Climatology of the instantaneous phase of Hilbert transform in intra-seasonal oscillations

Alvarez-Gasca, O.[1], Contreras-Hernández, A. D.[1] y Morales-Acoltzi, T.[2,3]

Facultad de Instrumentación Electrónica, Carrera de Ciencias Atmosféricas, Universidad Veracruzana, Jalapa, Veracruz
[2] Meteorología Tropical, Centro de Ciencias de la Atmósfera, **UNAM**, Ciudad Universitaria, México, D. F. 04510
[3] Facultad de Biología, Universidad Autónoma de Tlaxcala, Campus Ixtacuixtla (profesor invitado).
alvarezgasca@hotmail.com acoltzi@atmosfera.unam.mx

RESUMEN

Se presenta una revisión de los conceptos fundamentales de la climatología de la precipitación, a partir del enfoque estadístico tradicional y a través la fase instantánea de la transformada de Hilbert (FIH) en series de tiempo de lluvia. Los conceptos de anomalía, variabilidad, estacionalidad y estacionariedad son integrados a un enfoque matemático de variable compleja. La construcción de climatologías sustitutas y de la climatología dual, se apoyan en la nueva climatología de las fases. Esto a través del análisis del ciclo anual de la precipitación en una estación climatológica del norte de Veracruz (México). Finalmente, se explora el comportamiento casi-lineal de la FIH en una trayectoria de 103 km sobre el sureste de México, hallándose la máxima trasgresión a la linealidad sobre el valle de Oaxaca.

ABSTRACT

Here we present a review of the fundamental concept of the climatology of precipitation, taking a traditional statistical approach and by applying the instantaneous phase of Hilbert transform (IPH) to the time series representing rainy weather. The concepts of anomaly, variability, seasonality and steadiness are integrated into a mathematical approach to the complex variable. The construction of surrogate and dual climatology is upheld by the new climatology of phases. This is created through the analysis of the annual cycle of precipitation, in a climatologic station in Northern Veracruz (Mexico). Finally, the quasilinear behavior of the IPH along a trajectory of 103 km, crossing through southeastern Mexico was explored, revealing greatest linear transgression in the Oaxaca Valley.

Keywords: Rainfall climatology, intra-seasonal variability, Hilbert phase analysis

1. Introduction

Rain is a highly complex hydrometeor (Ogallo, 1979; Nicholls and Wong, 1990). This complexity was found, concerning both spatial (Haslet, 1989) and temporal (Krepper et al., 1989) variations. This is caused by irregularity in variations (Kumerow et al., 2004; L'Ecuyer et al., 2004), gradients between events of close proximity (Varma and Liu, 2006) and a lack of homogeneity with respect to time (Hosking and Stow, 1987).

Standards for the observation of precipitation (WMO, 2003) establish that at a given point on the surface (longitude =λ, latitude=φ, altitude = ξ) rain can be measured as the liquid water which accumulates in a unitary area (Horton, 1919). This quantification is known as accumulated rain (mm). Another measurement frequently employed is that of the intensity of precipitation per time unit; termed as the precipitation rate (mm/hr). In both cases it is considered that the rain gauge registers the amount of rain by scalar magnitude.

Rain is a nonnegative variable (L>0 or L = 0) and often considered as a function of time (mathematically represented as L = L (t)). The statistical concept of climatological mean value (Bilham, 1917) is aptly and widely used. It is applied as a reference for average rainy conditions. This average is calculated, employing a sufficiently long time interval, so as to guarantee that possible fortuitous events are statistically submerged (Willmott et al., 1996), in favor of a statistical theory referring to normal distribution (Carleton, 1999).

In order to discover the rainfall distribution over a season (Kincer, 1919), typically consisting of three months, one requires averages of L, referring to a great number of precipitation events (depressions, storms, tropical storms, cold fronts, tropical waves), which last for less than a month. If we choose to analyze the time series of accumulated rain over a year (four seasons) this would require averaging a great number of meteorological events: convective systems, instability associated with **ITCZ** oscillation and contingency systems of high and low pressure (Ali et al., 2003; Baldwin et al., 2005). These meteorological events typically last for less time than a season.

The traditional concept of rain climatology is statistical and relative to the "time of life" of the precipitation system (Epstein and Barnston, 1990). This concept is common in systematic applications of climatology for other variables (Von Storch and Zwiers, 1999). The construction of various climatologies $\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_m,$ associated with specific meteorological phenomena is a function of the sampling intervals in space and time (Hulme and New, 1997).

The construction of an objective climatology analysis, based on the complexity of precipitation has followed logical progressive formats. Following this approach, the next stage is to tackle the problem of climatic variability. Concerning this aspect of climate, it is necessary to have precise knowledge of tendencies (space, temporal or space-temporal), located within a time domain, this generally be oriented according to the extent of the available data base (Daley, 1999), although not necessarily delimited by this factor (Hartmann and Gross, 1988).

