Iniciación a la QCD

◇ ◇ ◇ Tercera Sesión ◇ ◇ ◇

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XI Escuela de Física Fundamental

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3 lectures: *three quarks for Muster Mark!*

Lecture 1:

- QCD at the LHC
- Gauge invariance and Feynman rules for QCD

Lecture 2:

- Renormalization and running α_s
- pQCD in e^+e^--collisions: from partons to hadrons, jets, shape variables

Lecture 3:

- pQCD in lepton-hadron collisions: DIS and parton evolution
- pQCD at the LHC

$\surd$ Overlap with other lectures is unavoidable.

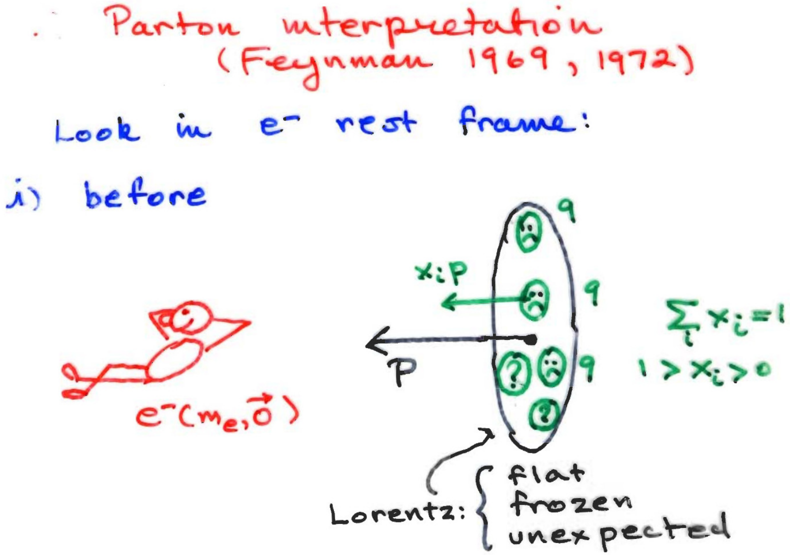
$\surd$ As with other things in life, the U and everything:

overlap is good :)

The parton model

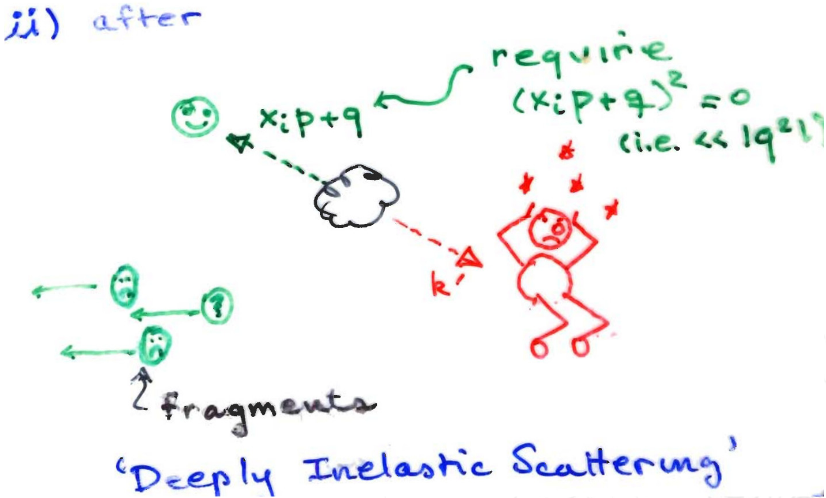
- quarks' binding forces that confine them are due to soft gluon exchange
- a hard virtual gluon exchange would break the proton apart! ($Q^* > Q \longleftrightarrow m_p/Q$ and $\alpha_s \to 0$)
- time scale for qq interaction $> 1/m_p$
- off-shell photon can probe the proton with limited lifetime
- photon can probe incoherent quark: "free" quark
- inner structure of the proton probed with off-shell photon is "universal" (wave function of the proton determined by soft-gluon dynamics)
- so simplest way to probe: deeply inelastic scattering (DIS) (what historically led to idea of partons)

