

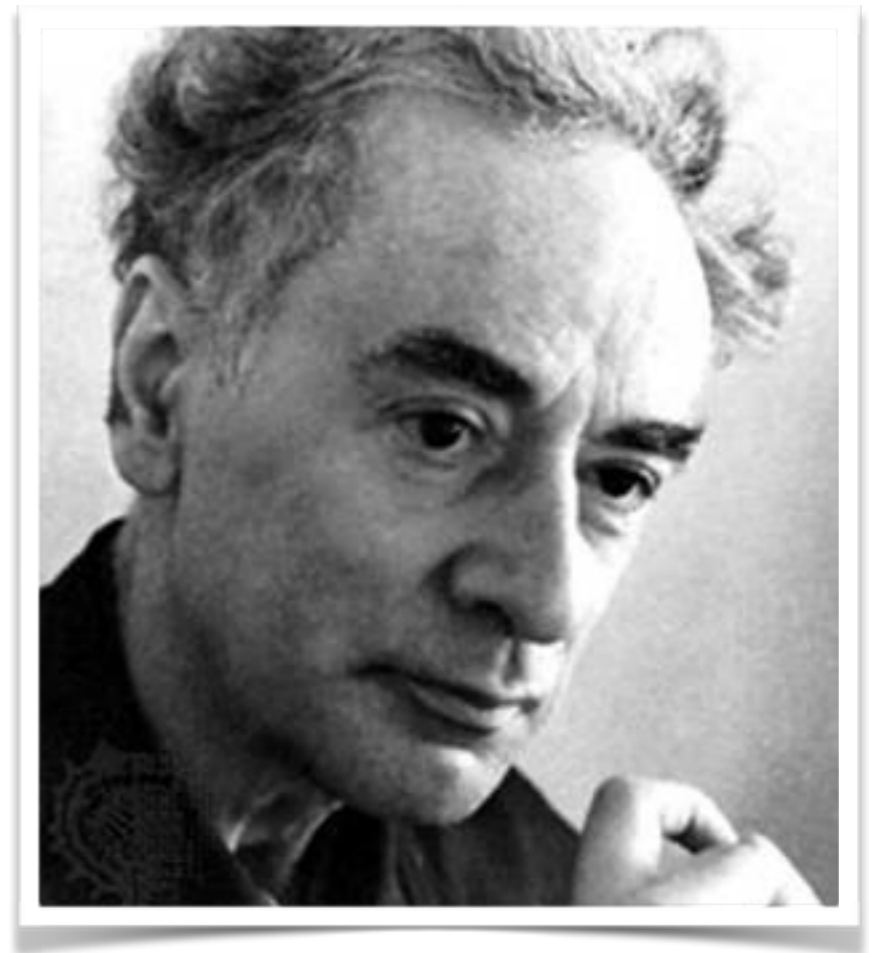
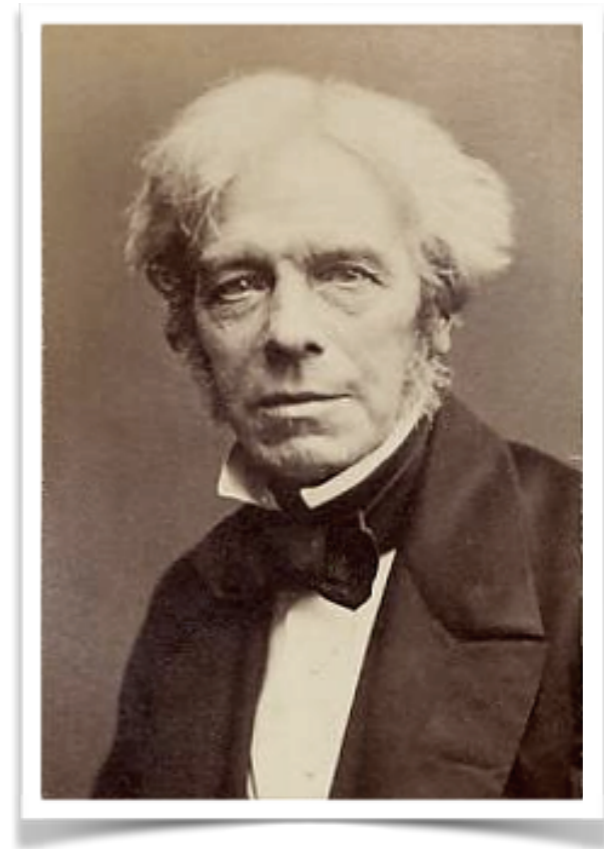
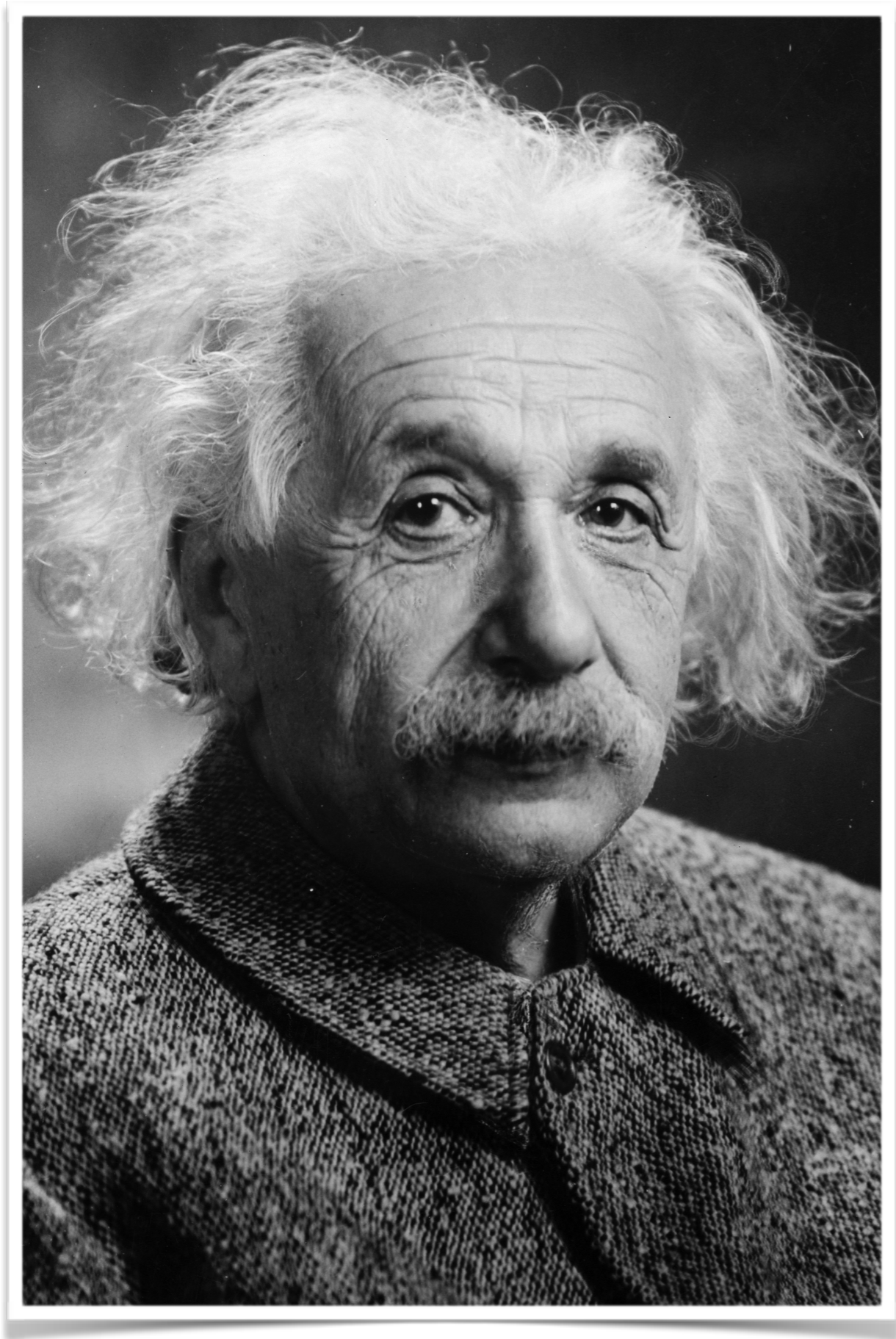


XI Escuela de Física Fundamental

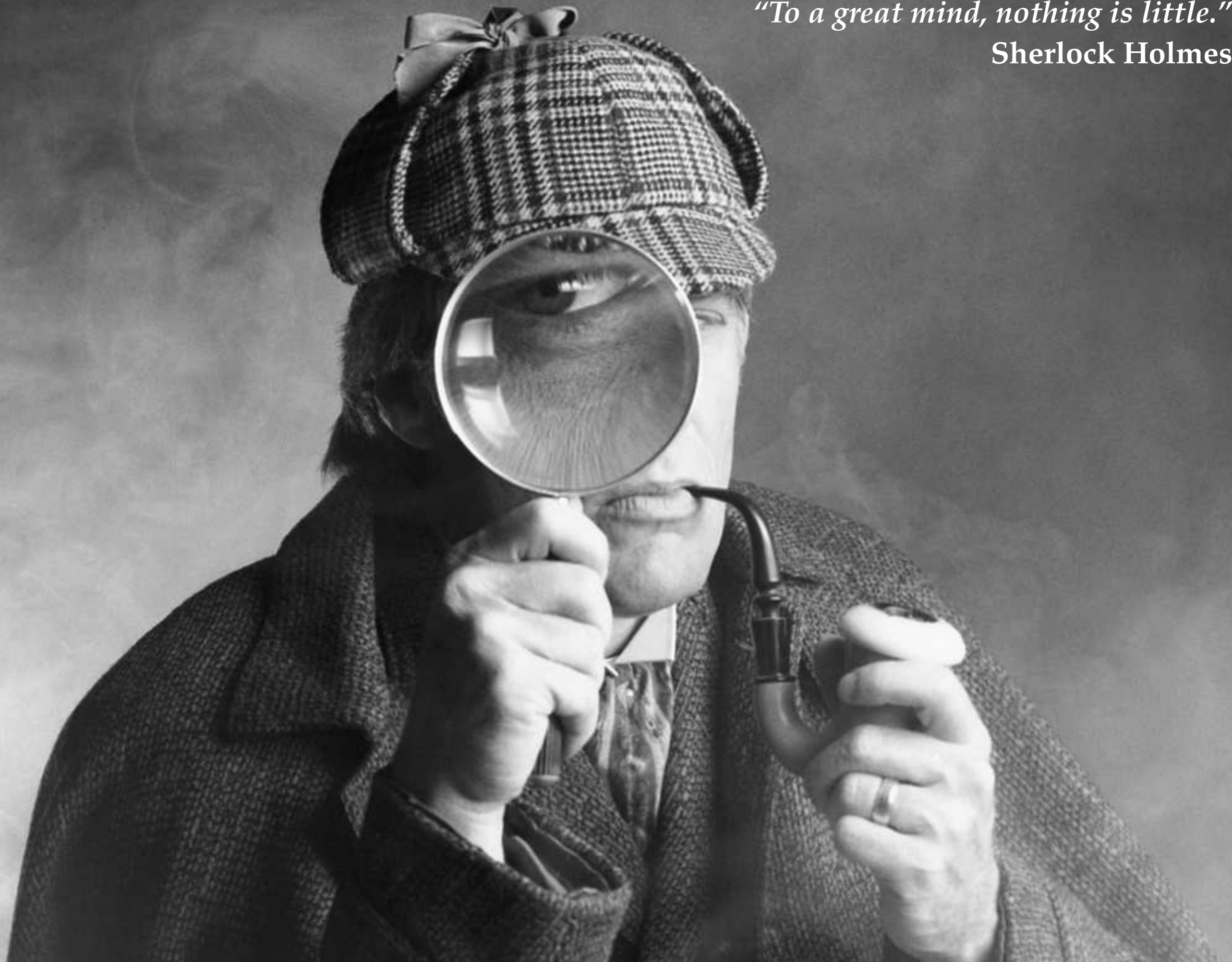
Las Ideas se trabajan: ¿Cómo es eso?