A simple approach for examining rainfall tendencies at any time, consists of calculating anomalies and then giving these statistical or even physical meaning. Continuing within our frame of reference, located at a given point on the surface we find that anomaly in precipitation (Bärring and Hulme, 1991) is a straight forward calculation, represented as $L - < L >_T$. This means that rain accumulated over a period T, minus the average rainfall over this time, calculated by using a great number of observations over the same time period can be represented as $< L >_T$. The anomaly with respect to this time period is represented as $L_{anom} = L - < L >_T$.

In order to include the full scope of climatic variability, it is often necessary to employ other objective parameters, such as recurrence. We understand recurrence (Michelangeli et al., 1995) as a property of the meteorological system, associated with greater probability. This density of probability is defined using a given set of planetary waves. Recurrence permits us to predict the probability that an anomaly with previously calculated dimensions will occur in the future.

While attempting to resolve the problem of calculating the probability of recurrence (Nicolis, 1998), we noticed that systems associated with precipitation manifest a constant state of continuous progression. This progression has existed since the earth's remote past (Jiang and Zhang, 2006), and consists of wet and dry periods (Shin et al., 2006). Uncertainty prevails concerning the moment when progressive rain systems formed the atmosphere as we know it today (Kuiper, 1952).

Lack of definition, concerning the limits of the temporary domain, wherein a process is developed is a characteristic factor in many scientific areas (Robert et al., 1996). This restricts methods for controlling process and requires analytical and experimental methods which vary from the standards of "laboratory sciences", such as physics, chemistry and biology; where under controlled initial conditions, the processes reproduce themselves voluntarily (Terenziani and Torasso, 1995).

2.0 Mathematical and heuristic model

In mathematics, the lack of definition concerning the limits of a time domain, concerning a function leads us to define elementary and transcendental functions as periodic, considering the Cartesian plane XY as an extended and "richer" plane (Schiffer and Bowden, 1984), endowed with a point which represents the infinite (Needham, 1999) and which is projected on Riemann's sphere (Freitag and Busam, 2005). Thus this offers a more apt mathematical representation of the rainfall variable.

The development of a consistent mathematical model (Fowler, 2004) for analyzing precipitation requires the following hypotheses:

a) The time series for rainfall $S_m = \{L_1, L_2, \ldots, L_m\}$ contains an implicit representation of the evolution of a phase during a periodic process, essential for the purpose of statistical seasonal comprehension.

b) Precipitation data from site $X = (\lambda, \varphi, \xi)$ contains time-dependent intrinsic mathematical properties.

c) The dataset S_m contains information about the recurrence of dry and humid episodes, within the intra-seasonal cycle.

Introducing rainfall, not as a real variable but as a complex variable $\mathcal{L} = \mathcal{L}_o\, e^{i\phi(t)}$, where amplitude is represented as $\mathcal{L}_o = |\mathcal{L}| = |\mathcal{L}_o\, e^{i\phi(t)}| = |\mathcal{L}_o| \cdot |e^{i\phi(t)}|$, $i^2 = -1$ and $\phi(t)$ represents the complex functional time phase (Wikle, 2002), which has time "t" as a real independent variable.

The functional form $\phi(t)$ is also the determining factor for describing two fundamental properties of temporal variability: seasonality and steadiness. The concept of seasonality is now precise (Pezzulli et al., 2005) but in spite of this has been treated qualitatively (Finkelstein and Truppi, 1991). The dominant quality during each season (spring, summer, autumn, winter) is relative with respect to specific variables.

Seasonality in the variable L implies coherence (Haylock and McBride, 2001; Davis et al., 2003) concerning the progression of data. Taking the observations from a location in a Central or Southeastern region from Mexico as an example, in spring we will have a period defined by intermittent light rain, in summer intense rain with possible interruptions (Mendoza et al., 2006), in autumn a tendency towards increased rainfall interruptions and in winter, not very frequent rain (Kousky and Srivatsangam, 1983). This coherence can be "lost" when climatic fluctuations or anomalies occur. Assuming that seasonality behaves as a conserving principle, it might be expected that climatic conditions would be shown to stabilize during a time τ consisting of less than three months.

Steadiness is a well-known property within other areas of the knowledge, linked to the dynamics of processes. Steadiness is a quality of stationary processes. Similarly, as in the case of seasonality, steadiness is now a precise term (Jameson and Kostinski, 2002) whereby it is possible to establish fundamental principles. A precipitation process is stationary when the progression of rainfall proceeds at a constant rate. Because of factors associated with the distortion of the climate and interactions with processes of varying scales, it is possible that the rate and the coherence of rain events, with respect to time may be lost.