The parton model



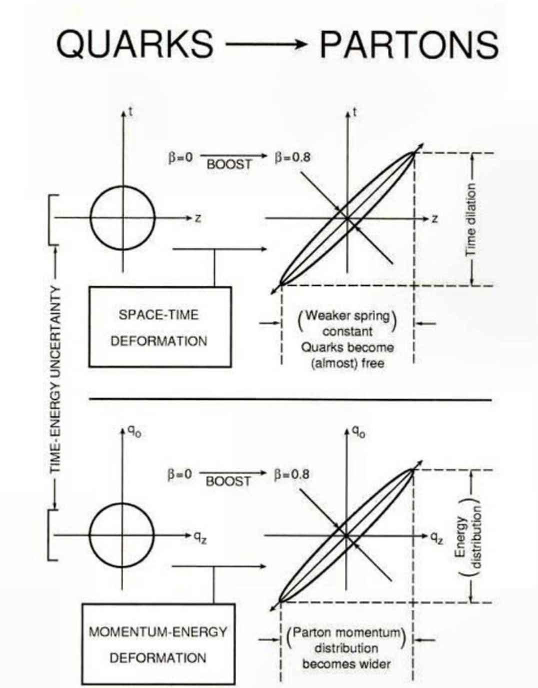
G. Sterman, CTEQ (2004)

The parton model



G. Sterman, CTEQ (2004)

The partor

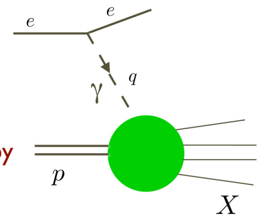

QUARKS → PARTONS
t
β=0
BOOST
β=0.8
z
Time dilation
SPACE-TIME
DEFORMATION
(Weaker spring) constant
Quarks become (almost) free
TIME-ENERGY UNCERTAINTY
q_0
q_z
Energy (distribution)
MOMENTUM-ENERGY
DEFORMATION
(Parton momentum) distribution
becomes wider

Deep Inelastic Scattering (DIS)

Deep inelastic scattering

$$ep \to eX$$

$$Q^2 = -q^2 \qquad x = \frac{Q^2}{2\,p\cdot q}$$



If $Q^2 < M_Z^2$ the cross section is dominated by one-photon exchange

$$k'_0 \frac{d\sigma}{d^3k'} = \frac{1}{k\cdot p}\left(\frac{\alpha}{q^2}\right)^2 L^{\mu\nu} W_{\mu\nu}$$

Leptonic tensor: computable QED

Hadronic tensor

sum over final states

$$L^{\mu\nu} = \frac{1}{4}\mathrm{tr}[\not{k}\gamma^\mu \not{k}'\gamma^\nu] = k^\mu k'^\nu + k'^\mu k^\nu - g^{\mu\nu} k\cdot k'$$

D. de Florian, CLASHEP (2015)

The parton model and DIS

x-sec for intn of the virtual photon with proton at LO:

$$\sigma_0 = \int_0^1 dx \sum_i \; e_i^2 \; f_i(x) \hat{\sigma}_0(\gamma^* q_i \longrightarrow q_i', x) \tag{1}$$

$f_i(x)$ density of quarks of flavour i carrying a fraction x of the proton momentum

$\hat{\sigma}_0$ intn between the photon and a free (massless) quark:

$$\begin{aligned} \hat{\sigma}_0 &= \frac{1}{flux} \overline{\sum} |M_0(\gamma^* q \longrightarrow q')|^2 \frac{d^2 p'}{(2\pi)^3 2p_0'} (2\pi)^4 \delta(p' - q - p) \\ &= \frac{1}{flux} \overline{\sum} |M_0|^2 2\pi \delta(p'^2) \end{aligned} \tag{2}$$

The parton model and DIS

Using $p' = xP + q$, where P is the proton momentum, we get

$$(p')^2 = 2xP \cdot q + q^2 \equiv 2xP \cdot q - Q^2 \tag{3}$$

"infinite momentum frame" $P^\mu \sim (P, 0, 0, P)$ with $P >> M$.