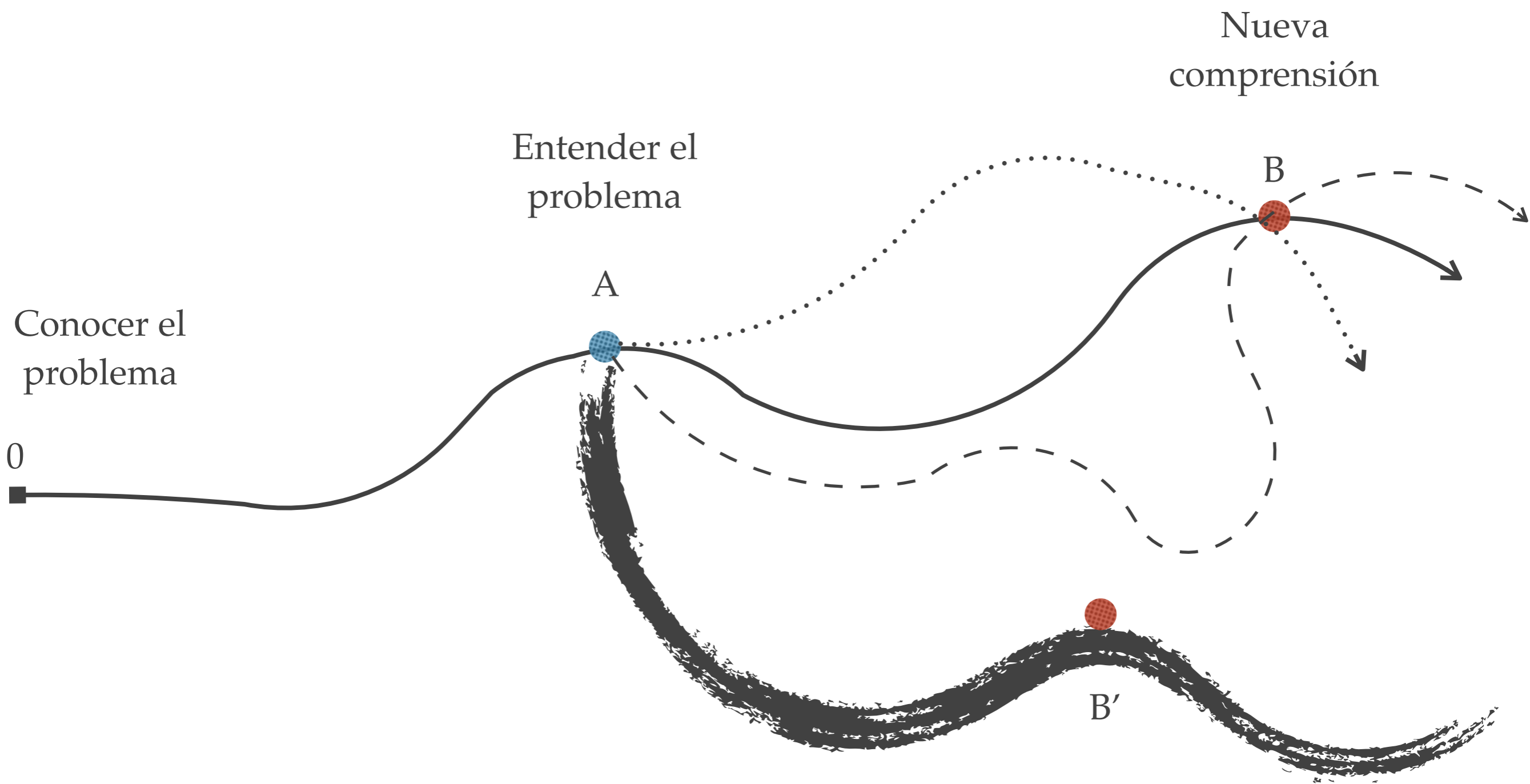
U. J. Saldaña Salazar
FCFM-BUAP

Xalapa, Veracruz
27 de octubre de 2016



"To a great mind, nothing is little."
Sherlock Holmes







“Querer entender”

Conocer el problema

0



“Querer publicar”

Entender el problema

A

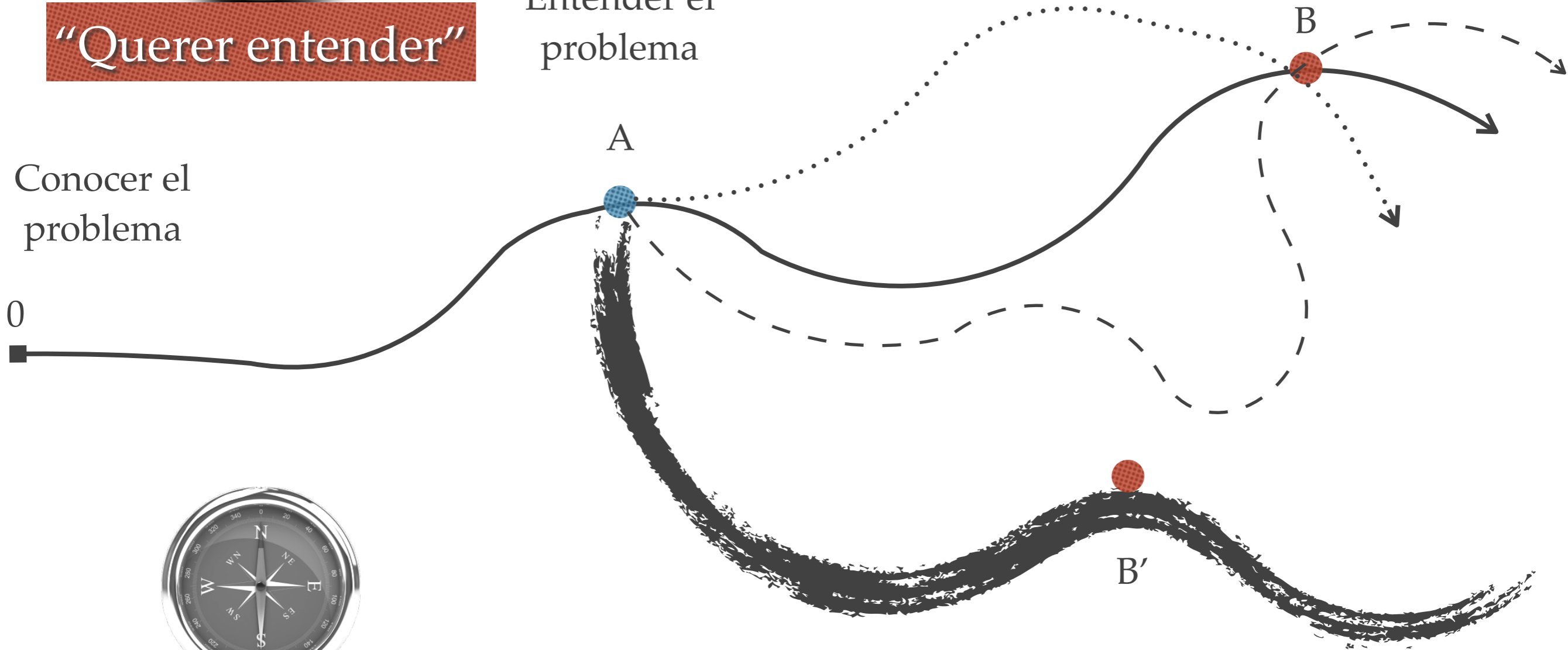


Nueva comprensión

B



B'





“Querer entender”

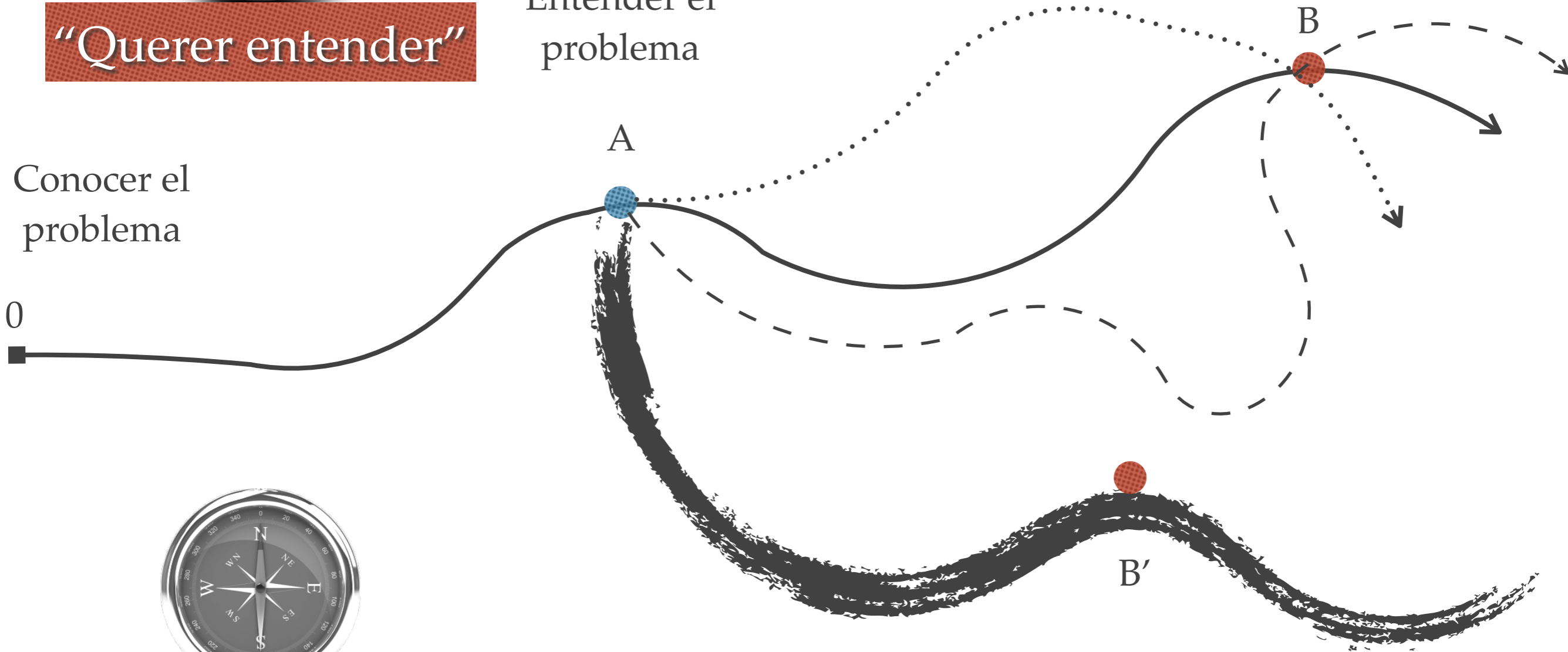
Loop de Conocimiento
 $A1 \rightarrow B = A2$

B -> A
Nueva
comprensión

Conocer el
problema

0

Entender el
problema



“Querer publicar”



“¿Ves la Lagrangiana del ME?”



“¿Ves la Lagrangiana del ME?
Pues algún día todos sus
parámetros serán tuyos”



“¿Ves la Lagrangiana del ME?
Pues algún día todos sus
parámetros serán tuyos.”

¿Y mi futuro, Apa?



Contenido

LO ACADÉMICO

1. El problema del sabor
2. Un modelo con S_3 y $4H$
3. La parametrización de masas
4. Principio de Ceguera del Sabor
5. Conclusiones

Contenido

LO ACADÉMICO

1. El problema del sabor
2. Un modelo con S3 y 4H
3. La parametrización de masas
4. Principio de Ceguera del Sabor
5. Conclusiones

LO PERSONAL


1. ¿Qué elegir? ¡Sí, señor!
2. “La Iluminación”
3. ¡Crisis! ¡Crisis!
4. Conociendo a Hitler
5. ¡Tu trabajo es una basura!
6. ¡Cuánto calienta el sol, aquí en la playa!