A precipitation process is deemed more stationary, the lesser the number of transgressions of the stationary state, measured over a long period (with statistical significance). Thus for example, a perfect stationary process (PSTP), would imply a regime free of fluctuations and anomalies, with an undefined recurrence and infinite seasonality (since $\tau = 0$). In a perfect seasonal process (PSEP) fluctuations and anomalies would evidently occur, but, with the limitation that the duration of the anomalies should not exceed certain time frames. Terming as T_α the time that a summer anomaly might persist, would strictly fulfill the inequality $T_\alpha < T_{verano}^{max}$. The limit value T_{verano}^{max} would be calculated as a function of the characteristic spectral coherence, associated with the site under observance and during a given season. Unlike the PSTP, the recurrence of a PSEP of rain can be defined. Finally, a PSEP might contain only a certain number of episodes of steadiness.

3.0 "Instantaneous phase of Hilbert transform"

We can formalize statistical knowledge referring to precipitation, by means of a complex analytical process (Feldman, 2001) known as the "instantaneous phase of Hilbert transform" (FIH). FIH is an integral transformation which permits elucidating the behavior implied by the functional form $\phi(t)$. In order to arrive at the formal expression in a heuristic way, we initiate by using a linear elementary case: $\phi(t) = \omega t + \delta$, with ω and δ representing real constants and shown in the complex variable as:

$$\mathcal{L}(t) = L_o\left(Cos(\omega t+\delta)+i\,Sin(\omega t+\delta)\right) \quad (1.1)$$

Taking the real elements, we derive L(t) = *Im* {$\mathcal{L}$ (t)} = L_o Sin (ωt + δ) and then introducing an arbitrary additive constant c, we get L(t) = L_o Sin (ωt + δ) + c, where an arbitrary constant has been consistently introduced in the model (Fowler, 2004). The instantaneous frequency associated with L(t) = L_o Sin (ωt + δ) + 1 coincides with the time derivative of the phase ϕ'(t)= (ωt + δ)'= ω. This means that the phase remains constant and corresponds to a PSTP. In order to obtain an adequate mathematical representation of fluctuations and anomalies, it is possible to progress from a PSTP to a PSEP, by adding together the harmonic terms of a set of frequencies $\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$, thus arriving at:

$$L_n(t) = c_k + \sum_{k=1}^{n} L_k\, e^{i\phi_k} \tag{1.2}$$

This represents a new way of expressing the rainfall variable. The parameter *n* can be taken as arbitrarily large, representing the limit, so that it will eventually result in the optimal adjustment for a series, as is the case in harmonic analysis (James Arthur, 2006). That is:

$$L_\infty(t) = \lim_{n\to\infty} L_n(t) \tag{1.3}$$

This representation of rain in time accedes to a more generalized definition of this phase (Goswami and Hoefel, 2004), which is termed the "instantaneous phase of Hilbert transform":

$$\phi(t) = \tan^{-1} \frac{H[L_\infty(t)]}{L_\infty(t)} \tag{1.4}$$

Where the expression

$$H[L_\infty(t)] = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{L_\infty(s)}{t-s}\, ds \tag{1.5}$$

represents the Hilbert transform for rain and P represents Cauchy's main value. The instantaneous frequency is also well-known as the modulation frequency:

$$\nu(t) = \frac{1}{2\pi} \frac{d\phi(t)}{dt} \tag{1.6}$$

4.0 Surrogate climatology

Let us consider the analysis of the mean monthly accumulated rain at the *Joloapan* weather station as a concrete case (λ = 20º 14' N, φ = 97º 16' W, ξ= 125). In Fig. 1.0 the rain histogram can be viewed, as well as the well-known midsummer deficit (MSD) known in Spanish as the "canícula". The behavior phase of this time series (Fig. 2.0) is harmonious for almost all the year, with the exception of summer. Calculations using the numerical algorithm for FIH (Vandenhouten and Grebe, 1996) specifically show that a non-linear intra-seasonal oscillation occurs during August.

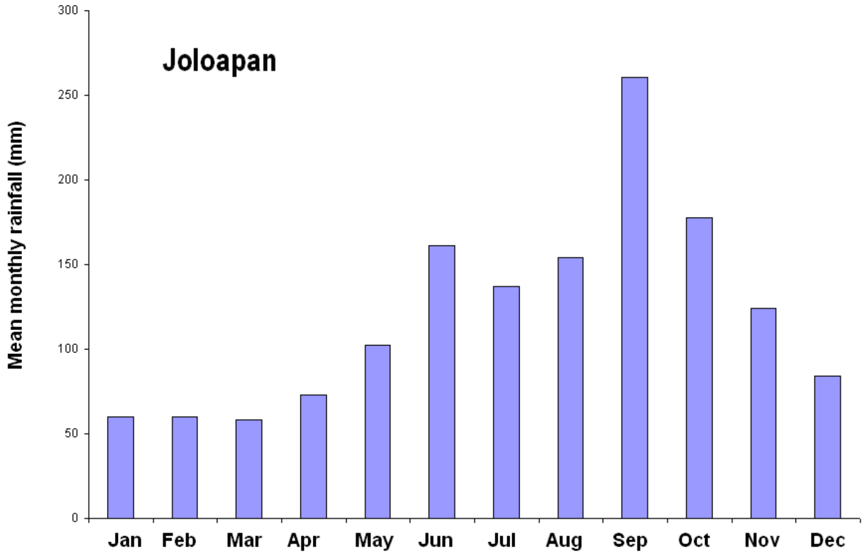

Fig. 1 Mean monthly rainfall climatology for *Joloapan* (33 years average). The deficit during July and August is a well know minimum, associated with the midsummer decline (MSD). Source: Mexican Weather Service.