$$\hat{\sigma}_0(\gamma^* q \longrightarrow q') = \frac{2\pi}{flux} \overline{\sum} |M_0|^2 \frac{1}{2P \cdot q} \delta(x - x_{bj}) \tag{4}$$

where $x_{bj} = \frac{Q^2}{2P \cdot q}$ is the so-called Bjorken-x variable. Finally:

$$\sigma_0 = \frac{2\pi}{flux} \frac{\overline{\sum}|M_0|^2}{Q^2} \sum_i x_{bj} e_i^2 f_i(x_{bj}) \equiv \frac{2\pi}{flux} \frac{\overline{\sum}|M_0|^2}{Q^2} F_2(x_{bj}) \tag{5}$$

So, measurement of inclusive *ep* x-sec as function of Q^2 and $P \cdot q$ ($= m_p(E' - E)$ in the proton rest frame) probes the quark momentum distribution inside the proton.

contd.

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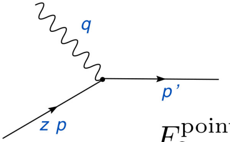
At lowest order

What happens if photon interacts with pointlike particle?



$$(p')^2 = (zp+q)^2 = 2z\ p\cdot q - Q^2 = 0 \quad \rightarrow \quad z = x$$

only couples to quark with mom. fraction *x*!

$$F_2^{\text{pointlike}} \sim e_q^2\, x\, \delta(z-x)$$

no Q : scaling!

- Point-like interaction ➡ scaling (and "direct" access to *x*)

$$F_2(x, \cancel{Q}^2) = \sum_q e_q^2\, x\, f_q(x)$$

- Quarks are fermions ➡ no coupling to longitudinal photons, only transverse polarization (Callan-Gross relation)

$$F_L(x,Q^2) = F_2(x,Q^2) - 2xF_1(x,Q^2) = 0\,!$$

If quarks were scalars F_1=0

contd.

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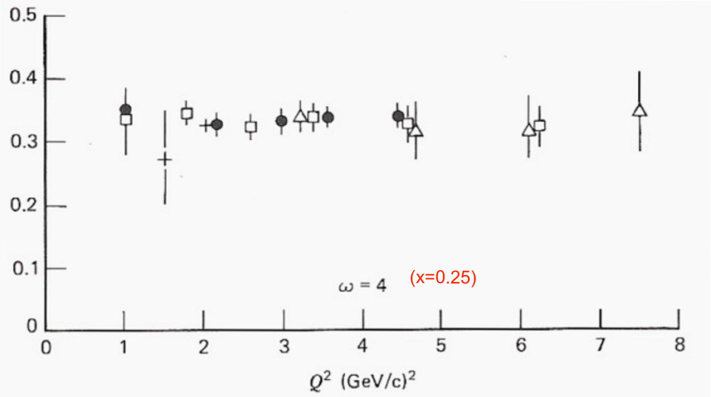
Cross section at lowest order: only F_2

$$\frac{d^2\sigma}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[(1+(1-y)^2)F_2(x) - y^2 F_L(x)\right]$$

Scaling (Bjorken 1968, SLAC data)



contd.

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Proton structure function (with electron scattering) is

$$F_2^{ep}/x = \frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x) + \frac{1}{9}s(x) + \frac{1}{9}\bar{s}(x) + \frac{4}{9}c(x) + \frac{4}{9}\bar{c}(x)$$

Same applies for neutron but with "neutron parton distributions"

Actually, can relate neutron to proton PDFs using isospin symmetry

$$f_{u/n}(x) = f_{d/p}(x) \equiv d(x)$$
$$f_{\bar{u}/n}(x) = f_{\bar{d}/p}(x) \equiv \bar{d}(x)$$
$$f_{d/n}(x) = f_{u/p}(x) \equiv u(x)$$
$$f_{s/n}(x) = f_{s/p}(x) \equiv s(x)$$

