Nonminimal derivative coupling scalar-tensor theories: odd-parity perturbations and black hole stability

A. Cisterna[1] M. Cruz[2] T. Delsate[3] J. Saavedra[4]

[1]Universidad Austral de Chile
[2]Facultad de Física, Universidad Veracruzana
[3]University of Mons - UMONS, Bélgica
[4]PUCV-Chile

Escuela de Física Fundamental, 2016

Outline

Horndeski theory

Inputs

- Modify gravity theory.
- Scalar-tensor theory.
- Non-minimal kinetic scalar field.

Modify gravity theory

As we know GR enjoys the following main properties

Einstein equations

$$G_{\mu\nu} = \kappa T_{\mu\nu}.$$

- Is invariant under diffeomorphisms

$$\nabla_{\mu} G^{\mu\nu} = 0$$

- Possesses second order and symmetric EOM

$$\frac{\partial^2 g_{\mu\nu}(x)}{\partial x^2}$$

- The spacetime is four-dimensional
- Only one field enters in the purely gravitational description of the theory, the metric field, $g_{\mu\nu}(x)$

In order to describe *new* gravitational phenomena through *modifications* of GR, we need to relax at least one of the previous features. Many examples of modified theories are known today:

- Higher dimensional theories as Lovelock
- Higher derivative theories
- Massive gravity theories, bigravity, $f(R)$-gravity...
- Brans-Dicke theory

Let us describe gravity with one extra degree of freedom

$$S[g_{\mu\nu},\phi,\psi]=\int\left(\phi R-\frac{\omega(\phi)}{\phi}\nabla_\mu\phi\nabla^\mu\phi-V(\phi)\right)d^4x+S_m[g,\psi],$$

Here the gravity sector is described by $g_{\mu\nu}$ and $\phi(x)$. The matter field are coupled only with the metric tensor.
Now we can ask ourself, which is the most general scalar-tensor theory which yields second order equations of motion, for both, the metric and the scalar field?
This question was answered by Horndeski 40 years ago.

$$L_H = k_1(\rho,\phi)\delta^{\mu\nu\rho}_{\alpha\beta\gamma}\nabla^\mu\nabla_\alpha\phi R^{\nu\sigma}_{\beta\gamma} - \frac{4}{3}k_{1,\rho}(\phi,\rho)\delta^{\mu\nu\rho}_{\alpha\beta\gamma}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi$$
$$+ \quad k_3(\phi,\rho)\delta^{\mu\nu\rho}_{\alpha\beta\gamma}\nabla_\alpha\phi\nabla^\mu\phi R^{\nu\sigma}_{\beta\gamma} + ...$$

- Here $\rho = \nabla_\mu\phi\nabla^\mu\phi$ is the standard kinetic term of the scalar field.

Was proven that this Lagrangian is equivalent to the Lagrangian coming from covariantized Galileons

$$L = K(\phi,\rho) - G_3(\phi,\rho)\Box\phi + G_4(\phi,\rho)R + G_{4,\rho}(\phi,\rho)[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2]$$
$$+ G_5(\phi,\rho)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,\rho}}{6}[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + (\nabla_\mu\nabla_\nu\phi)^3],$$

- Now commun sectors of the theory can be recognized easily, Brans-Dicke theory, K-essence, GR, etc.

In particular our interest is focused on the theory described by the following term

$$G_5(\phi,\rho)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi$$

which gives us non minimally kinetic coupled scalar fields

$$G_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi$$

Why second order theories?

For the *free particle we have*:

$$L_{fp} = \frac{1}{2}m\dot{x}^2 - V \underbrace{\Rightarrow}_{\text{Legendre transform}} H_{fp} = \frac{p^2}{2m} + V.$$

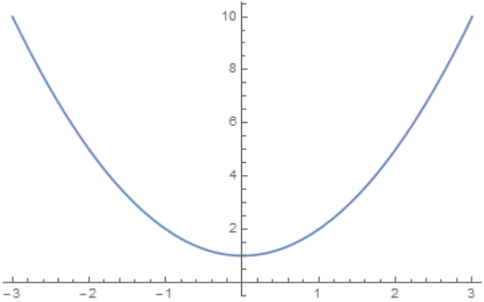

Figure: Energy of particle

Ghost states...

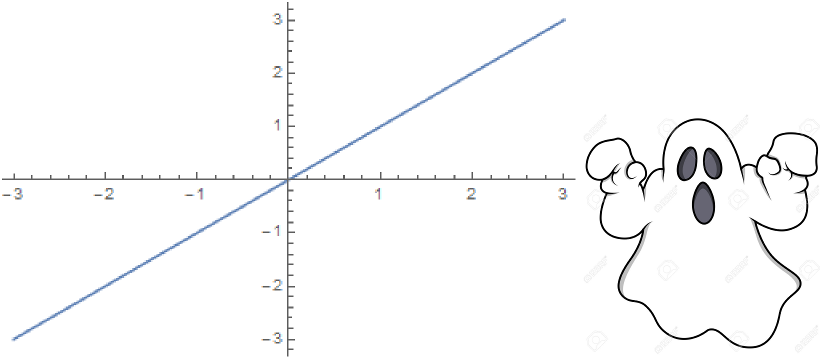

Figure: Energy of particle

Towards odd parity perturbations

- The model
- Equations of motion
- Odd-parity perturbations

The model

The action we are working with take the form

$$S[g_{\mu\nu},\phi] = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa}(R-2\Lambda) - \frac{1}{2}\left(\alpha g^{\mu\nu} - \beta G^{\mu\nu}\right)\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right]$$