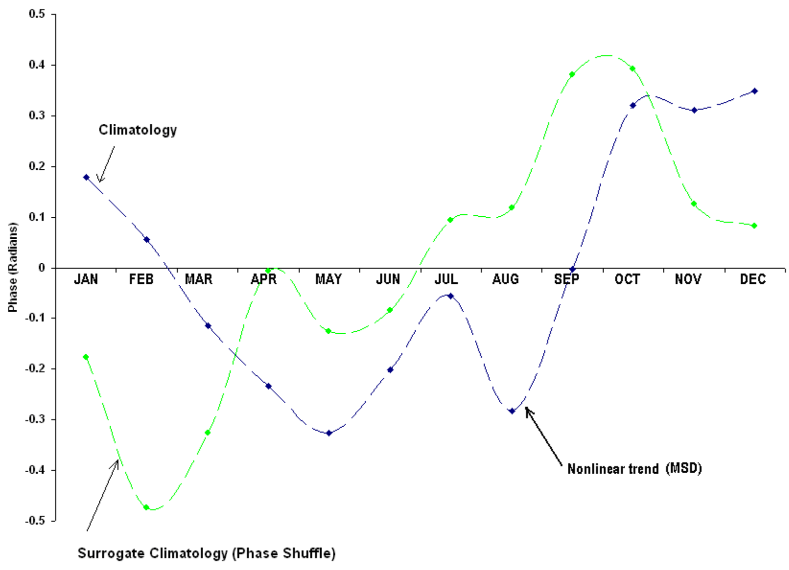

Fig. 2 Mean monthly phase associated with the rainfall climatology for *Joloapan*. The seasonality is modulated by a nonlinear trend during August. A surrogate climatology constructed by a phase shuffling algorithm results in quasilinear behavior.

More extensive numerical experiments using the FIH (Vandenhouten and Grebe, 1995) permitted the generation of a large number ($M \cong 5x10^8$) of sets $\Phi_1,\ldots,\Phi_M$. Each set Φ_k contains twelve elements ϕ_{Jan}, $\phi_{Feb},\ldots,\phi_{Dec}$ which represent the monthly average phases. The expedient election of the set Φ_k leads to the construction of a surrogate climatology $\mathscr{C}_{surrogate}$. The surrogate climatology refers to any set $\mathscr{L}_k$, made up of normal monthly rain averages L_{Jan}, L_{Feb}...,L_{Dec}, corresponding to the set Φ_k. The election of a sequence of phases may follow certain random rational and standard principles. Let us apply the term surrogate dual climatology ($\mathscr{C}_{Dual}$) to a climatology which transforms a given phase, into a linear formation within the time series. The intention of this dual climatology is to practically suppress the effect of the MSD as it manifests itself statistically; as this represents an intrinsic state. When comparing this with a traditional histogram of rain (Figure 3.0) we note that it does not appear to be represented.

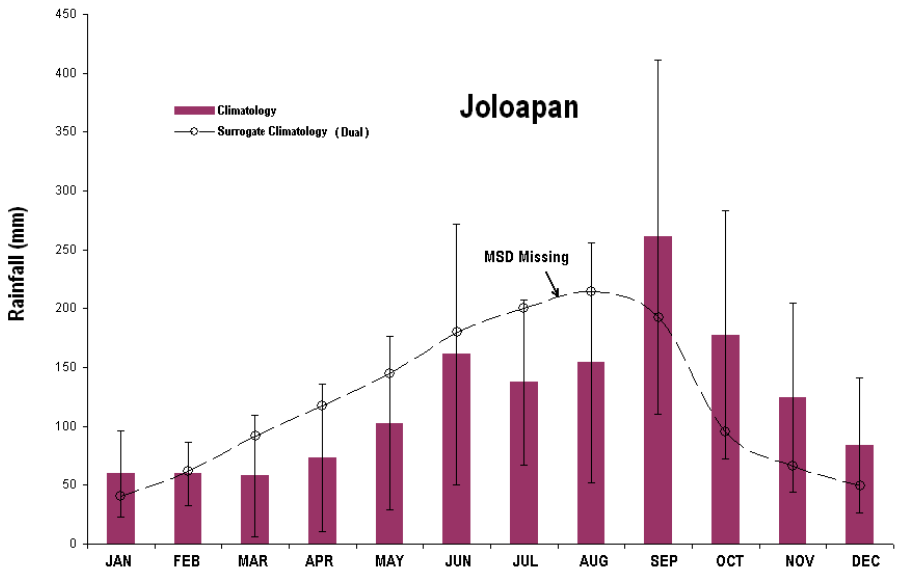

Fig. 3 Standard climatology of precipitation (bars) and dual climatology (broken line) for *Joloapan*. The statistical signature of MSD is suppressed. Dual climatology possesses the same descriptive values (mean and variance) as standard climatology. Vertical segments represent the radius of variance for standard climatology.

