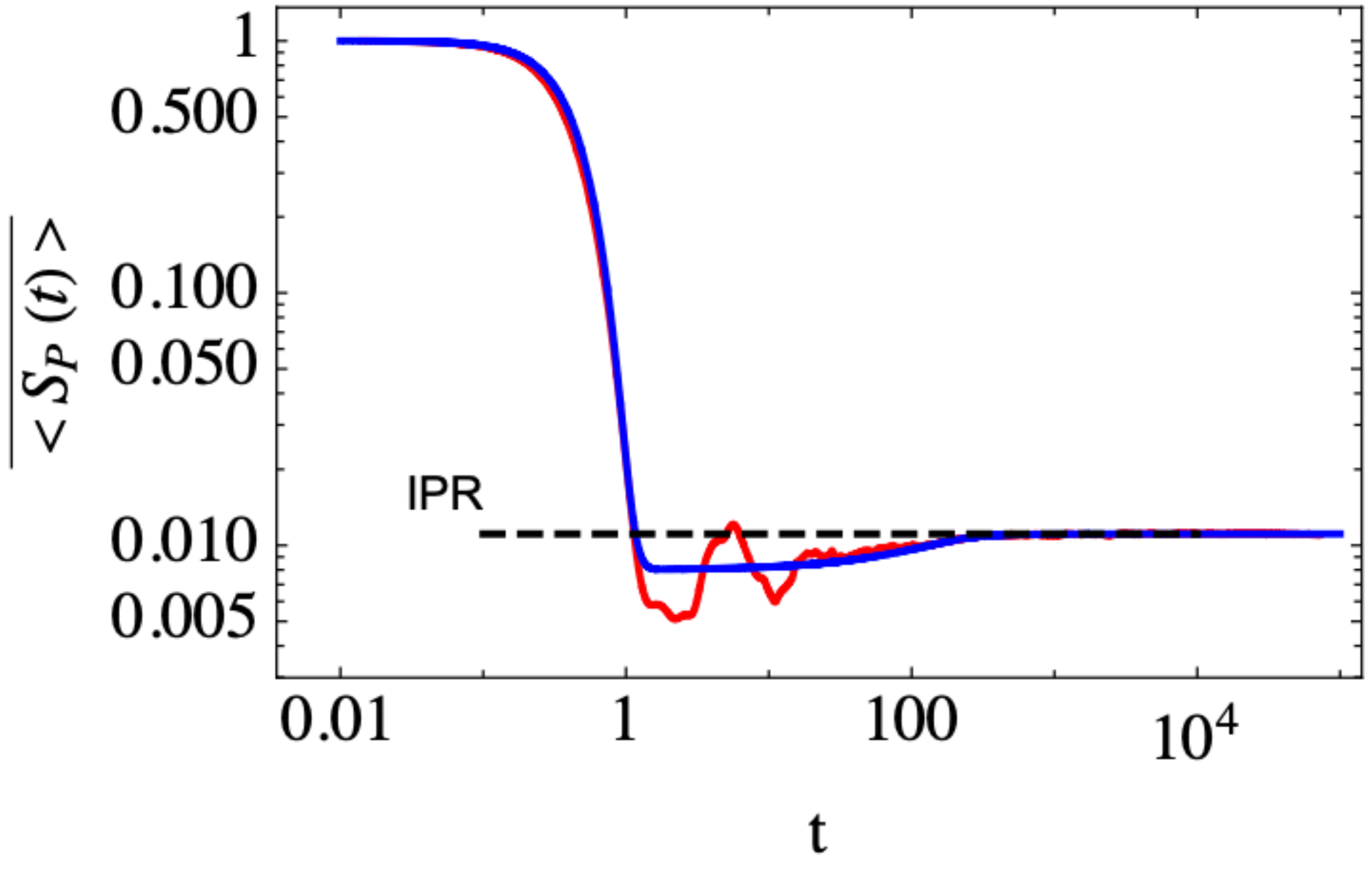
Introducing disorder in the BHM

$N = L = 6$ Initial state $|\Psi(0)> = |1,1,1,1,1,1>$ and 500 realizations taking random ϕ