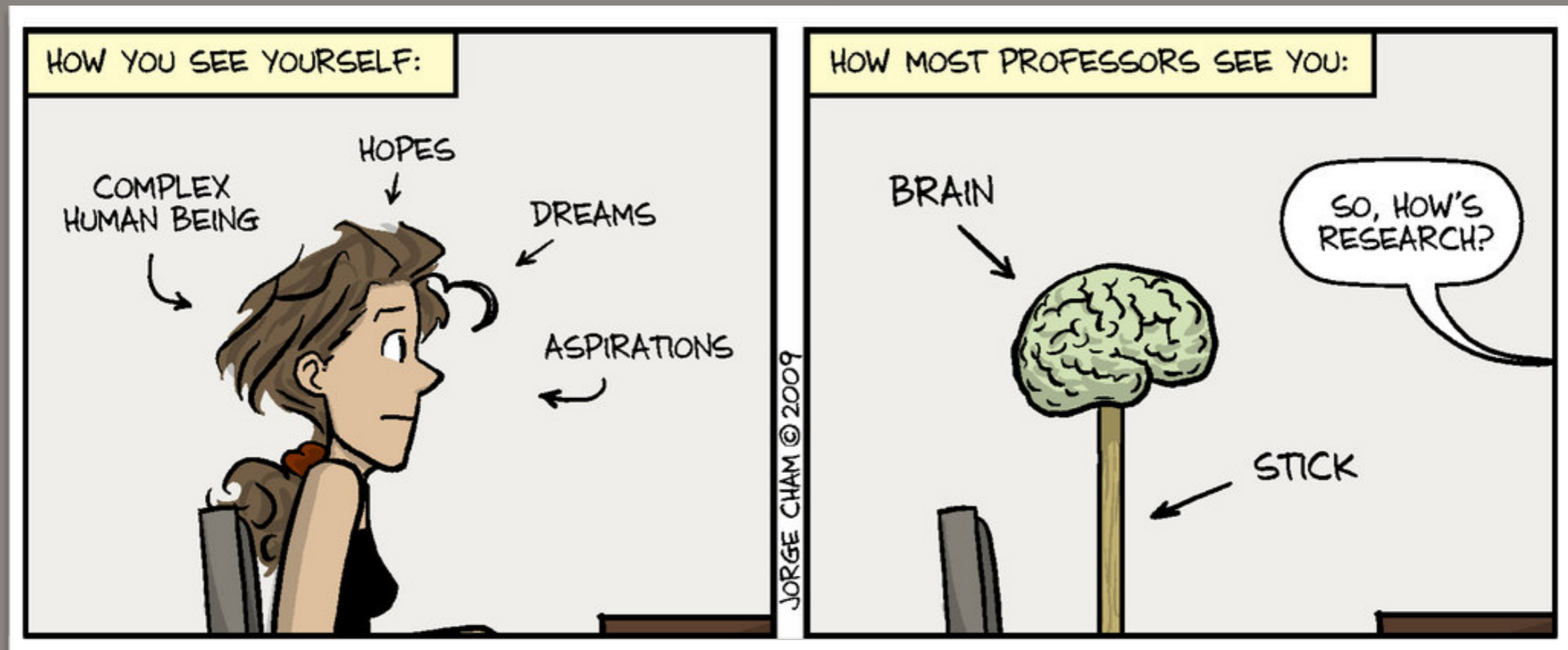
Contenido

LO ACADÉMICO

1. El problema del sabor
2. Un modelo con S3 y 4H
3. La parametrización de masas
4. Principio de Ceguera del Sabor
5. Conclusiones

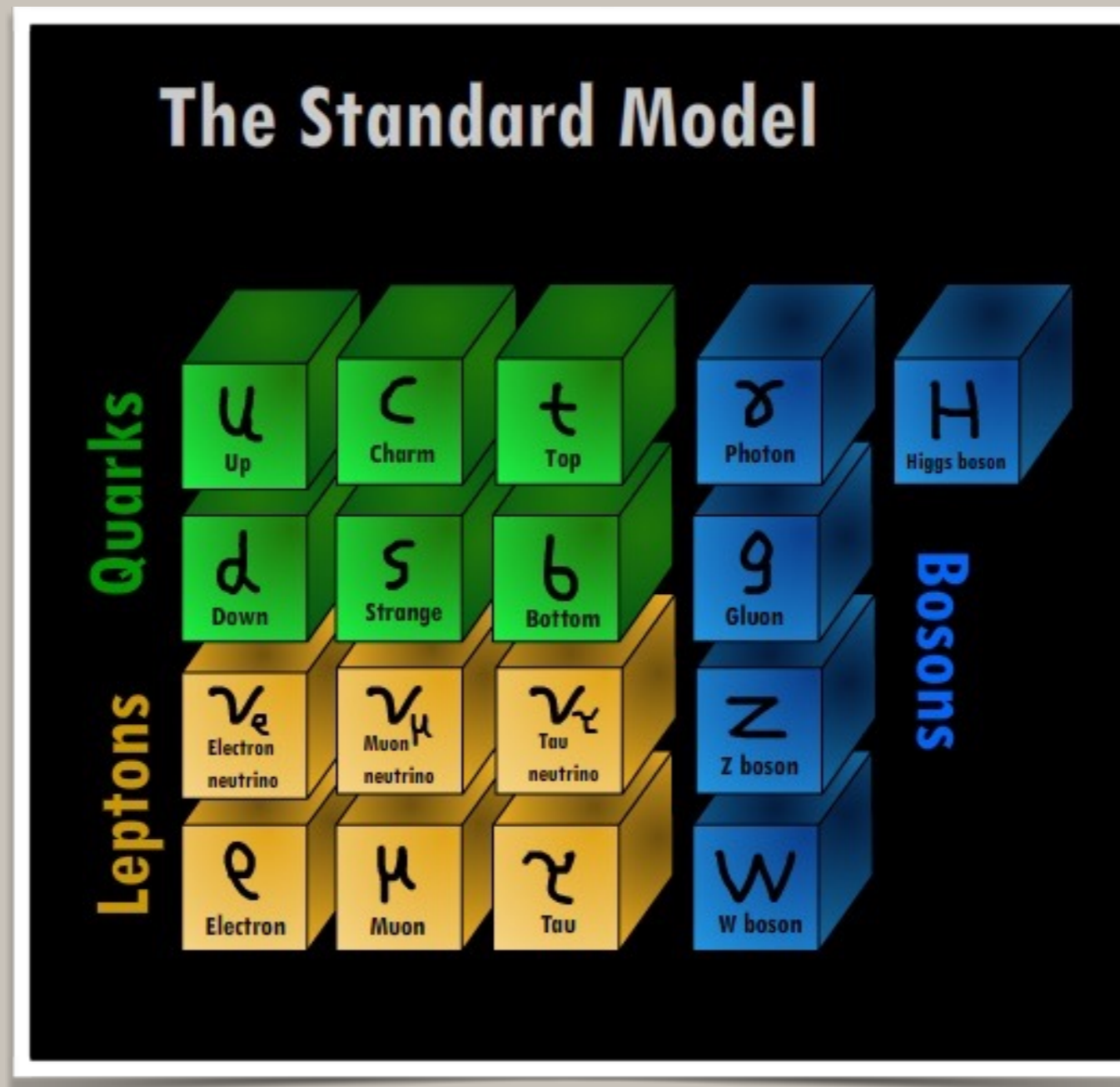
LO PERSONAL

1. ¿Qué elegir? ¡Sí, señor! **Punto 0**
 2. “La Iluminación” **Punto A**
 3. ¡Crisis! ¡Crisis!
 4. Conociendo a Hitler **Punto B**
 5. ¡Tu trabajo es una basura!
 6. ¡Cuánto calienta el sol, aquí en la playa! **Punto A'**
- 

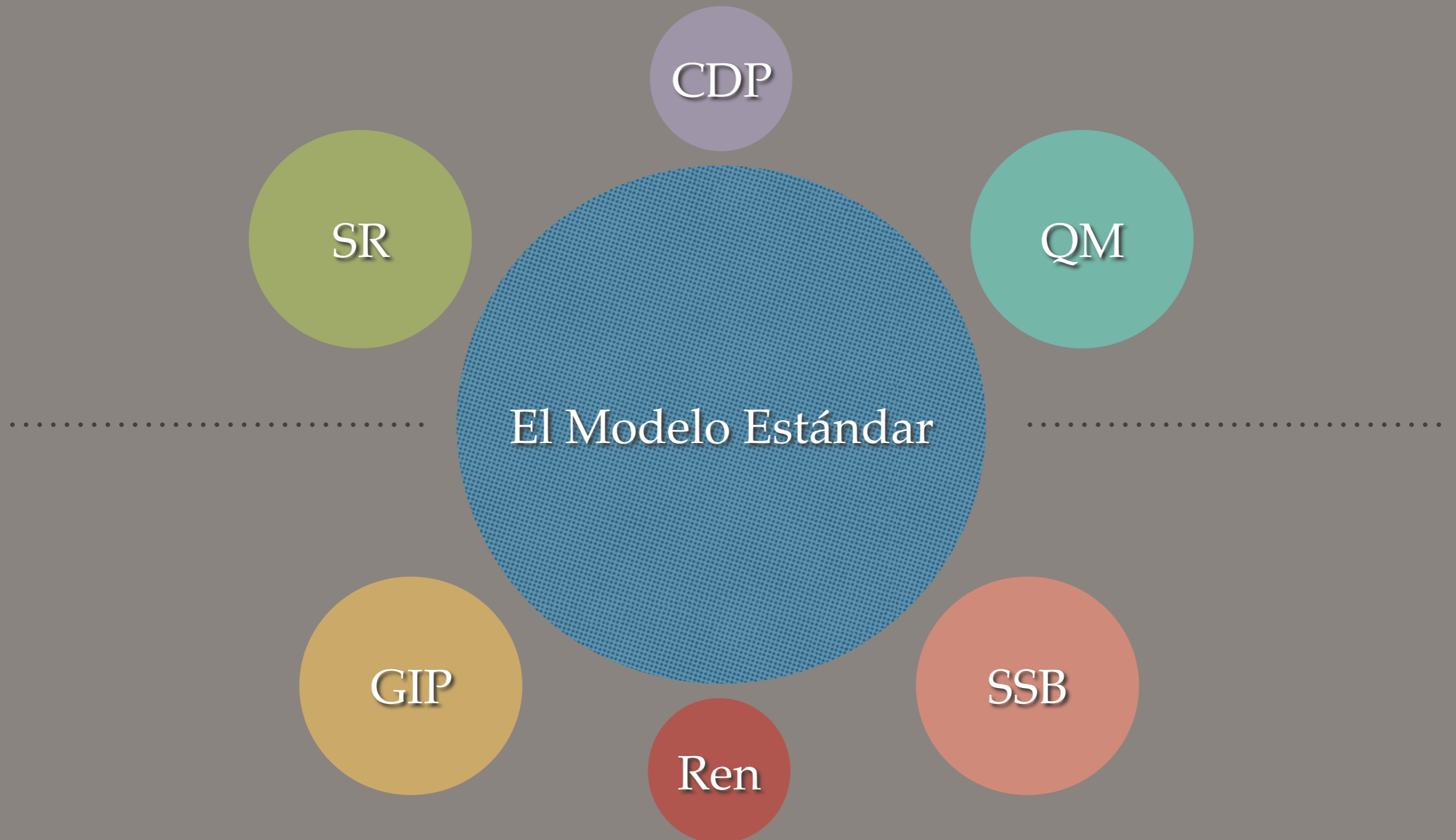


¿Qué elegir? ¡Sí, señor!
(Conocer el problema)

Un modelo con S_3 y $4H$ (durante)