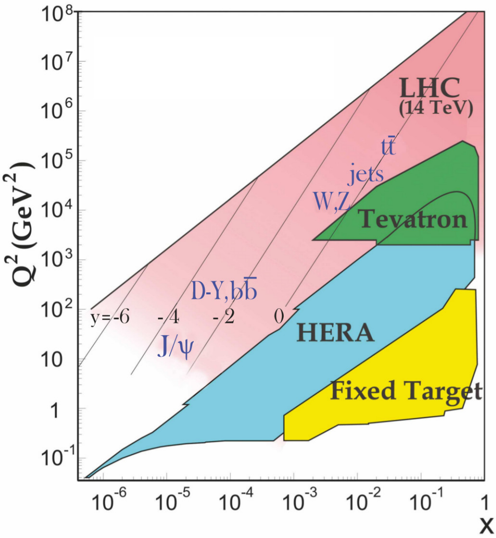
(p ↔ n)

(usually better than % accuracy)

$$F_2^{en}/x = \frac{1}{9}u(x) + \frac{4}{9}d(x) + \frac{1}{9}\bar{u}(x) + \frac{4}{9}\bar{d}(x) + \frac{1}{9}s(x) + \frac{1}{9}\bar{s}(x) + \frac{4}{9}c(x) + \frac{4}{9}\bar{c}(x)$$

In real life one measures deuteron (p+n) structure functions

Parameter space exploration (PDG)



contd.

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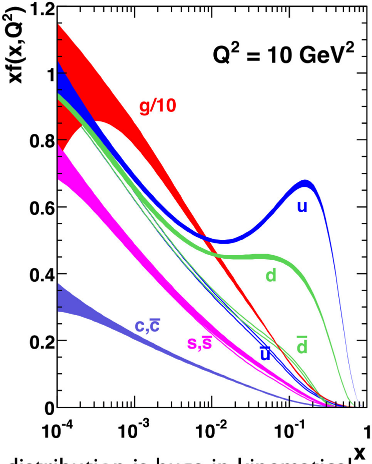
Parton Distributions

How do they look like?



- Vanish when $x \to 1$
- "Quark" peak at $x \sim 1/3$
- Gluon and "sea" rise as $x \to 0$

radiation of soft particles

Notice gluon divided by 10 ! : gluon distribution is huge in kinematical region relevant for LHC

LHC is a "gluon Collider"

contd.

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What does it mean that proton has two up and one down quark?

Valence distributions
$$u_v(x) = u(x) - \bar{u}(x)$$
$$d_v(x) = d(x) - \bar{d}(x)$$

Sum Rules
$$\int_0^1 dx\, u_v(x) = 2$$
$$\int_0^1 dx\, d_v(x) = 1 \qquad \int_0^1 dx\, [u(x) + \bar{u}(x)] = \infty$$
$$s(x) \neq \bar{s}(x) \qquad \int_0^1 dx\, s_v(x) = 0$$

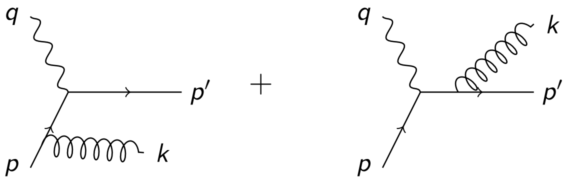
Notice that number of quarks plus antiquarks can be infinity!

Momentum of the proton distributed among components

50%
$$\int_0^1 dx \sum_q [x\, q(x) + x\, \bar{q}(x)] + \int_0^1 dx\, x\, g(x) = 1$$

Parton evolution

Go beyond LO parton-model and add real-emissions
Real-emission corrections to the Born level process:



(6)

The first diagram is proportional to $1/(p-k)^2 = 1/(2pk)$, which diverges when k is emitted parallel to p:

$$p \cdot k = p^0 k^0 (1 - \cos\theta) \longrightarrow 0 \quad \text{when} \quad \cos\theta \longrightarrow 1 \qquad (7)$$

The second diagram is also divergent, if k is emitted parallel to p': harmless! summing over all possible final states for inclusiveness
coll div cancelled in final-state q self-energy corrections