In figure 4.0 the normal mean monthly precipitation levels are compared with a trial for surrogate climatology $\mathscr{C}_{Trial}$ .This alternative climatology is similar to the rainfall data observed during the years 1985 and 1997 in *Joloapan*. The summer intra-seasonal oscillations of those years appear to have reached a quasilinear phase, very similar to that of alternative climatology.

An interesting case is the evolution of the FIH in a set of five stations in Southeast Mexico, located in the states of Veracruz and Oaxaca (Figure 5.0). The selection of the stations was made on the basis of their geographic location, with the aim of understanding the variability in oscillations representing a phase in summer rainfall which is induced by orographic forcing.

The stations indicate strong changes in precipitation during summer (Figure 6.0). However, the behavior of this phase is quasilinear progressive (Vandenhouten et al., 2000), reaching its maximum within the Valley of Oaxaca (Figure 7.0).

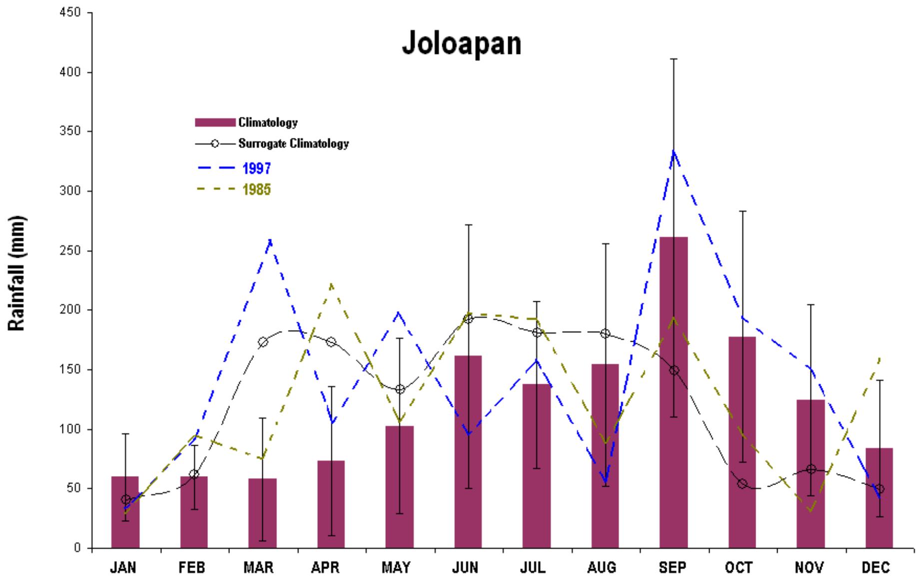

Fig. 4 Standard climatology of precipitation (bars) as well as surrogate climatology (smooth and broken line) for *Joloapan*. Mean monthly historical records are shown only for comparative purposes. During the summer of 1985 and 1997 the rainfall deficit associated with MSD did not occur.

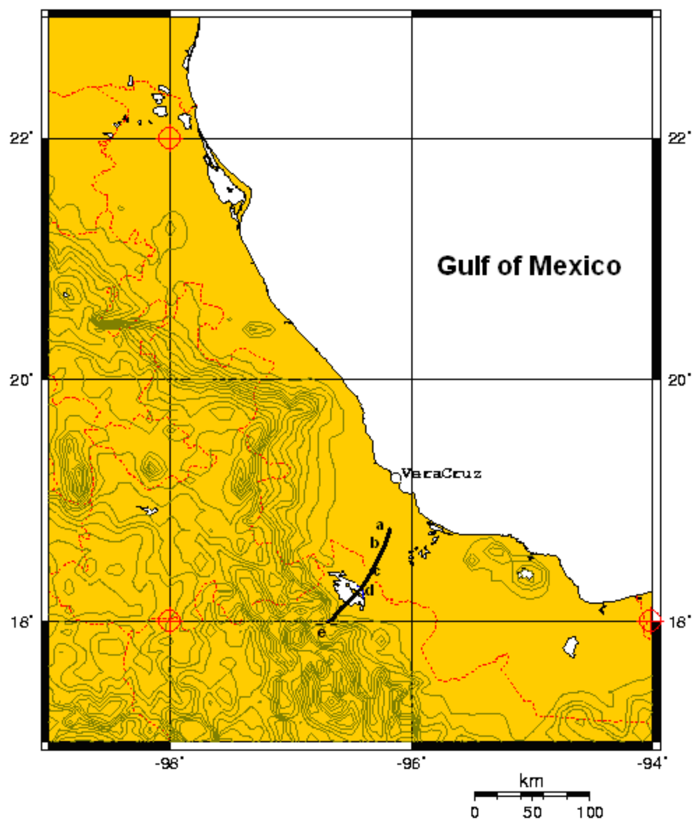

Fig. 5 The exploration of the mean monthly phase of precipitation during summer is made along a short trajectory of 103 km. Five stations (**a** to **d)** are considered, ranging from coastal lowlands in Veracruz State through to higher land just within the Valley of Oaxaca.