El problema del sabor



El problema del sabor

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

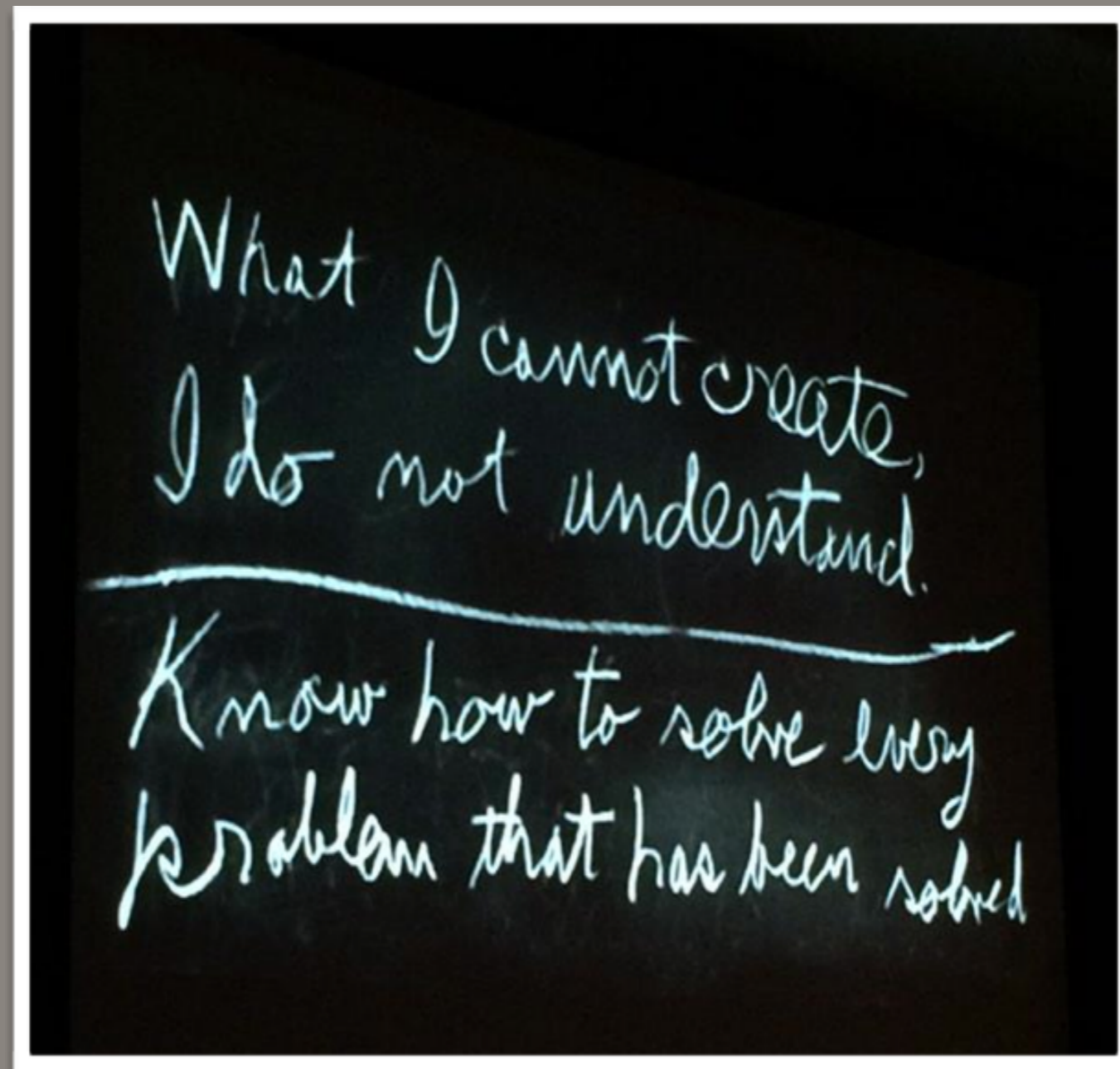
$$|U_{\text{PMNS}}^{\text{th}}| = \begin{pmatrix} 0.83^{+0.04}_{-0.05} & 0.54^{+0.06}_{-0.09} & 0.14 \pm 0.03 \\ 0.38^{+0.04}_{-0.06} & 0.57^{+0.03}_{-0.04} & 0.73 \pm 0.02 \\ 0.41^{+0.04}_{-0.06} & 0.61^{+0.03}_{-0.04} & 0.67 \pm 0.02 \end{pmatrix} ;$$

El problema del sabor

$$|V_{CKM}| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

$$|U_{PMNS}| = \begin{pmatrix} 0.801 \rightarrow 0.845 & 0.514 \rightarrow 0.580 & 0.137 \rightarrow 0.158 \\ 0.225 \rightarrow 0.517 & 0.441 \rightarrow 0.699 & 0.614 \rightarrow 0.793 \\ 0.246 \rightarrow 0.529 & 0.464 \rightarrow 0.713 & 0.590 \rightarrow 0.776 \end{pmatrix}$$

- ¿Por qué tres familias?
- ¿Por qué V y U son tan diferentes?
- ¿De dónde viene la violación de CP ?
- ¿Por qué existe una jerarquía en las masas?
- ¿Por qué la masa del top es tan grande? $m_t \gg m_i$
- ...



“La Iluminación”
(Entender el problema)

“La Iluminación”

An heuristic S_3 flavour model

A. Mondragón^a and U. J. Saldaña Salazar^a

(*a*) Instituto de Física, Universidad Nacional Autónoma de México,
Apdo. Postal 20-364, 01000, México D.F., México.

May 14, 2013

Abstract

The Standard Model (SM) is extended to include, besides the massive nature of neutrinos, analytical and exact formulae for the mixing angles, of both quarks and leptons, in terms of the corresponding mass ratios. In order to do so, first we add a discrete and non-Abelian symmetrical group, S_3 , which describes a feature of indistinguishability of three objects, in this case all three flavours of each family couple equally to the gauge bosons.

$$\tan^2 \theta_c \approx \frac{m_d}{m_s}$$

Gatto, Sartori, Tonin (1968),
Cabibbo (1968), Tanaka (1969),
Mohapatra (1977), Weinberg (1977),
Fritzsch (1977), Ramond (1993), Xing (1996),
Rasin (1997), Chkareuli (1998), Mondragón (1998),
Tanimoto (1999), Fritzsch, Xing (1999), King,
Valle, Peinado, Spinrath, Antusch...

2 Mixing angles in terms of mass ratios

In the 1980's, it was experimentally noticed that the Higgs mechanism gives masses and properties of the weak bosons. Since then, this was already taken by people as an indication that the Higgs particle should exist. Now that it has been mostly confirmed by the LHC then one is forced to ask questions about the possibility of all the renormalizable, gauge invariant, and Lorentz invariant new terms.

“La Iluminación”

An heuristic S_3 flavour model

A. Mondragón^a and U. J. Saldaña Salazar^a

(*a*) Instituto de Física, Universidad Nacional Autónoma de México,
Apdo. Postal 20-364, 01000, México D.F., México.

May 14, 2013

Abstract

The Standard Model (SM) is extended to include, besides the massive nature of neutrinos, analytical solutions for the quark masses and mixing in terms of the correlation of the S_3 Abelian symmetry objects, in this

$$\tan^2 \theta_c \approx \frac{m_d}{m_s}$$

Gatto, Sartori, Tonin (1968),

PHYSICAL REVIEW D **88**, 096004 (2013)

Quark sector of S_3 models: Classification and comparison with experimental data

F. González Canales,^{1,2} A. Mondragón,^{1,*} M. Mondragón,¹ U. J. Saldaña Salazar,¹ and L. Velasco-Sevilla^{1,3}

¹*Instituto de Física, Universidad Nacional Autónoma de México, Apartado Postal 20-364, 01000 México DF, México*

²*Facultad de Ciencias de la Electrónica, Benemérita Universidad Autónoma de Puebla, Apartado Postal 157, 72570 Puebla, Puebla, México*

³*University of Hamburg, II. Institute for Theoretical Physics, Luruper Chaussee 149, 22761 Hamburg, Germany*

(Received 29 April 2013; published 8 November 2013)

S_3 models offer a low energy approach to describe the observed pattern of masses and mixing, of both quarks and leptons. In this work, we first revisit an S_3 model with only one Higgs electroweak doublet, where the flavor symmetry must be broken in order to produce an acceptable pattern of masses and mixing

2 M

In the
masses
people
been m
of all th

**The awkward moment when
you realise**

**WTF have I done with
my life**



¡Crisis! ¡Crisis!

Quark sector of S_3 models: Classification and comparison with experimental data

 F. González Canales,^{1,2} A. Mondragón,^{1,*} M. Mondragón,¹ U. J. Saldaña Salazar,¹ and L. Velasco-Sevilla^{1,3}
¹*Instituto de Física, Universidad Nacional Autónoma de México, Apartado Postal 20-364, 01000 México DF, México*
²*Facultad de Ciencias de la Electrónica, Benemérita Universidad Autónoma de Puebla, Apartado Postal 157, 72570 Puebla, Puebla, México*
³*University of Hamburg, II. Institute for Theoretical Physics, Luruper Chaussee 149, 22761 Hamburg, Germany*

(Received 29 April 2013; published 8 November 2013)

S_3 models offer a low energy approach to describe the observed pattern of masses and mixing, of both quarks and leptons. In this work, we first revisit an S_3 model with only one Higgs electroweak doublet, where the flavor symmetry must be broken in order to produce an acceptable pattern of masses and mixing

$$\begin{aligned}
 V_{ud}^{\text{th}} &= \sqrt{\frac{\tilde{\sigma}_c \tilde{\sigma}_s \xi_1^u \xi_1^d}{\mathcal{D}_{1u} \mathcal{D}_{1d}}} + \sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_d}{\mathcal{D}_{1u} \mathcal{D}_{1d}}} \left(\sqrt{(1 - \delta_u)(1 - \delta_d) \xi_1^u \xi_1^d} + \sqrt{\delta_u \delta_d \xi_2^u \xi_2^d} e^{i\phi_2} \right) e^{i\phi_1}, \\
 V_{us}^{\text{th}} &= -\sqrt{\frac{\tilde{\sigma}_c \tilde{\sigma}_d \xi_1^u \xi_2^d}{\mathcal{D}_{1u} \mathcal{D}_{2d}}} + \sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_s}{\mathcal{D}_{1u} \mathcal{D}_{2d}}} \left(\sqrt{(1 - \delta_u)(1 - \delta_d) \xi_1^u \xi_2^d} + \sqrt{\delta_u \delta_d \xi_2^u \xi_1^d} e^{i\phi_2} \right) e^{i\phi_1}, \\
 V_{ub}^{\text{th}} &= \sqrt{\frac{\tilde{\sigma}_c \tilde{\sigma}_d \tilde{\sigma}_s \delta_d \xi_1^u}{\mathcal{D}_{1u} \mathcal{D}_{3d}}} + \sqrt{\frac{\tilde{\sigma}_u}{\mathcal{D}_{1u} \mathcal{D}_{3d}}} \left(\sqrt{(1 - \delta_u)(1 - \delta_d) \delta_d \xi_1^u} - \sqrt{\delta_u \xi_2^u \xi_1^d \xi_2^d} e^{i\phi_2} \right) e^{i\phi_1}, \\
 V_{cd}^{\text{th}} &= -\sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_s \xi_2^u \xi_1^d}{\mathcal{D}_{2u} \mathcal{D}_{1d}}} + \sqrt{\frac{\tilde{\sigma}_c \tilde{\sigma}_d}{\mathcal{D}_{2u} \mathcal{D}_{1d}}} \left(\sqrt{(1 - \delta_u)(1 - \delta_d) \xi_2^u \xi_1^d} + \sqrt{\delta_u \delta_d \xi_1^u \xi_2^d} e^{i\phi_2} \right) e^{i\phi_1},
 \end{aligned}$$



Conociendo a Hitler



Karlsruher Institut für Technologie

KIT-Campus Süd | Karlsruhe House of Young Scientists (KHYS) |
Straße am Forum 3 | 76131 Karlsruhe

Herrn
Wolfgang Hollik
Institut für Theoretische Teilchenphysik
(TTP)
Campus Süd
Karlsruher Institut für Technologie (KIT),
Geb. 30.23
76128 Karlsruhe

Karlsruhe House of Young Scientists (KHYS)

Kollegiale Geschäftsleitung:

Dr. Britta Trautwein
Dr. Gabriele Gregolec

Straße am Forum 3, Geb. 30.96
76131 Karlsruhe

Telefon: 0721 608-42593
Fax: 0721-608-46222
E-Mail: jana.schmitt@kit.edu
Web: www.khys.kit.edu

Jana Schmitt M.A.

15.01.2014

Scholarship Approval for Mr. Saldaña-Salazar

Dear Mr. Hollik, dear Mr. Saldaña-Salazar,

We are pleased to inform you that the Karlsruhe House of Young Scientists (KHYS) will fund the visiting re-
search period of Mr. Saldaña-Salazar at the Karlsruhe Institute of Technology (KIT) for a total of 4 months (the
funding must take place within the period from 01.03.2014 until 31.10.2014).

On the possible relation of fermion masses and mixing

U. J. Saldaña Salazar

June 18, 2014

Abstract

We study the structure of the Yukawa interactions. The mathematical meaning of the masses, and the possible mathematical origin of the mixing matrix reparametrized in terms of the masses. Somehow, with this, we try to address under what conditions we can reparametrize the unitary matrices that diagonalizes the mass matrices with the masses, as their singular values. Then, the latter would mean that mixing angles and even possibly the Dirac CPV phase could be predicted by a simple extension of the SM (at low energies).

under chiral symmetry transformations.

We wish to address now the question: given some structure of the masses what can we say about the mixing matrix or other known phenomena?