Parton evolution: gauge fixing + parametrization

The amplitude for the only diagram carrying the initial-state singularity is:

$$M_g = ig\lambda^a_{ij}\overline{u}(p')\Gamma\frac{\not{p}-\not{k}}{(p-k)^2}\hat{\epsilon}(k)u(p) \tag{8}$$

Squaring the most singular part of the amplitude, and summing over colours and spins, we get:

$$\sum_g |M_g|^2 = g\overbrace{\sum_a \mathrm{tr}(\lambda^a\lambda^a)}^{N\times C_F}\times\frac{1}{t^2}\times\sum_\epsilon \mathrm{tr}[\not{p}'\Gamma(\not{p}-\not{k})\not{\epsilon}\not{p}\not{\epsilon}^*(\not{p}-\not{k})\Gamma^\dagger] \tag{9}$$

with $t=(p-k)^2=-k_t^2/(1-z)$.

Parton evolution: AP splitting functions

So the one-gluon emission process factorizes in the collinear limit into the Born process times a factor which is independent of the beams nature! If we add the gluon phase-space:

$$[dk] \equiv \frac{d^3k}{(2\pi)^3 2k^0} = \frac{dk_{\parallel}}{k^0}\frac{d\phi}{2\pi}\frac{1}{8\pi^2}\frac{dk_bot^2}{2} = \frac{dz}{1-z}\frac{1}{16\pi^2}dk_{\perp}^2 \quad (10)$$

we get

$$\overline{\sum}|M_g|^2[dk] = \frac{dk_{\perp}^2}{k_{\perp}^2}dz\left(\frac{\alpha_s}{2\pi}\right)P_{qq}(z)\overline{\sum}|M_0|^2 \quad (11)$$

where

$$P_{qq}(z) = C_F\frac{1+z^2}{1-z} \quad (12)$$

is the so-called Altarelli-Parisi splitting function for the $q \longrightarrow q$ transition (z is the momentum fraction of the original quark taken away by the quark after gluon emission).

Parton evolution: correction to parton-model x-sec

Ready to calculate the corrections to the LO parton-model x-sec:

$$\sigma_g = \int dx \; f(x) \frac{1}{flux} \int dz \frac{dk_\perp^2}{k_\perp^2} \left(\frac{\alpha_s}{2\pi}\right) P_{qq}(z) \overline{\sum} |M_0|^2 2\pi \delta(p'^2) \tag{13}$$

Using $(p'^2) = (p - k + q)^2 \sim (zp + q)^2 = (xzP + q)^2$ and

$$\delta(p'^2) = \frac{1}{2P \cdot q} \frac{1}{z} \delta\left(x - \frac{x_{bj}}{z}\right) = \frac{x_{bj}}{z} \delta(\left(x - \frac{x_{bj}}{z}\right) \tag{14}$$

So finally,

$$\sigma_g = \frac{2\pi}{flux} \left(\frac{\overline{\sigma} |M_0|^2}{Q^2}\right) \sum_i e_i^2 x_{bj} \frac{\alpha_s}{2\pi} \int \frac{dk_\perp^2}{k_\perp^2} \int \frac{dz}{z} P_{qq}(z) f_i\left(\frac{x}{z}\right) \tag{15}$$

Parton evolution: parton density and RGE

Inclusion of the $\mathcal{O}(\alpha_s)$ correction is equivalent to a contribution to the parton density:

$$f_i(x) \longrightarrow f_i(x) + \frac{\alpha_s}{2\pi} \int_{\mu_0^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_x^1 \frac{dz}{z} P_{qq}(z) f_i\left(\frac{x}{z}\right) \qquad (16)$$

The renormalized parton density:

$$f(x, Q^2) = f(x, \mu^2) + \log\left(\frac{Q^2}{\mu^2}\right) \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P_{qq}(z) f\left(\frac{x}{z}\right) \qquad (17)$$

Parton evolution: parton density and RGE

RGE condition:

$$\frac{df(x,Q^2)}{d\ln\mu^2} = \mu^2 \frac{df(x,\mu^2)}{d\mu^2} - \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z} P_{qq}(z) f\left(\frac{x}{z}\right) \equiv 0 \quad (18)$$

and then

$$\mu^2 \frac{df(x,\mu^2)}{d\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z} P_{qq}(z) f\left(\frac{x}{z},\mu^2\right) \quad (19)$$

This equation is the
DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) equation
In analogy to $R_{e^+e^-}$ where RGE induces resummation of leading logs, here the DGLAP equation resums full tower of leading logarithms of Q^2.