$u = 0.5; W = 0.4$



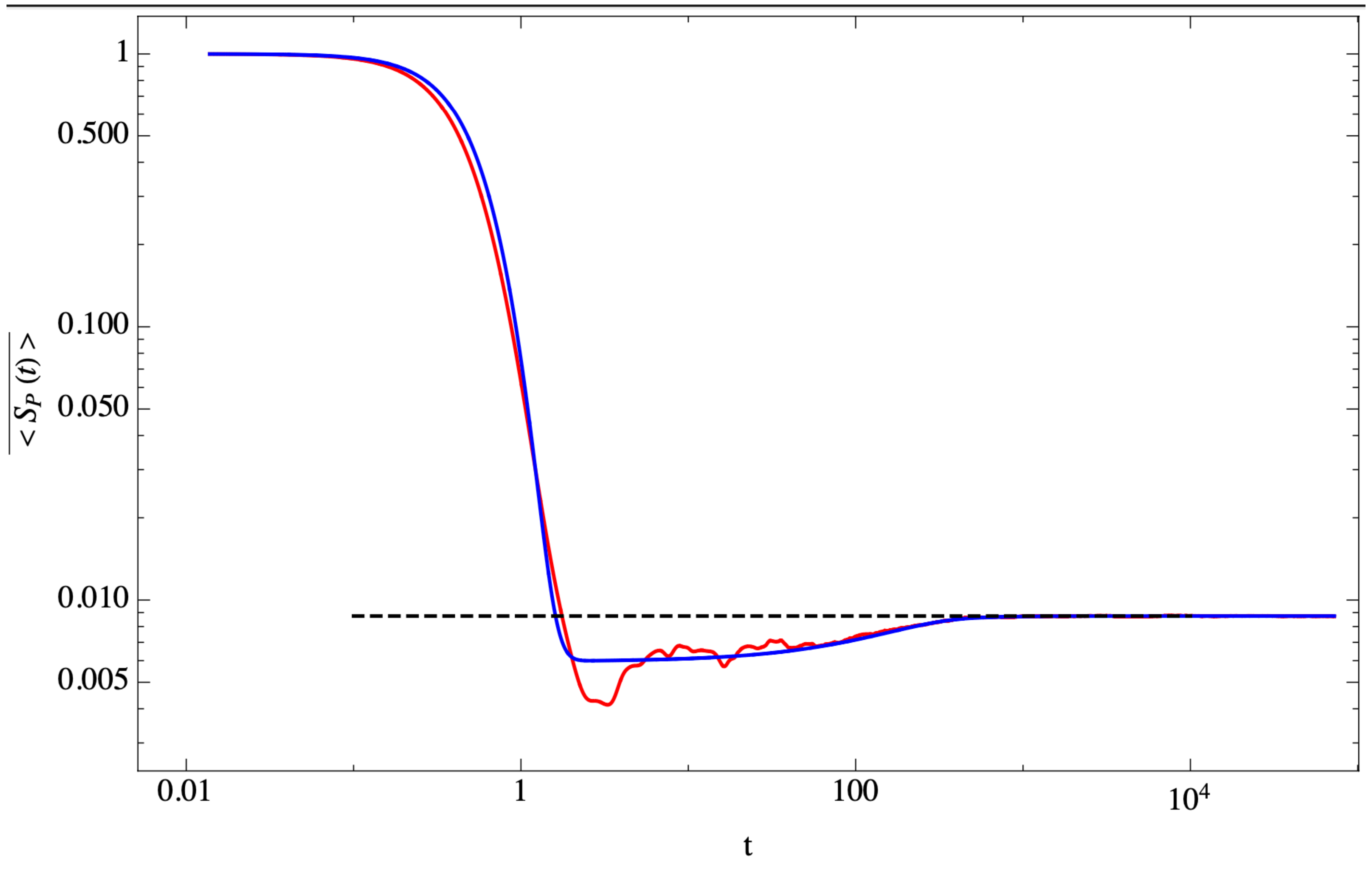
Average level spacing



The correlation hole is not clear.

Introducing disorder in the BHM

Initial state $|\Psi(0)> = |2,0,2,0,2,0>$



The correlación hole is clear

Thanks!!