On the possible relation of fermion masses and mixing

Mexican grants: PAPIIT-IN113712 and CONACyT-132059



U. J. Saldaña Salazar
IF-UNAM

Warsaw, Poland
PASCOS 2014
June 24



On

- **Strong mass hierarchy**

$$m_u : m_c : m_t \approx 10^{-6} : 10^{-3} : 1,$$

$$m_d : m_s : m_b \approx 10^{-4} : 10^{-2} : 1,$$

$$m_e : m_\mu : m_\tau \approx 10^{-5} : 10^{-2} : 1,$$

mion

- $\text{Rank}(X) = \#\{\sigma_X \neq 0\}$

$$L_X X X^\dagger L_X^\dagger = D_X^2$$

$$R_X X^\dagger X R_X^\dagger = D_X^2$$



under chiral symmetry transformations.

We wish to address now the question: given some structure of the masses what can we say about the mixing matrix or other known phenomena?

On the possible relation of fermion masses and mixing

U. J. Saldaña Salazar

July 12, 2014

Abstract

We study here all the properties, or at least those which more interest us, of complex matrices. Having in mind that we want to apply this knowledge to the understanding of a possible relation between the fermion masses and the mixing angles, that is, between the complex mass matrices and the left singular unitary matrices that help us to diagonalize them.

our mass matrix is now approximately a second rank matrix to order $\sigma_{f,2}/\sigma_{f,3}$. The final expression can be written as,

$$\mathcal{M}_f = \left[\left(l_{f,1} \frac{\sigma_{f,1}}{\sigma_{f,2}} r_{f,1}^\dagger + l_{f,2} r_{f,2}^\dagger \right) \frac{\sigma_{f,2}}{\sigma_{f,3}} + l_{f,3} r_{f,3}^\dagger \right] \sigma_{f,3}, \quad (16)$$

which clearly shows the hierarchy and its relation to the rank of the matrix, and therefore, its structure.

16 de Agosto de 2014

$$= \begin{pmatrix} c_{12}^u c_{12}^d + s_{12}^u s_{12}^d \sqrt{1 - \zeta_0^2} e^{i\delta_{\text{off}}^\infty} & -e^{-i\delta_{12}^d} (c_{12}^u s_{12}^d - s_{12}^u c_{12}^d \sqrt{1 - \zeta_0^2} e^{i\delta_{\text{off}}^\infty}) & s_{12}^u \zeta_0 e^{i(\delta_{12}^u - \delta_{\text{off}}^0)} \\ -e^{i\delta_{12}^u} (s_{12}^u c_{12}^d - c_{12}^u s_{12}^d \sqrt{1 - \zeta_0^2} e^{i\delta_{\text{off}}^\infty}) & c_{12}^u c_{12}^d \sqrt{1 - \zeta_0^2} + s_{12}^u s_{12}^d e^{i\delta_{\text{off}}^\infty} & c_{12}^u \zeta_0 e^{-i\delta_{\text{off}}^0} \\ -s_{12}^d \zeta_0 e^{i\delta_{\text{off}}^0} & -c_{12}^d \zeta_0 e^{i\delta_{\text{off}}^0} & \sqrt{1 - \zeta_0^2} \end{pmatrix} \quad (116)$$

$$= \begin{pmatrix} 0.9745 & 0.2242 & 0.0036 \\ 0.2236 & 0.9716 & 0.0771 \\ 0.0170 & 0.0753 & 0.9970 \end{pmatrix}, \quad (117)$$

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886_{-0.00032}^{+0.00033} & 0.0405_{-0.0012}^{+0.0011} & 0.99914 \pm 0.00005 \end{pmatrix}$$

16 de Agosto de 2014

5.5 Correcting rotations to the heavy limit

The uncertainty in the [23] element is small compare to 1, but not so small compare to the mean value (50%). Nevertheless, the way in which it appears suggests, as these numbers refer to angles, that, we could take it into account as a correcting rotation. That is, in the light mass limit, the corrections coming from neglecting the first family mass, could be taken into account by adding a rotation in the [23] sector by the angle

$$\sin \theta_{13}^f = \sqrt{\frac{m_{f,1}}{m_{f,1} + m_{f,3}}} \quad (132)$$

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

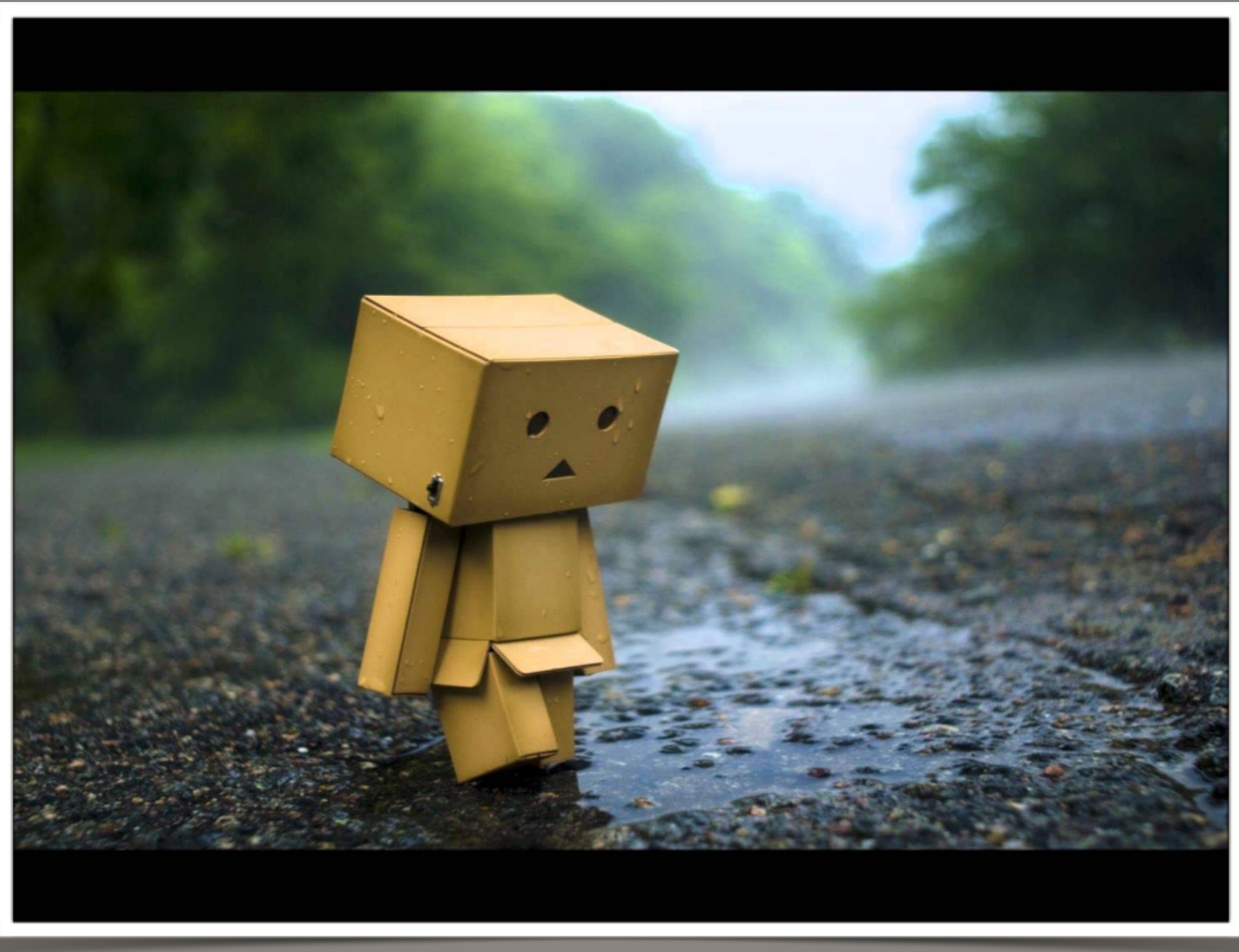
5.5 Correcting rotations to the heavy limit

The uncertainty in the [23] element is small compare to 1, but not so small compare to the mean value (50%). Nevertheless, the way in which it appears suggests, as these numbers refer to angles, that, we could take it into account as a correcting rotation. That is, in the light mass limit, the corrections coming from neglecting the first family mass, could be taken into account by adding a rotation in the [23] sector by the angle

$$\sin \theta_{13}^f = \sqrt{\frac{m_{f,1}}{m_{f,1} + m_{f,3}}} \quad (132)$$

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

$$\begin{aligned} \varsigma_0^- &= 0.0492 \pm 0.0051, \\ \varsigma_0^+ &= 0.0438 \pm 0.0051. \end{aligned}$$



¡Tu trabajo es una basura!

Mass ratios

$$\tan^2 \theta_c \approx \frac{m_d}{m_s}$$

Gatto, Sartori, Tonin (1968),
Cabibbo (1968), Tanaka (1969),
Mohapatra (1977), Weinberg (1977),
Fritzsch (1977), Ramond (1993), Xing (1996),
Rasin (1997), Chkareuli (1998), Mondragón (1998),
Tanimoto (1999), Fritzsch, Xing (1999), King,
Valle, Peinado, Spinrath, Antusch...

$$m_1^a, m_2^a, m_3^a, m_1^b, m_2^b, m_3^b,$$

$$\Rightarrow 2(n - 1)$$

$$\Rightarrow n \leq 3$$

$$V = V\left(\frac{m_1^a}{m_2^a}, \frac{m_2^a}{m_3^a}, \frac{m_1^b}{m_2^b}, \frac{m_2^b}{m_3^b}\right)$$

Mass ratios

$$\tan^2 \theta_c \approx \frac{m_d}{m_s}$$

Gatto, Sartori, Tonin (1968),
Cabibbo (1968), Tanaka (1969),

But, is it even possible?

Tanimoto (1999), Fritzsch, Xing (1999), King,
Valle, Peinado, Spinrath, Antusch...

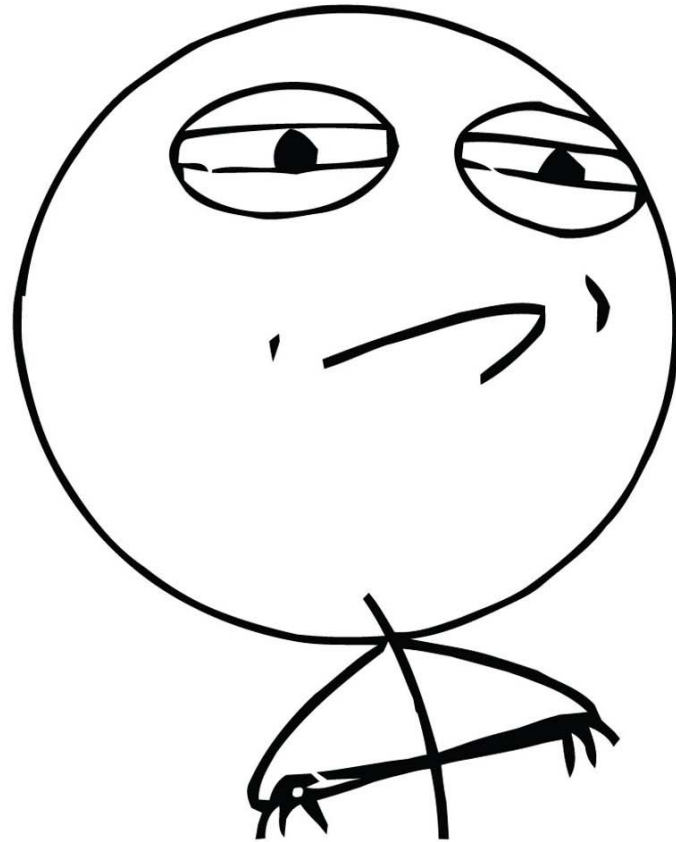
$$m_1^a, m_2^a, m_3^a, m_1^b, m_2^b, m_3^b,$$

$$\Rightarrow 2(n - 1)$$

$$\Rightarrow n \leq 3$$

$$V = V\left(\frac{m_1^a}{m_2^a}, \frac{m_2^a}{m_3^a}, \frac{m_1^b}{m_2^b}, \frac{m_2^b}{m_3^b}\right)$$

Mass ratios



CHALLENGE ACCEPTED

$$V = V \left(\frac{m_1^a}{m_2^a}, \frac{m_2^a}{m_3^a}, \frac{m_1^b}{m_2^b}, \frac{m_2^b}{m_3^b} \right)$$

Complex phases I

$$V = V\left(\frac{m_i}{m_j}, \delta_1, \delta_2, \dots, \delta_k\right)$$

$$\Rightarrow \delta_k = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

(Masina, Savoy)

$$V = L_a L_b^\dagger = V\left(\frac{m_1^a}{m_2^a}, \frac{m_2^a}{m_3^a}, \frac{m_1^b}{m_2^b}, \frac{m_2^b}{m_3^b}\right)$$

$$\Rightarrow L_f = L_f\left(\frac{m_1^f}{m_2^f}, \frac{m_2^f}{m_3^f}\right)$$

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

Complex phases I

$$V = V\left(\frac{m_i}{m_j}, \delta_1, \delta_2, \dots, \delta_k\right)$$

$$\Rightarrow \delta_k = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

(Masina, Savoy)

$$V = L_a L_b^\dagger = V\left(\frac{m_1^a}{m_2^a}, \frac{m_2^a}{m_3^a}, \frac{m_1^b}{m_2^b}, \frac{m_2^b}{m_3^b}\right)$$

$$\Rightarrow L_f = L_f\left(\frac{m_1^f}{m_2^f}, \frac{m_2^f}{m_3^f}\right)$$

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

No exact solution.