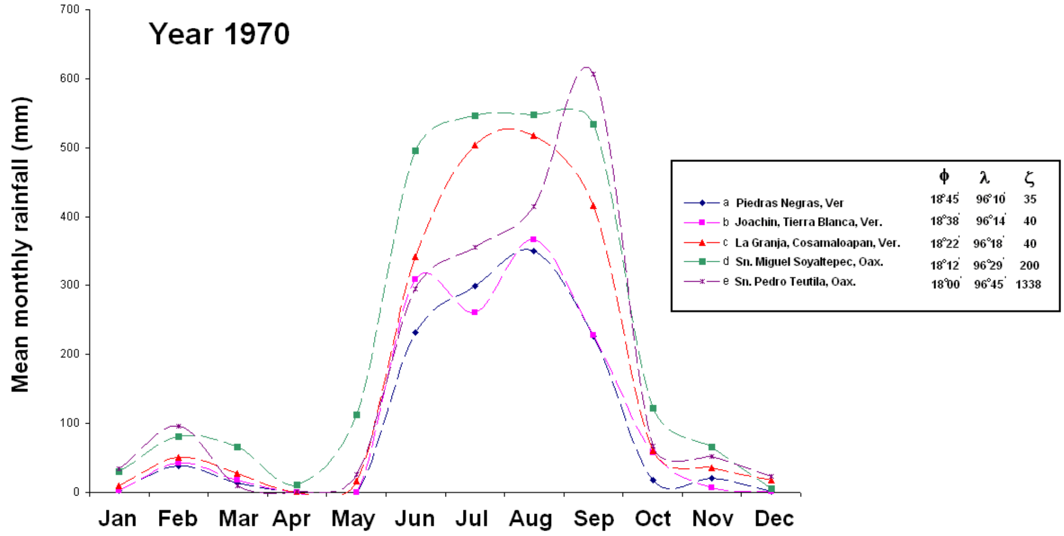

Fig. 6 Standard mean monthly rainfall climatology along the trajectory formed by five stations in Southern Mexico.

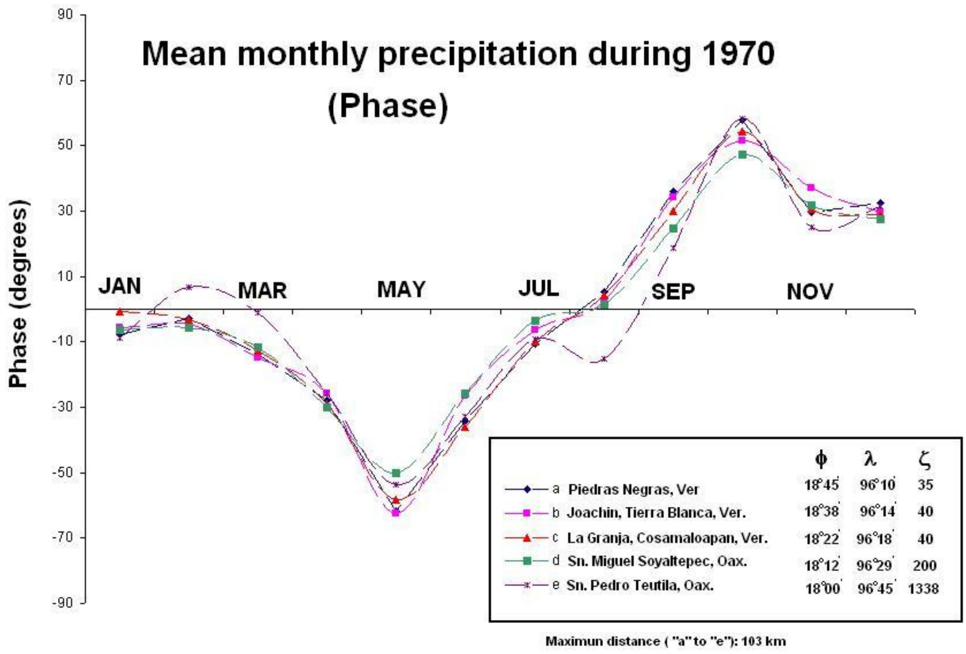

Fig. 7 Cuasilinear behavior of the phase reaching its maximum within the valley of Oaxaca (stations d and e).