We have:

$$E_{\mu\nu} = G_{\mu\nu} + \Lambda g_{\mu\nu} - \kappa\left[\alpha T_{\mu\nu} + \beta\Theta_{\mu\nu}\right] = 0,$$

and

$$\nabla_\mu\left(\alpha g^{\mu\nu}\nabla_\nu\phi - \beta G^{\mu\nu}\nabla_\nu\phi\right) - \frac{dV(\phi)}{d\phi} = 0,$$

For simplicity

$$\begin{aligned}
T_{\mu\nu} &= \nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}\nabla_\alpha\phi\nabla^\alpha\phi - \frac{1}{\alpha}g_{\mu\nu}V(\phi), \\
\Theta_{\mu\nu} &= \frac{1}{2}\nabla_\mu\phi\nabla_\nu\phi R - 2\nabla_\alpha\phi\nabla_{(\mu}\phi R_{\nu)}{}^\alpha - \nabla^\alpha\phi\nabla^\beta\phi R_{\mu\alpha\nu\beta} + \frac{1}{2}G_{\mu\nu}\nabla_\alpha\phi\nabla^\alpha\phi \\
&- (\nabla_\mu\nabla^\alpha\phi)(\nabla_\nu\nabla_\alpha\phi) + (\nabla_\mu\nabla_\nu\phi)\Box\phi \\
&+ g_{\mu\nu}\left[-\frac{1}{2}(\Box\phi)^2 + \nabla_\alpha\phi\nabla_\beta\phi R^{\alpha\beta} + \frac{1}{2}(\nabla^\alpha\nabla^\beta\phi)(\nabla_\alpha\nabla_\beta\phi)\right].
\end{aligned}$$

The perturbed metric

$$
\begin{aligned}
ds^2 &= -A(r)dt^2 + B(r)dr^2 \\
&+ C(r)\left[\frac{dz^2}{1-kz^2} + (1-kz^2)(d\varphi + k_1 dt + k_2 dr + k_3 dz)^2\right],
\end{aligned}
$$

- $\phi = \phi_0(r) + \epsilon\Phi(t, r, z)$

Considering the Einstein field equations only at first order in ϵ, we find that

$$E_r^t = \epsilon\kappa\frac{d\phi_0}{dr}\left[\alpha\frac{\partial_t\Phi}{A} - \frac{\beta}{ABC}\left(\frac{1}{2}\frac{A_rC_r}{A} + \frac{1}{4}\frac{C_r^2}{C} - kB - C_r\partial_r\right)\partial_t\Phi\right] + \mathcal{O}(\epsilon^2) = 0,$$

$$E_z^r = -\epsilon\kappa\frac{d\phi_0}{dr}\left[\alpha\frac{\partial_z\Phi}{B} - \frac{\beta}{B^2C}\left(\frac{1}{2}\frac{A_rC_r}{A} + \frac{1}{4}\frac{C_r^2}{C} - \frac{1}{2}\left(\frac{AC_r + CA_r}{A}\right)\partial_r\right)\partial_z\Phi\right]$$
$$+ \mathcal{O}(\epsilon^2) = 0,$$

- $\Phi = 0$

BUT!

$$E^r_\varphi = \frac{\partial}{\partial z}\left[\frac{A}{C}\left\{1+\frac{\beta\kappa}{2B}\left(\frac{d\phi_0}{dr}\right)^2\right\}(1-kz^2)^2\left(\partial_z k_2-\partial_r k_3\right)\right]$$
$$+\frac{\partial}{\partial t}\left[\left\{1+\frac{\beta\kappa}{2B}\left(\frac{d\phi_0}{dr}\right)^2\right\}(1-kz^2)\left(\partial_r k_1-\partial_t k_2\right)\right]=0,$$
$$E^t_\varphi = \frac{\partial}{\partial z}\left[C\sqrt{\frac{B}{A}}\left\{1-\frac{\beta\kappa}{2B}\left(\frac{d\phi_0}{dr}\right)^2\right\}(1-kz^2)^2(\partial_z k_1-\partial_t k_3)\right]$$
$$+\frac{\partial}{\partial r}\left[\frac{C^2}{\sqrt{AB}}\left\{1+\frac{\beta\kappa}{2B}\left(\frac{d\phi_0}{dr}\right)^2\right\}(1-kz^2)(\partial_r k_1-\partial_t k_2)\right]=0,$$
$$E^z_\varphi = \frac{\partial}{\partial r}\left[C\sqrt{\frac{A}{B}}\left\{1+\frac{\beta\kappa}{2B}\left(\frac{d\phi_0}{dr}\right)^2\right\}\left(\partial_z k_2-\partial_r k_3\right)\right]$$
$$+\frac{\partial}{\partial t}\left[C\sqrt{\frac{B}{A}}\left\{1-\frac{\beta\kappa}{2B}\left(\frac{d\phi_0}{dr}\right)^2\right\}\left(\partial_t k_3-\partial_z k_1\right)\right]=0.$$