Complex phases I

$$V = V\left(\frac{m_i}{m_j}, \delta_1, \delta_2, \dots, \delta_k\right)$$

$$\Rightarrow \delta_k = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

(Masina, Savoy)

$$V = L_a L_b^\dagger = V\left(\frac{m_1^a}{m_2^a}, \frac{m_2^a}{m_3^a}, \frac{m_1^b}{m_2^b}, \frac{m_2^b}{m_3^b}\right)$$

$$\Rightarrow L_f = L_f\left(\frac{m_1^f}{m_2^f}, \frac{m_2^f}{m_3^f}\right)$$

An approximated solution?

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

No exact solution.

Hierarchical masses

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

+

$$m_1 \ll m_2 \ll m_3$$

$$m_t : m_c : m_u = 1 : 10^{-3} : 10^{-5}$$

$$m_b : m_s : m_d = 1 : 10^{-2} : 10^{-4}$$

$$m_\tau : m_\mu : m_e = 1 : 10^{-2} : 10^{-4}$$

$$\Delta m_{31(32)}^2 : \Delta m_{21}^2 = 1 : 10^{-2}$$

Schmidt-Mirsky approximation theorem

(Schmidt, Mirsky, Eckart, Young)

$$\text{rank}[A] = n \quad \sigma_n > \sigma_{n-1} > \cdots > \sigma_2 > \sigma_1 > 0$$

$$s_k = \{\sigma_k, \sigma_{k-1}, \dots, \sigma_1\} \ll \sigma_{k+1}$$

$$\|A - B\|_X \geq \|A - A(s_k = 0)\|_X$$

$$\text{rank}[B] = n - k$$

$$\|A\|_F = \sqrt{\text{Tr}(AA^\dagger)}$$

Hierarchical masses

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$$m_t : m_c : m_u = 1 : 10^{-3} : 10^{-5}$$

$$m_b : m_s : m_d = 1 : 10^{-2} : 10^{-4}$$

$$m_\tau : m_\mu : m_e = 1 : 10^{-2} : 10^{-4}$$

$$m_{1,2} = 0$$

+

$$\begin{array}{l} \text{rank 1} \\ \text{rank 2} \end{array} \quad m_1 \ll m_2 \ll m_3$$

$$\Delta m_{31(32)}^2 : \Delta m_{21}^2 = 1 : 10^{-2}$$

$$m_1 = 0$$

Schmidt-Mirsky approximation theorem

(Schmidt, Mirsky, Eckart, Young)

$$\text{rank}[A] = n \quad \sigma_n > \sigma_{n-1} > \cdots > \sigma_2 > \sigma_1 > 0$$

$$s_k = \{\sigma_k, \sigma_{k-1}, \dots, \sigma_1\} \ll \sigma_{k+1}$$

$$\|A - B\|_X \geq \|A - A(s_k = 0)\|_X$$

$$\text{rank}[B] = n - k$$

$$\|A\|_F = \sqrt{\text{Tr}(AA^\dagger)}$$

Minimal flavor violation (MFV)

$$\mathcal{L}_\psi = \sum_\psi \bar{\psi} i \gamma^\mu \partial_\mu \psi$$

$$\mathcal{L}_\psi = \sum_\psi \bar{\psi} i \gamma^\mu D_\mu \psi$$

$$\mathcal{L}_\psi = \sum_f \bar{\psi}_f (i \gamma^\mu D_\mu^f - \mathcal{M}_f) \psi_f$$

$$U(48) \longrightarrow U(3)^Q \times U(3)^u \times U(3)^d \times U(3)^\ell \times U(3)^e \times U(3)^\nu \longrightarrow U(1)_B \times U(1)_L$$

Rank 0 $U(3)^Q \times U(3)^u \times U(3)^d \times U(3)^\ell \times U(3)^e \times U(3)^\nu$ $m_1, m_2, m_3 = 0$

Rank 1 $U(2)^Q \times U(2)^u \times U(2)^d \times U(2)^\ell \times U(2)^e \times U(2)^\nu$ $m_1, m_2 = 0$

Rank 2 $U(1)^Q \times U(1)^u \times U(1)^d \times U(1)^\ell \times U(1)^e \times U(1)^\nu$ $m_1 = 0$

Rank 3 $U(1)_B \times U(1)_L$

Electroweak basis*

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$$a = u, e$$

$$b = d, \nu$$

$$M_a, M_b \Rightarrow M_f \neq \Sigma_f$$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f M_{f,r=2} M_{f,r=2}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f M_{f,r=1} M_{f,r=1}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Electroweak basis*

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$$a = u, e$$

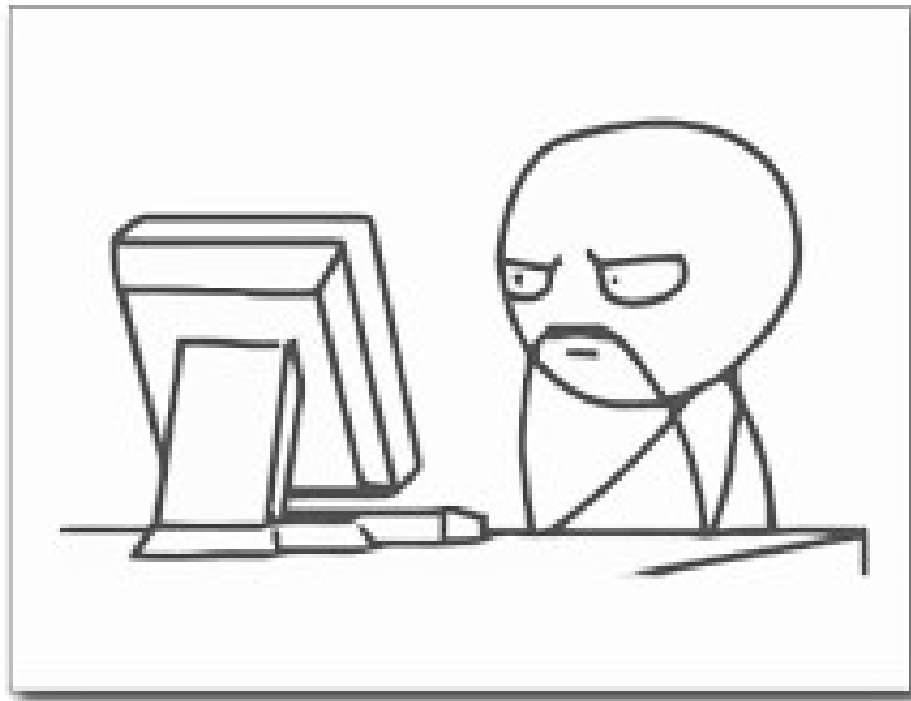
$$b = d, \nu$$

$$M_a, M_b \Rightarrow M_f \neq \Sigma_f$$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f M_{f,r=2} M_{f,r=2}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 \\ 0 & \hat{m}_2^2 \\ 0 & 0 \end{pmatrix}$$

$$L_f M_{f,r=1} M_{f,r=1}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$



Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$$a = u, e$$

$$b = d, \nu$$

$$M_a, M_b \Rightarrow M_f \neq \Sigma_f$$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f M_{f,r=2} M_{f,r=2}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f M_{f,r=1} M_{f,r=1}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$$a = u, e$$

$$b = d, \nu$$

$$M_a, M_b \Rightarrow M_f \neq \Sigma_f$$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f M_{f,r=2} M_{f,r=2}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f M_{f,r=1} M_{f,r=1}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f = L_f(0, 0) = 1_{3 \times 3}$$

Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$$a = u, e$$

$$b = d, \nu$$

$$M_a, M_b \Rightarrow M_f \neq \Sigma_f$$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f M_{f,r=2} M_{f,r=2}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_{f,r=1} M_{f,r=1}^\dagger = m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f = L_f(0, 0) = 1_{3 \times 3}$$

Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$$a = u, e$$

$$b = d, \nu$$

$$M_a, M_b \Rightarrow M_f \neq \Sigma_f$$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f M_{f,r=2} M_{f,r=2}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad L_f \left(0, \frac{m_2}{m_3}\right) = L_{23} \left(\frac{m_2}{m_3}\right)$$

$$M_{f,r=1} M_{f,r=1}^\dagger = m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f = L_f(0, 0) = 1_{3 \times 3}$$

Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$$a = u, e$$

$$b = d, \nu$$

$$M_a, M_b \Rightarrow M_f \neq \Sigma_f$$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_{f,r=2} M_{f,r=2}^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \theta_{23}^2 & \theta_{23} \\ 0 & \theta_{23} & 1 + \theta_{23}^2 \end{pmatrix}$$

$$L_f \left(0, \frac{m_2}{m_3}\right) = L_{23} \left(\frac{m_2}{m_3}\right)$$

$$M_{f,r=1} M_{f,r=1}^\dagger = m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f = L_f(0, 0) = 1_{3 \times 3}$$

Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$$a = u, e$$

$$b = d, \nu$$

$$M_a, M_b \Rightarrow M_f \neq \Sigma_f$$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

?

$$\tan^2 \theta_{23} = \frac{m_2}{m_3}$$

$$M_{f,r=2} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & m_{23} \\ 0 & m_{23} & m_{33} \end{pmatrix}$$

$$L_f \left(0, \frac{m_2}{m_3}\right) = L_{23} \left(\frac{m_2}{m_3}\right)$$

$$|M_{f,r=1}| = m_3 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f = L_f(0, 0) = 1_{3 \times 3}$$

Complex phases II

$$V = L_a L_b^\dagger \rightarrow V_{23} = L_{23}^a L_{23}^b{}^\dagger$$

$$V_{ij} = \sqrt{\frac{\hat{m}_{ij}^a + \hat{m}_{ij}^b - 2\hat{m}_{ij}^a \hat{m}_{ij}^b \cos(\delta_{ij}^a - \delta_{ij}^b)}{(1 + \hat{m}_{ij}^a)(1 + \hat{m}_{ij}^b)}}$$

