PARADOXICAL GAMES AS A DIDACTIC TOOL TO TRAIN TEACHERS IN PROBABILITY

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In this paper the contents needed in the didactical preparation of teachers to teach probability are described and the possibility that classical paradoxes of probability offer to organise didactic activities that can help carry out this training are suggested. Results of working with one such activity in courses directed to teachers in México, Portugal and Spain are presented.

INTRODUCTION

Although probability is included in primary and secondary school levels in the mathematics curriculum, mathematics teachers frequently lack specific preparation in probability education. Even when many prospective secondary teachers have a major in mathematics, they usually need some additional professional knowledge related to the teaching of probability, where general principles that are valid for other areas of mathematics cannot always be applied. The situation is even more challenging for primary teachers, few of whom have had suitable training in either probability or probability education. Consequently, it is urgent to offer these teachers a better prior training as well as continuous support from University departments and research groups (Franklin & Mewborn, 2006).

In this paper the components in the professional knowledge these teachers need to teach probability are described and a possible didactical activity based on a paradoxical game that was suggested by Batanero, Godino and Roa (2004) is summarised. We also present new data collected in working with this activity in some workshops directed to in-service teachers in Mexico, Portugal and Spain. These data suggest change in the teachers' initial misconceptions as well as an increase in their professional knowledge.

TEACHER KNOWLEDGE TO TEACH PROBABILITY

Although teachers do not need high levels of mathematical knowledge, they however require a profound understanding of the basic mathematics they teach at school level, including a deep grasp of the interconnections and relationships among different aspects of this knowledge (Ma, 1999). In Batanero, Godino and Roa (2004), the following additional components needed in the professional knowledge of teachers were described: a) *Epistemology:* Epistemological reflection on the meaning of concepts to be taught (e.g., different meanings of probability; see Batanero, Henry & Parzysz, 2005); b) *Cognition:* Prediction of students' learning difficulties, errors, obstacles and strategies; c) *Teaching resources and techniques:* Experience with good examples of teaching situations, didactic tools; critical capacity to analyse textbooks, curricular documents and to adapt statistics knowledge to different teaching levels; d) *Affect:* Ability to engage students' interest and take into account the students' attitudes and beliefs; e) *Interaction:* Ability to create good communication in the classroom and to use assessment as a way to guide instruction. These types of knowledge are revisited by Godino, Batanero, Roa and Wilhelmi (2008) who include the following components in their model of teachers' professional knowledge:

- *Epistemic component:* Knowledge of mathematical or statistical content, that is, the set of problems, procedures, concepts, properties, language, and arguments included in the teaching of a given topic and its distribution over the teaching time.
- *Cognitive component:* Knowledge of students' levels of development and understanding of the topic, and students' strategies, difficulties, and errors as regards the intended content.
- *Affective facet:* Knowledge of students' attitudes, emotions, and motivations regarding the content and the study process.

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- *Media component:* Knowledge of didactic and technological resources available for teaching and the possible ways to use and distribute these resources over time.
- *Interactional component*: Managing the possible organisations of the classroom discourse and the interactions between the teacher and the students that help solve the students' difficulties and conflicts.
- *Ecological component:* Knowledge of the relationships of the topic with the official curriculum, other mathematical or statistical themes and with the social, political and economical settings that support the teaching and learning.

As argued by Ponte and Chapman (2006), we should view teachers as professionals, and ground teacher education in professional practice, making all elements of practice (preparing lessons, tasks and materials, carrying out lessons, observing and reflecting on lessons) a central element in the teacher education process. In the next section we suggest the interest of classical paradoxes in the history of probability to organise some didactic activities directed to train teachers in probability. The aim is provoking teachers' reflection about the meaning of elementary stochastic notions, helping them realise the students' difficulties and obstacles, and providing them with examples of didactical methodology and materials.

PARADOXES AS A TOOL TO DEVELOP TEACHERS' KNOWLEDGE

The above models of professional knowledge suggest that statistical knowledge alone is not enough for teachers to be able to teach probability. An implication of the analysis is the need to develop and assess teachers' professional knowledge and competencies, which takes into account the different components of pedagogical knowledge. A possible didactic tool is to use some classical paradoxes that appeared in the history of Probability to design activities directed to training teachers. We can take advantage of some of these classical paradoxes (see Székely, 1986) to build didactic situations that serve to provoke the teachers' didactic reflection, as it is described below.