Parton evolution: parton density evolution

$\mathcal{O}(\alpha_s)$ parton evolution equation for the density of the ith quark flavour:

$$\frac{df_q(x,t)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[P_{qq}(z) f_i\left(\frac{x}{z},t\right) + P_{qg}(z) f_g\left(\frac{x}{z},t\right)\right] \tag{20}$$

$\mathcal{O}(\alpha_s)$ parton evolution equation for the density of gluons:

$$\frac{df_g(x,t)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[P_{gq}(z)\sum_{i=q,\overline{q}} f_i\left(\frac{x}{z},t\right) + P_{gg}(z) f_g\left(\frac{x}{z},t\right)\right] \tag{21}$$

with

$$\begin{aligned} P_{qg} &= \frac{1}{2}\left[z^2 + (1-z)^2\right] \\ P_{gq}(z) &= P_{qq}(1-z) = C_F\frac{1+(1-z)^2}{z} \\ P_{gg}(z) &= 2C_A\left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right] \end{aligned} \tag{22}$$

Parton evolution: valence/singlet densities

Define moments of an arbitrary function $g(x)$ as:

$$g_n = \int_0^1 \frac{dx}{x} x^n g(x) \tag{23}$$

Evol eqns turn into ordinary linear differential equations:

$$\frac{df_i^{(n)}}{dt} = \frac{\alpha_s}{2\pi}[P_{qq}^{(n)} f_i^{(n)} + P_{qg}^{(n)} f_g^{(n)}] \tag{24}$$

$$\frac{df_g^{(n)}}{dt} = \frac{\alpha_s}{2\pi}[P_{gg}^{(n)} f_g^{(n)} + P_{gq}^{(n)} f_i^{(n)}] \tag{25}$$

Take *valence* ($V(x,t)$) and *singlet* ($\Sigma(x,t)$) densities:

$$V(x) = \sum_i f_i(x) - \sum_{\bar{i}} f_{\bar{i}}(x) \tag{26}$$

$$\Sigma(x) = \sum_i f_i(x) + \sum_{\bar{i}} f_{\bar{i}}(x) \tag{27}$$

contd.

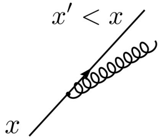
D. de Florian, CLASHEP (2015)

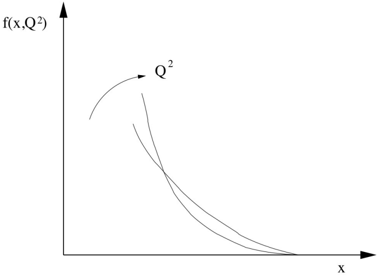
Scaling violations are:

- Positive at small *x* *(more partons with smaller energy)*
- Slightly negative at large *x*

Main effect of increase in Q^2 is shift of partons from larger to smaller *x*

Resolve shorter distances in the proton: quark with fraction x can be resolved as a qg pair (quark with smaller momentum)





contd.

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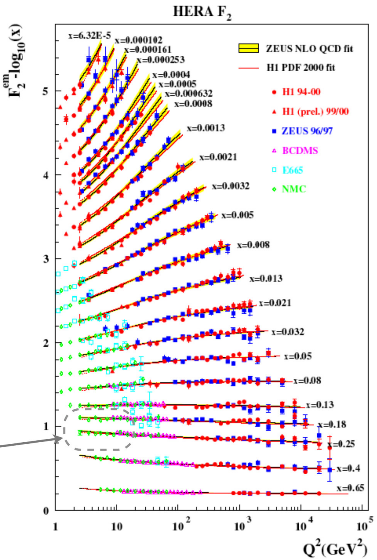
AP Evolution equations allow to predict the Q^2 dependence of DIS data

And very well!