- Minimal mixing

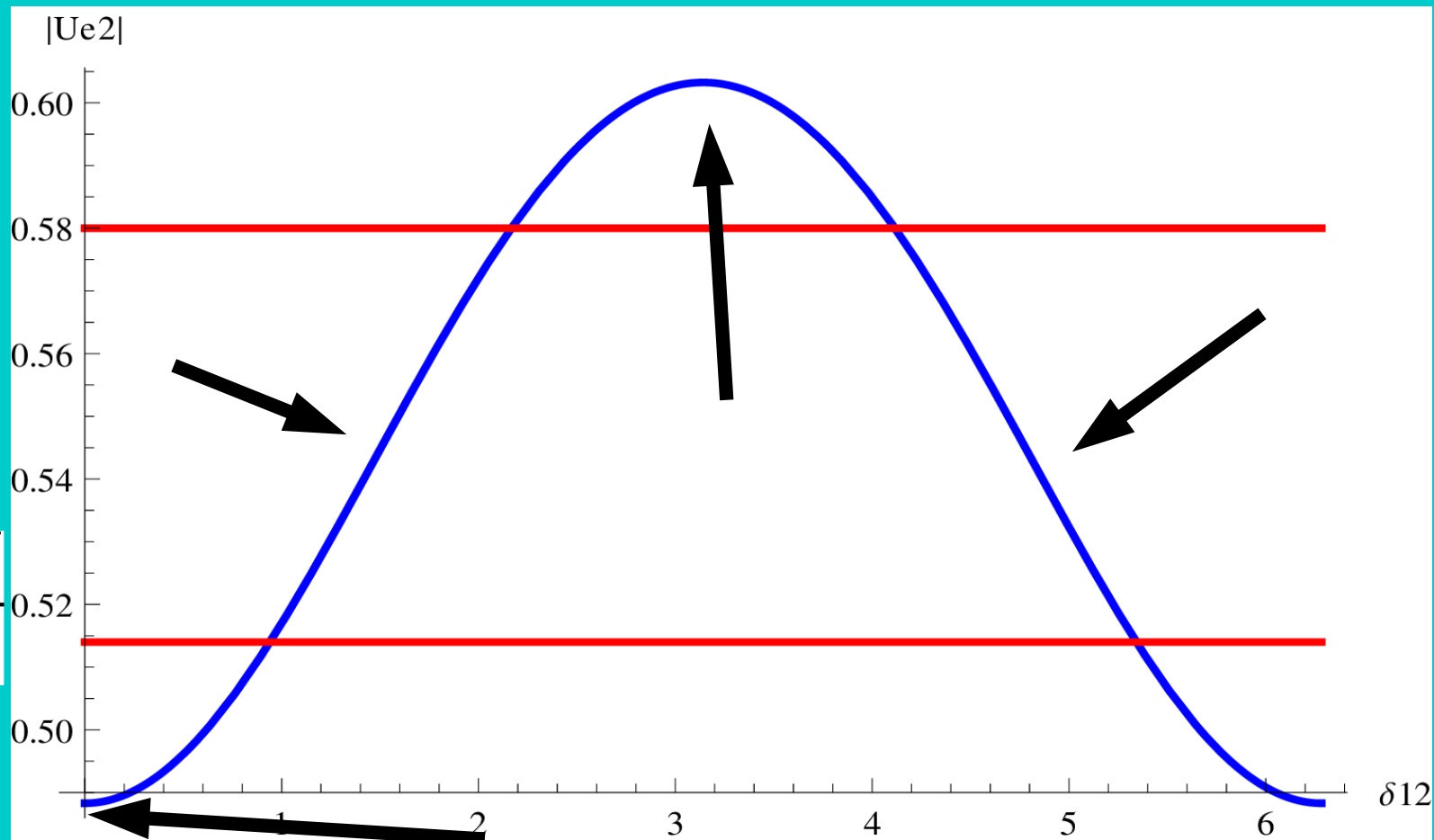
$$\Delta\delta_{ij} = 0$$

- Maximal mixing

$$\Delta\delta_{ij} = \pi$$

- CP Violation

$$\Delta\delta_{ij} = (3) \frac{\pi}{2}$$



Ansatz

$$\mathcal{M}_f = \begin{pmatrix} f_1(m_1) & f_2(m_1) & f_3(m_1) \\ f_4(m_1) & m_2 & 0 \\ f_7(m_1) & 0 & m_3 \end{pmatrix}$$

(Fritzsch, Xing, Chkareuli, Froggatt, Nielsen, Rasin, Hall)

$$\Rightarrow \mathcal{M}_f = \begin{pmatrix} f_1(m_1) & f_2(m_1) & f_3(m_1) \\ f_4(m_1) & m_2 + f_5(m_1) & f_6(m_1) \\ f_7(m_1) & f_8(m_1) & m_3 + f_9(m_1) \end{pmatrix}$$

$$L_{23}^f = L_{23}^{(2)}\left(\frac{m_1 m_2}{m_3^2}\right) L_{23}^{(1)}\left(\frac{m_1}{m_3}\right) L_{23}^{(0)}\left(\frac{m_2}{m_3}\right)$$

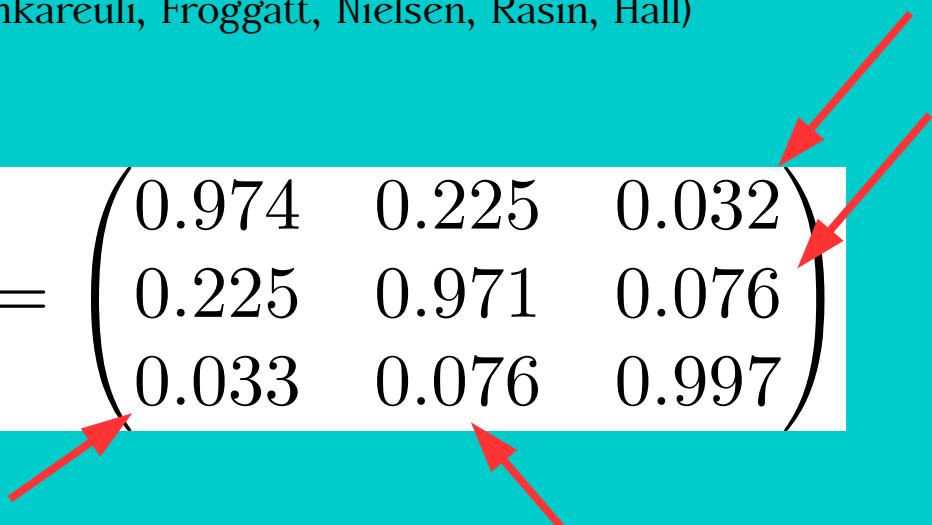
$$L_f = L_{12}(\theta_{12}^f, \pi/2) L_{13}(\theta_{13}^f, 0) L_{23}(\theta_{23}^f, 0)$$



CKM (other authors)

$$\mathcal{M}_f = \begin{pmatrix} f_1(m_1) & f_2(m_1) & f_3(m_1) \\ f_4(m_1) & m_2 & 0 \\ f_7(m_1) & 0 & m_3 \end{pmatrix}$$

(Fritzsch, Xing, Chkareuli, Froggatt, Nielsen, Rasin, Hall)

$$V_{CKM}^{\text{th}} = \begin{pmatrix} 0.974 & 0.225 & 0.032 \\ 0.225 & 0.971 & 0.076 \\ 0.033 & 0.076 & 0.997 \end{pmatrix}$$


$$|V_{CKM}| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

PDG 2014

CKM (ours)

Nucl. Phys. B892 (2015) 364-389

W. G. Hollik & UJSS

$$\Rightarrow \mathcal{M}_f = \begin{pmatrix} f_1(m_1) & f_2(m_1) & f_3(m_1) \\ f_4(m_1) & m_2 + f_5(m_1) & f_6(m_1) \\ f_7(m_1) & f_8(m_1) & m_3 + f_9(m_1) \end{pmatrix}$$

$$L_{23}^f = L_{23}^{(2)}\left(\frac{m_1 m_2}{m_3^2}\right) L_{23}^{(1)}\left(\frac{m_1}{m_3}\right) L_{23}^{(0)}\left(\frac{m_2}{m_3}\right)$$

$$|V_{\text{CKM}}^{\text{th}}| = \begin{pmatrix} 0.974_{-0.003}^{+0.004} & 0.225_{-0.011}^{+0.016} & 0.0031_{-0.0015}^{+0.0018} \\ 0.225_{-0.011}^{+0.016} & 0.974_{-0.003}^{+0.004} & 0.039_{-0.004}^{+0.005} \\ 0.0087_{-0.0008}^{+0.0010} & 0.038_{-0.004}^{+0.004} & 0.9992_{-0.0001}^{+0.0002} \end{pmatrix}$$

$$J_q^{\text{th}} = (2.6_{-1.0}^{+1.3}) \times 10^{-5}$$

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886_{-0.00032}^{+0.00033} & 0.0405_{-0.0012}^{+0.0011} & 0.99914 \pm 0.00005 \end{pmatrix}$$

$$J_q = (3.06_{-0.20}^{+0.21}) \times 10^{-5}$$

PDG 2014

CKM (ours)

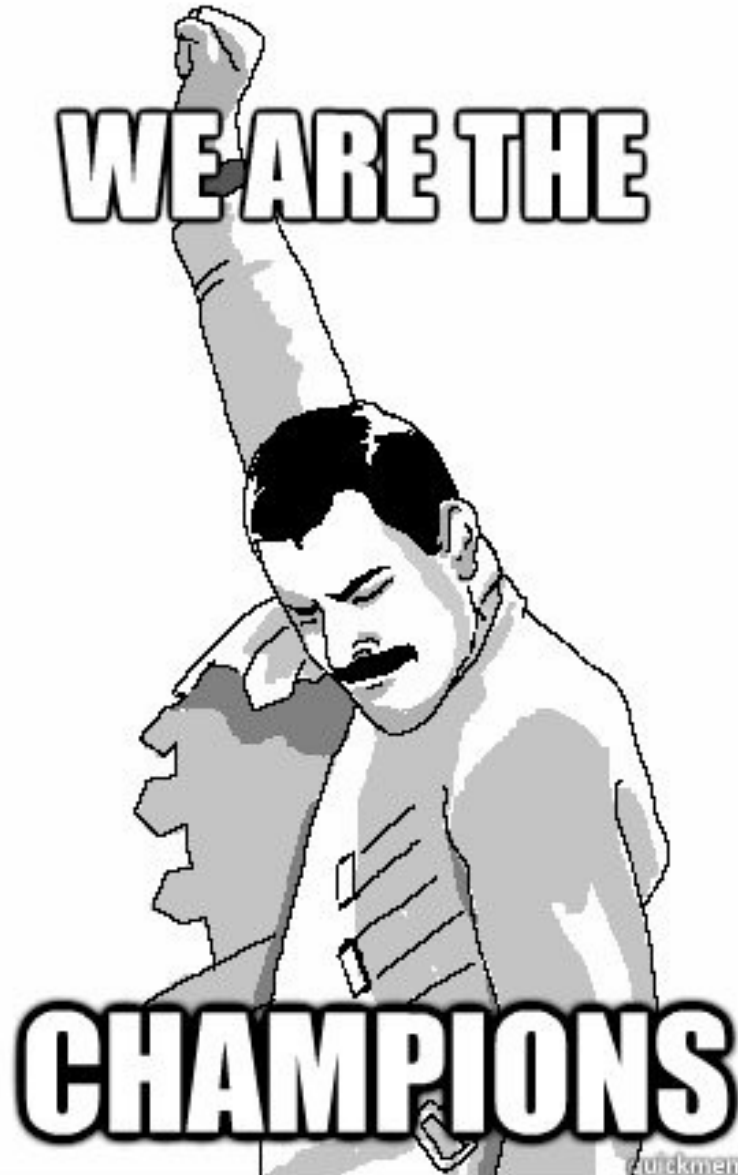
Neel, Phys. Rev. D (2015) 96: 090

$$\Rightarrow \mathcal{M}_f =$$

I

$$|V_{CKM}^{\text{th}}| =$$

$$|V_{CKM}| = \begin{pmatrix} 0.974 \\ 0.225 \\ 0.00 \end{pmatrix}$$



$$J_q = (3.00 - 0.20) \times 10$$

$$\begin{pmatrix} n_1 \\ n_1 \\ g(m_1) \end{pmatrix}$$

$$\begin{pmatrix} +0.0018 \\ -0.0015 \\ +0.005 \\ -0.004 \\ +0.0002 \\ -0.0001 \end{pmatrix}$$

$$\begin{pmatrix} 10355 \pm 0.00015 \\ .0414 \pm 0.0012 \\ 19914 \pm 0.00005 \end{pmatrix}$$