5.0 Conclusions

The phase of Hilbert turned out to be a powerful resource for analysis, concerning steadiness and seasonality, during the summer rainy season. This permitted us to extract a maximum amount of information as available from climatological processes. In terms of the dynamic implications, these were found to concur with qualitative models, indicating ocean-atmosphere interaction (Magaña et al., 1999).

This concurrence upholds the application of high resolution numerical models in order to explore the trajectory in Southeastern Mexico and for the possible construction of dynamic schemes on a meso-scale. The construction of surrogate and dual climatology models offer us an alternative interpretation of the midsummer rainfall deficit. From the viewpoint of this alternative interpretation, the rain deficit is represented as an intrinsic and relative effect, which does not uphold the migration thesis. The validity of applying the algorithm of the phase of Hilbert to other intra-seasonal oscillations remains open to debate.

Acknowledgements: The elaboration of this work was possible thanks to the support of the higher authorities of Veracruz University, who granted a sabbatical year. We also, express our sincere thanks to those from Tropical Meteorology at the Centro de Ciencias de la Atmósfera of the National University of Mexico and from the University of Tlaxcala, in the campus of Ixtacuixtla, who gave permission to use their facilities.

References

Ali, A., T. Lebel and A. Amani 2003: Invariance in the Spatial Structure of Sahelian Rain Fields at Climatological Scales. Journal of Hydrometeorology. Vol. 4, No. 6 pp. 996–1011.

Baldwin, M.E., J.S. Kain, and S. Lakshmivarahan 2005: Development of an Automated Classification Procedure for Rainfall Systems. Monthly Weather Review: Vol. 133, No. 4, 844–862.

Bärring, L. and M. Hulme 1991: Filters and Approximate Confidence Intervals for Interpreting Rainfall Anomaly indices. Journal of Climate: **4**, 8, 837–847.

Bilham E.G. 1917: Use of monthly mean values in climatological analysis. Monthly Weather Review: **45**, 12, 602.

Carleton, A.M. 1999: Methodology in climatology. Annals AAG 89:713-735.

Daley, R. 1999: Atmospheric Data Analysis, Cambridge University Press.

Davis, C.A., K.W. Manning, R.E. Carbone, S.B. Trier and J.D. Tuttle 2003: Coherence of Warm-Season Continental Rainfall in Numerical Weather Prediction Models. Monthly Weather Review: Vol. 131, No. 11, 2667–2679.

Epstein, E. and A. Barnston 1990: A Precipitation Climatology of 5-Day Periods. Journal of Climate: **3**, 2, 218–236

Feldman, M. 2001: Hilbert Transforms. Encyclopedia of Vibration, Academic Press, pp. 642-648.

Finkelstein, P.L. and L.E. Truppi 1991: Spatial Distribution of Precipitation Seasonality in the United States. Journal of Climate: Vol. 4, No. 4 pp. 373–385

Fowler, A. C. 2004: Mathematical Models in the Applied Sciences. Cambridge texts in applied mathematics, 418 pp.

Freitag, E. and R. Busam 2005: Complex Analysis. Springer, first edition, 552 pp.

Goswami. J.C. and A. E. Hoefel 2004: Algorithms for estimating instantaneous frequency. Signal Processing 84, 1423-1427.

Hartmann, D. L. and J. R. Gross 1988: Seasonal Variability of the 40–50 Day Oscillation in Wind and Rainfall in the Tropics. Journal of the Atmospheric Sciences, **45**, 19, 2680-2702.

Haslet, J. 1989: Space time modeling in meteorology: A review. Bulletin International Statistics Institute 51, 229-246.

Haylock M. and J. McBride 2001: Spatial Coherence and Predictability of Indonesian Wet Season Rainfall. Journal of Climate: Vol. 14, No. 18 pp. 3882–3887.

Horton R. E. 1919: The measurement of rainfall and snow. Monthly Weather Review: **47**, 5, 294–296.

Hosking, J.G. and C.D. Stow 1987: Ground-Based, High-Resolution Measurements of the Spatial and Temporal Distribution of Rainfall. Journal of Applied Meteorology: Vol. 26, No. 11 pp. 1530–1539.

Hulme, M. and M. New 1997: Dependence of Large-Scale Precipitation Climatologies on Temporal and Spatial Sampling. Journal of Climate: Vol. 10, No. 5 pp. 1099–1113.

James, J. F. 2006: A Student's Guide to Fourier Transforms, Cambridge University Press, Second Edition, 142 pp.

Jameson, A.R. and A.B. Kostinsk 2002: When is Rain Steady? Journal of Applied Meteorology: Vol. 41, No. 1 pp. 83–90.

Jiang, D. and Z. Zhang 2006: Paleoclimate Modelling at the Institute of Atmospheric Physics, Chinese Academy of Sciences Advances in Atmospheric Sciences, Vol. 23, No. 6, 1040–1049.