Region studied to find scaling!



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pQCD vocabulary: LO-NLO-NNLO-...

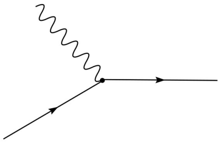
Improved (factorized) Parton Model

$$\sigma(ep \to eX) = \int_0^1 dz \sum_{i=q,\bar{q},g} f_i(z,\mu_F^2)\ \hat{\sigma}^{\rm hard}(ei \to eX)$$

Factorized

LO   Leading Order: Born partonic cross-section
+ LO evolution of pdfs

$$F_2(x,Q^2) = \sum_q e_q^2\, x\, f_q(x,Q^2)$$



contd.

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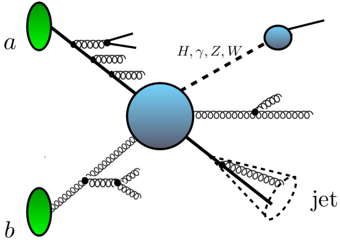
Factorization Formula

non-perturbative parton distributions

$$d\sigma = \sum_{ab} \int dx_a \int dx_b \, f_a(x_a, \mu_F^2) f_b(x_b, \mu_F^2) \times d\hat{\sigma}_{ab}(x_a, x_b, Q^2, \alpha_s(\mu_R^2)) + \mathcal{O}\left(\left(\frac{\Lambda}{Q}\right)^m\right)$$

perturbative partonic cross-section



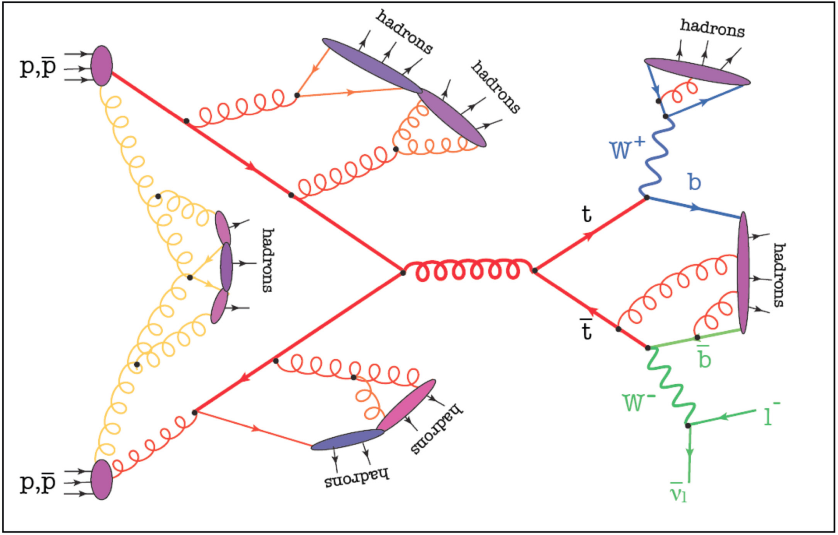
Partonic cross-section:
expansion in $\alpha_s(\mu_R^2) \ll 1$

$$d\hat{\sigma} = \alpha_s^n \, d\hat{\sigma}^{(0)} + \alpha_s^{n+1} \, d\hat{\sigma}^{(1)} + \ldots$$

Expression relies on factorization theorem : HT, mass corrections, etc. not trivial

Need precision for both perturbative and non-perturbative components!

QCD at the LHC



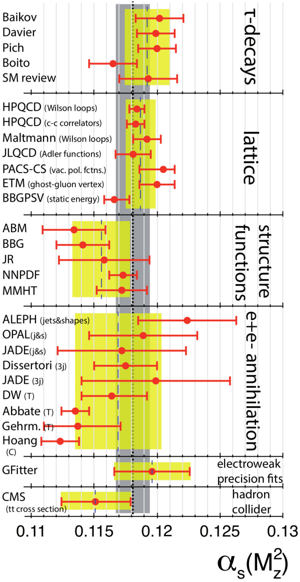
Drawing by K. Hamilton

Juan Rojo La Thuile, 25/02/2014

Why do we keep QCDing?