PDG 2014

PMNS (Neutrino masses)

$$m_{\nu 2} = \sqrt{\Delta m_{21}^2 / (1 - \hat{m}_{\nu 12}^2)},$$

$$m_{\nu 1} = \sqrt{m_{\nu 2}^2 - \Delta m_{21}^2},$$

$$m_{\nu 3} = \sqrt{\Delta m_{31}^2 - \Delta m_{21}^2 + m_{\nu 2}^2}.$$

$$|V_{12}^{f=q,\ell}| \approx \sqrt{\frac{\hat{m}_{12}^a + \hat{m}_{12}^b}{(1 + \hat{m}_{12}^a)(1 + \hat{m}_{12}^b)}}$$

$$\frac{m_e}{m_\mu}, |U_{e2}|, \Delta m_{21}^2, \Delta m_{31}^2$$

$$\text{NH: } \Delta m_{31}^2 = +2.457 \pm 0.002 \times 10^{-3} \text{ eV}^2,$$

$$\text{IH: } \Delta m_{32}^2 = -2.448 \pm 0.047 \times 10^{-3} \text{ eV}^2,$$

$$\Delta m_{21}^2 = 7.50_{-0.17}^{+0.19} \times 10^{-5} \text{ eV}^2, \text{ NuFit14}$$

$$m_{\nu 1} = (0.0041 \pm 0.0015) \text{ eV},$$

$$m_{\nu 2} = (0.0096 \pm 0.0005) \text{ eV},$$

$$m_{\nu 3} = (0.050 \pm 0.001) \text{ eV}.$$

$$m_{\nu 3} > m_{\nu 2} > m_{\nu 1}$$

PMNS (our predictions)

$$|U_{PMNS}| = \begin{pmatrix} 0.801 \rightarrow 0.845 & 0.514 \rightarrow 0.580 & 0.137 \rightarrow 0.158 \\ 0.225 \rightarrow 0.517 & 0.441 \rightarrow 0.699 & 0.614 \rightarrow 0.793 \\ 0.246 \rightarrow 0.529 & 0.464 \rightarrow 0.713 & 0.590 \rightarrow 0.776 \end{pmatrix}$$

$$J_\ell = -0.033 \pm 0.010$$

NuFit14

$$|U_{PMNS}^{\text{th}}| = \begin{pmatrix} 0.83_{-0.05}^{+0.04} & 0.54_{-0.09}^{+0.06} & 0.14 \pm 0.03 \\ 0.38_{-0.06}^{+0.04} & 0.57_{-0.04}^{+0.03} & 0.73 \pm 0.02 \\ 0.41_{-0.06}^{+0.04} & 0.61_{-0.04}^{+0.03} & 0.67 \pm 0.02 \end{pmatrix},$$

$$J_\ell = -0.031_{-0.007}^{+0.006}$$

$$m_{\nu 1} = (0.0041 \pm 0.0015) \text{ eV},$$

$$m_{\nu 2} = (0.0096 \pm 0.0005) \text{ eV},$$

$$m_{\nu 3} = (0.050 \pm 0.001) \text{ eV}.$$

$$m_{\nu 3} > m_{\nu 2} > m_{\nu 1}$$

Nucl. Phys. B892 (2015) 364-389
W. G. Hollik & UJSS

PMNS (without fine tuning)

$$\sin^2 \theta_{12}^\ell = 0.323 \pm 0.016, \quad \sin^2 \theta_{23}^\ell = 0.567_{-0.128}^{+0.032}, \quad \sin^2 \theta_{13}^\ell = 0.0234 \pm 0.0020,$$

Forero et al

$$\frac{\delta_{\text{CP}}}{\pi} = 1.34_{-0.38}^{+0.64}$$

$$\sin^2 \theta_{13}^\ell = 0.020 \pm 0.001$$

Daya Bay 2016

$$\sin^2 \theta_{12}^{\ell, \text{th}} = 0.30_{-0.09}^{+0.07},$$

$$\sin^2 \theta_{23}^{\ell, \text{th}} = 0.54 \pm 0.03,$$

$$\sin^2 \theta_{13}^{\ell, \text{th}} = 0.020_{-0.007}^{+0.009},$$

$$\frac{\delta_{\text{CP}}^{\text{th}}}{\pi} = 1.36_{-0.16}^{+0.05}$$

$$m_{\nu 1} = (0.0041 \pm 0.0015) \text{ eV},$$

$$m_{\nu 2} = (0.0096 \pm 0.0005) \text{ eV},$$

$$m_{\nu 3} = (0.050 \pm 0.001) \text{ eV}.$$

$$m_{\nu 3} > m_{\nu 2} > m_{\nu 1}$$

Nucl. Phys. B892 (2015) 364-389
W. G. Hollik & UJSS

PMNS (without fine tuning)

$$\sin^2 \theta_{12}^{\ell} = 0.323 :$$

$$\frac{\delta_{\text{CP}}}{\pi}$$

$$\sin^2 \theta_{12}^{\ell, \text{th}} = 0.3$$

YES WE ARE
SUPERIOR



$$0.0234 \pm 0.0020,$$

Forero et al

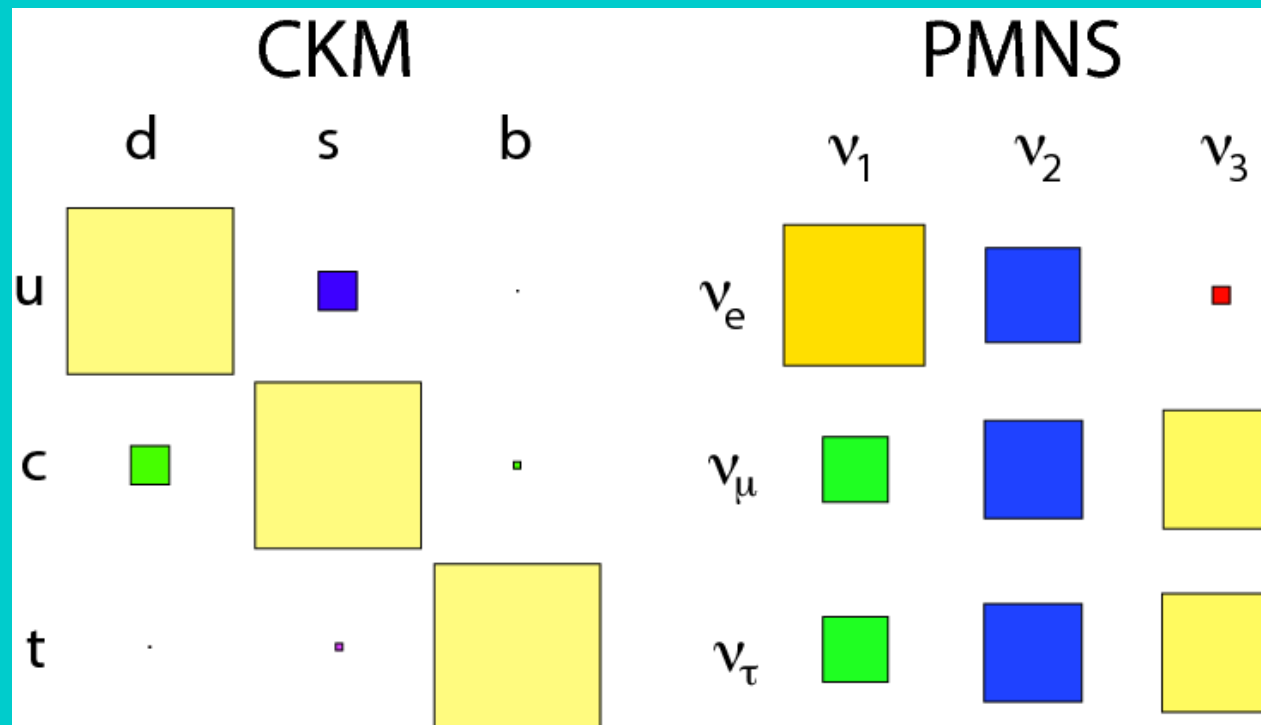
2011
2016

$$= 0.020^{+0.009}_{-0.007},$$

$$m_{\nu 3} > m_{\nu 2} > m_{\nu 1}$$

Nucl. Phys. B892 (2015) 364-389
W. G. Hollik & UJSS

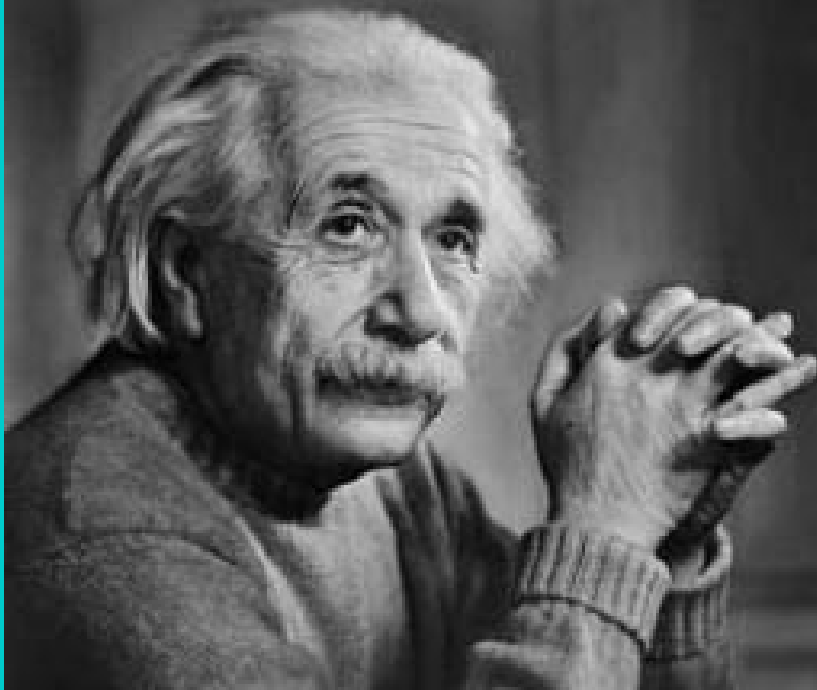
Some insight into the *flavor puzzle*



- Strong hierarchical masses
- Minimal mixing in the 1-3 and 2-3 sectors
- CP Violation in the 1-2 sector
- Weak hierarchy in neutrino masses
- Minimal mixing in the 1-3 sector
- Maximal mixing in the 2-3 sector
- CP Violation in the 1-2 sector

If you can't explain it **simply**, you don't understand it well enough.

– Albert Einstein



Conclusions

- The hierarchy in the masses provides the *simplest* way to study fermion mixing
- We have built a new mixing parametrization using four mass ratios
- The flavor puzzle is understood as a direct consequence of the fermion masses
- For the parametrization it was necessary to use the Schmidt-Mirsky approximation theorem
- Application of this theorem was equivalent to ask Minimal Flavor Violation
- We found an excellent agreement in the quark mixing sector (CKM)
- Application to the lepton sector provided the absolute value of neutrino masses (which gave an excellent agreement to the PMNS matrix) and pointed to which 2-3 octant



¡Cuánto calienta el sol,
aquí en la playa!
(Nueva comprensión)

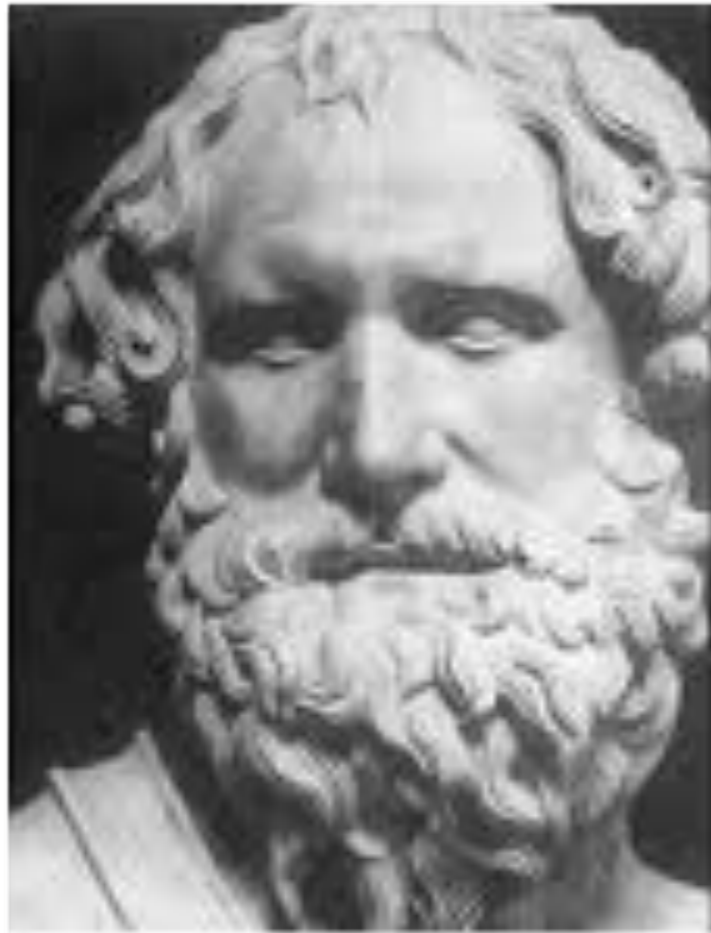


Conclusiones

Conclusiones

- ❖ La actitud de “querer entender” lo vuelve a uno INVESTIGADOR.
- ❖ Enfrentarse a un problema es un proceso:
 - ❖ Conocerlo,
 - ❖ Entender su objetivo
 - ❖ Trabajarlo queriendo entenderlo.
- ❖ Hay que buscar un equilibrio entre la vida académica y la personal.
- ❖ El Modelo Estándar tiene mucho aún por ser entendido, solamente pide que quien lo mire lo haga con nuevos ojos.

¡Gracias por su atención!



Nature (Physics)

Mathematics reveals its secrets only to those who approach it with pure love, for its own beauty.

Archimedes

AZ QUOTES