Kincer, J.B. 1919: The seasonal distribution of precipitation and its frecuency and intensity in the United States. Monthly Weather Review: Vol. 47, No. 9 pp. 624–631.

Kousky, V.E. and S. Srivatsangam 1983: The Seasonal Cycle over the United States and Mexico. Monthly Weather Review: Vol. 111, No. 1, 165–171.

Krepper, C. M., B. V. Scia, and J. O. Pierini 1989: Time and space variability of rainfall in central-east Argentina. Journal of Climate: **2**, 1, 39-47.

Kuiper, G. P. 1952: Planetary atmosphere and their origin. Atmospheres of the Earth and planets, University of Chicago Press, 306-405.

Kummerow, C., P. Poyner, W. Berg and J. Thomas-Stahle 2004: The Effects of Rainfall Inhomogeneity on Climate Variability of Rainfall Estimated from Passive Microwave Sensors. Journal of Atmospheric and Oceanic Technology: Vol. 21, No. 4 pp. 624–638.

L'Ecuyer T.S., C. Kummerow and W. Berg 2004: Toward a Global Map of Raindrop Size Distributions. Part I: Rain-Type Classification and Its Implications for Validating Global Rainfall Products. Journal of Hydrometeorology 5(5): 831.

Magaña V., J.A. Amador and S. Medina 1999: The Midsummer Drought over Mexico and Central America. Journal of Climate: Vol. 12, No. 6 pp. 1577–1588.

Mendoza, B., V. Velasco and E. Jáuregui 2006: A Study of Historical Droughts in Southeastern Mexico. Journal of Climate: Vol. 19, No. 12 pp. 2916–2934.

Michelangeli P.A., R. Vautard and B. Legras 1995: Weather Regimes: Recurrence and Quasi Stationarity. Journal of the Atmospheric Sciences: Vol. 52, No. 8 pp. 1237–1256.

Needham, T. 1999: Visual complex analysis, Oxford University Press, 616 pp.

Nicholls, N. and K.K. Wong 1990: Dependence of Rainfall Variability on Mean Rainfall, Latitude, and the Southern Oscillation. Journal of Climate: **3**, 1, 163-170.

Nicolis, C. 1998: Atmospheric Analogs and Recurrence Time Statistics: Toward a Dynamical Formulation. Journal of the Atmospheric Sciences: Vol. 55, No. 3 pp. 465–475.

Ogallo, L. 1979: Rainfall variability in Africa. Monthly Weather Review: **107**, 9, 1133-1139

Pezzulli S., D.B. Stephenson and A. Hannachi 2005: The Variability of Seasonality. Journal of Climate: Vol. 18, No. 1, 71–88.

Robert, A. M., W. D. Shoaff and L. Khatib 1996: Domain-independent temporal reasoning with recurring events. Computational Intelligence 12 (3), 450–477.

Schiffer and Bowden 1984: The role of mathematics on science. The mathematical association of America. New mathematical library, 207 pp.

Shin S.I., P.D. Sardeshmukh, R.S. Webb, R.J. Oglesby and J. J. Barsugli 2006: Understanding the Mid-Holocene Climate. Journal of Climate: Vol. 19, No. 12 pp. 2801–2817.

Terenziani, P. and P. Torasso 1995: Time, Action-Types, and Causation: An Integrated Analysis. Computational Intelligence 11 (3), 529–552.

Vandenhouten, R. and R. Grebe. 1995: Phase reconstruction and unwrapping from holographic interferograms of partially absorbent phase objects. Applied Optics: 34 (8):1401.

Vandenhouten, R. and R. Grebe 1996: SANTIS (Systems ANalysis and TIme Series Processing) - ein Werkzeug zur Analyse von Biosignalen. Biomedizinische Technik. Biomedical Engineering. 41: 364.

Vandenhouten R., M. Lambertz, P. Langhorst, and R. Grebe 2000: Nonstationary time-series analysis applied to investigation of brainstem system dynamics. IEEE Transactions on Bio-Medical Engineering. 47 (6):729-37.

Varma A.K. and G. Liu, 2006: Small-Scale Horizontal Rain-Rate Variability Observed by Satellite. Monthly Weather Review: Vol. 134, No. 10 pp. 2722–2733.

Von Storch, H. and F. W. Zwiers 1999: Statistical analysis in climate research, Cambridge University Press.

Wikle, C. K. 2002: Spatio-temporal methods in climatology. Encyclopedia of Life Support Systems, EOLSS Publishers Co. Ltd.

Willmott, C.J., S.M. Robeson and M.J. Janis, 1996: Comparison of Approaches for Estimating Time-Averaged Precipitation Using Data from the USA. International Journal of Climatology, 16, 1103-1115.

WMO 2003: Manual on the global observing system volume I, Annex V to the Technical Regulation: Global Aspects, World Meteorological Organization Report WMO-No. 544.