Physics@Colliders
cannot be done with quantitative seriousness
without pQCD beyond LO
but also
QCD is at the heart of everything!

QCD reviews in PDG



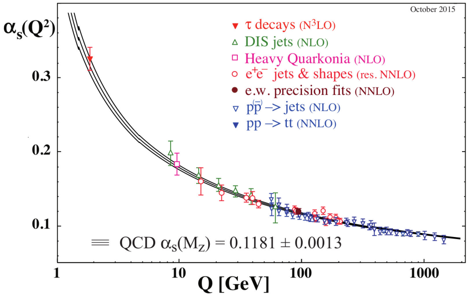
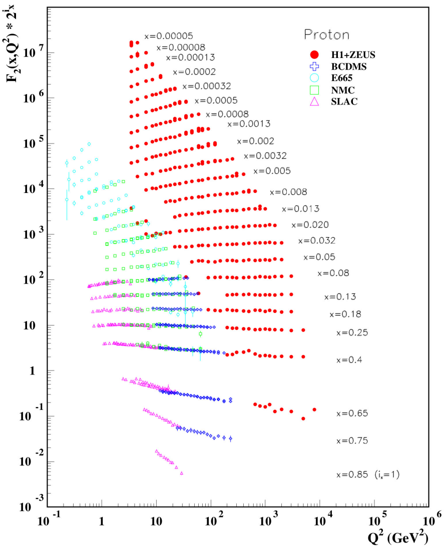


Figure 9.3: Summary of measurements of α_s as a function of the energy scale Q. The respective degree of QCD perturbation theory used in the extraction of α_s is indicated in brackets (NLO: next-to-leading order; NNLO: next-to-next-to leading order; res. NNLO: NNLO matched with resummed next-to-leading logs; N^{3}LO: next-to-NNLO).

QCD reviews in PDG



Higgs at hadron colliders

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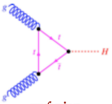
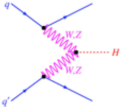
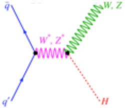


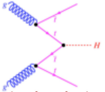
gg fusion



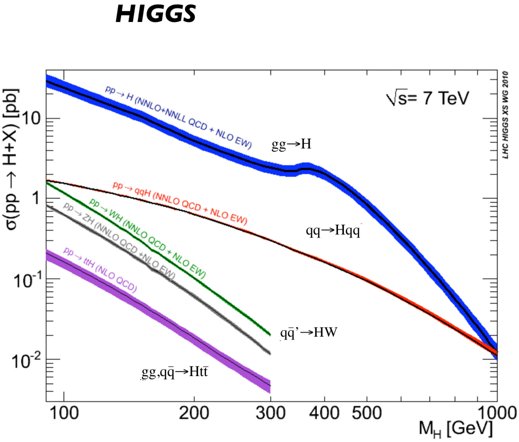
vector boson fusion



associated production with W,Z



associated production with heavy quarks



- Gluon-gluon fusion dominates due to large gluon luminosity
- Similar at other energies

Invitación

9TH CERN LATIN-AMERICAN SCHOOL OF HIGH-ENERGY PHYSICS

San Juan del Rio, Mexico, 8–21 March 2017

Deadline for applications: 11 November 2016
http://cern.ch/PhysicSchool/CLASHEP

Invitación

Scientific Programme

Heavy-Ion Physics
A. Ayala, UNAM, *Mexico*

Higgs Physics
L. Da Rold, CAB, CONICET/IB, *Argentina*

Field Theory and the E-W Standard Model
C. Garcia-Canal, UNLP, *Argentina*

Special lecture on gravitational waves
G. Gonzalez, Louisiana State U., *USA*

Probability and Statistics
C. Maña, CIEMAT, *Spain*

QCD
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