The important thing is...

$$\frac{C^2}{\sqrt{AB}}\mathcal{P}_{(+)}\frac{\partial}{\partial r}\left[\frac{1}{C}\sqrt{\frac{A}{B}}\frac{1}{\mathcal{P}_{(-)}}\frac{\partial\mathcal{Q}}{\partial r}\right]+(1-kz^2)^2\frac{\partial}{\partial z}\left[\frac{1}{(1-kz^2)}\frac{\partial\mathcal{Q}}{\partial z}\right]=\frac{C}{A}\partial_t^2\mathcal{Q}. \quad (1)$$

- $\mathcal{Q}=C\sqrt{\frac{A}{B}}\mathcal{P}_{(+)}(1-kz^2)^2(\partial_z k_2-\partial_r k_3)$,
- $\mathcal{P}(r)_{(\pm)}=1\pm\frac{\beta\kappa}{2B}\left(\frac{d\phi_0}{dr}\right)^2$.

The mathematical methods courses are useful!
$\to\mathcal{Q}=Q(r,t)D(z)$

The master equation

$$\Psi(r^*,t) = \left[C\mathcal{P}_{(-)}\right]^{-1/2} Q(r,t)$$

- Fourier decomposition: $\Psi = \int \Psi_\omega e^{i\omega t} dt$

$$\mathcal{H}\Psi_\omega := -\frac{\partial^2\Psi_\omega}{\partial r^{*2}} + \left(\gamma\frac{A}{C\mathcal{P}_{(+)}} + \frac{3}{4C^2}\left(\frac{dC}{dr^*}\right)^2 - \frac{1}{2C}\frac{d^2C}{dr^{*2}} + \frac{3}{4\mathcal{P}_{(-)}^2}\left(\frac{d\mathcal{P}_{(-)}}{dr^*}\right)^2 - \frac{1}{2\mathcal{P}_{(-)}}\frac{d^2\mathcal{P}_{(-)}}{dr^{*2}} + \frac{1}{2C\mathcal{P}_{(-)}}\frac{dC}{dr^*}\frac{d\mathcal{P}_{(-)}}{dr^*}\right)\Psi_\omega = \omega_{eff}^2\Psi_\omega,$$

$$= -\frac{\partial^2\Psi_\omega}{\partial r^{*2}} + V\Psi_\omega = \omega_{eff}^2\Psi_\omega,$$

- $\omega_{eff}^2 = \frac{\mathcal{P}_{(-)}}{\mathcal{P}_{(+)}}\omega^2$.

The spectrum

$$\int dr^* (\Psi_\omega)^* \mathcal{H} \Psi_\omega = \int dr^* \left[\mid D\Psi_\omega \mid^2 + V_S \mid \Psi_\omega \mid^2 \right] - (\Psi_\omega D\Psi_\omega) \mid_{Boundary},$$

where $D = \frac{\partial}{\partial r^*} + S$ and

$$V_S = V + \frac{dS}{dr^*} - S^2. \qquad (2)$$

If we choose $S = \frac{1}{2C}\frac{dC}{dr^*} + \frac{1}{2\mathcal{P}_{(-)}}\frac{d\mathcal{P}_{(-)}}{dr^*}$, we find

$$V_S = \gamma \frac{A}{C\mathcal{P}_{(+)}}. \qquad (3)$$

The solution

For spherically symmetric spacetimes

$$A(r) = \frac{r^2}{L^2} + \frac{k}{\alpha}\sqrt{\alpha\beta k}\left(\frac{\alpha+\beta\Lambda}{\alpha-\beta\Lambda}\right)^2 \frac{\arctan\left(\frac{\sqrt{\alpha\beta k}}{\beta k} r\right)}{r} - \frac{\mu}{r} + \frac{3\alpha+\beta\Lambda}{\alpha-\beta\Lambda}k,$$

and

$$B(r) = \frac{\alpha^2((\alpha-\beta\Lambda)\, r^2 + 2\beta k)^2}{(\alpha-\beta\Lambda)^2(\alpha r^2+\beta k)^2 A(r)} \ ,$$

- $C(r) = r,$
- $\alpha + \beta\Lambda < 0.$

Slowly-rotating objects

- $k_1 = \omega(r)$.

Then the frame dragging function satisfies

$$\frac{\partial}{\partial r}\left[\frac{C^2}{\sqrt{AB}}\left\{1+\frac{\beta\kappa}{2B}\left(\frac{d\phi_0}{dr}\right)^2\right\}(1-kz^2)\frac{\partial}{\partial r}\omega(r)\right]=0.$$

Solving we have

The GR case

$$\omega(r)=c_1+\frac{c_2}{r}.$$

Thank you for your attention!