Didactic Analysis of a Chance Game

Batanero et al. (2004) proposed an activity based on the Bertrand's box paradox that serves to compare the frequentist and Laplace's conceptions of probability, and to reflect on the concepts of dependent experiments and conditional probability, as well as on the role of problem-solving in the construction of mathematical knowledge. At the beginning of the activity, the lecturer proposes that the teachers experiment themselves with the following game:

> *Game: We take three counters of the same shape and size. One is blue on both sides, the second is red on both sides and the third is blue on one side and red on the other. We put the three counters into a box, and shake the box, before selecting a counter at random. After selecting the counter we show one of the sides. The aim of the game is to guess the color of the hidden side. We repeat the process, putting the counter again in the box before each new extraction. We make predictions about the hidden side color and win a point each time our prediction is right.*

Batanero et al. (2004) suggested that teachers first do some trials of the game and then the lecturer asks the teachers to find the strategy that produces the best chance to win over a long series of trials. After some repetitions of the game, all the strategies suggested by the teachers would be listed in the blackboard and a discussion to decide which the best strategy is would be organised by the lecturer. The aim of this discussion, where both correct reasoning and possible misconceptions will be revealed is to increase the teacher's probabilistic knowledge. In this game they can identify some fundamental ideas identified by Heitele (1975): event, probability and convergence, combinatorial operations, addition and multiplication rules, independence, conditional probability, random variable, equidistribution and symmetry, expectation and sampling. The activity will also help increase some components of professional knowledge: a) *epistemic* (subjective, frequentists and classical meanings of probability: b) *cognitive* (students' difficulties and of wrong strategies during the game); c) *affective* (students' interest and involvement in the activity) d) *media* (resources needed; use of computers or Internet applet to simulate the game); e) *interaction* (organisation of the classroom discourse); and f) *ecology* (social problems related to wrong misconceptions in chance games).

EVALUATION

Batanero et al. (2004) reported the evaluation of this activity in a sample of 47 undergraduate statisticians in Spain that were preparing to become statistics teachers at University level. In Table 1 we present some data taken from written solutions to the activity by 166 in-service teachers taking part in three different workshops (Spain, n=98 secondary school mathematics teachers; Portugal, n=27 secondary school mathematics teacher and Mexico, n=41 statistics lecturers at University level). Most Portuguese and Spanish participants in the workshops were graduates in Mathematics, while Mexican participants included a variety of backgrounds (statistics, mathematics, science, medicine or engineering). The average number of teaching experience in the whole sample was μ=12.6 years (σ=10.1). Due to the space limitations in this paper we only discriminate the country where the workshop was held, although complementary results will be presented in the conference. The aim of the current study is compare if the starting difficulties and the learning long the activity reported by Batanero et al. in pre-service teachers would reproduce in the case of in-service teachers. Another differences are the bigger sample size that multiply by four the sample used by Batanero *et al*. and the different cultural settings.

Table 1. Frequency of starting and final strategies

Strategies considered correct by the future teachers	Number of teachers choosing the strategy							
	Initial stage				Last stage			
	Spain	Port	Mex	Total	Spain	Port.	Mex	Total
Predicting the colour shown on the visible side (correct)	33	5	3	41	71	10	23	104
Predicting the colour contrary to that shown on the visible side	8	3	2	13	5	4	2	11
Considering there is no strategy or predicting at random	38	14	27	79	12	7	7	26
Choosing red (blue) in all the trials	3	1	1	5	1	2	1	4
Alternating colours	4	3	0	7	5	3	3	11
Using the previous outcomes before predicting	7	1	4	12	3	0	5	8
Changing strategies along the sequence of trials	4	0	4	8	0	1	0	1
Irrelevant physical properties of the cards	1	0	0	1	1	0	0	1

In agreement with previous results reported in Batanero, Godino and Roa (2004), less than half the in-service teachers chose the correct strategy at the beginning of the game. This suggests the difficulty of solving this problem in spite of the apparent simplicity, even when most participants had a bachelor in mathematics, statistics or sciences. A variety of starting strategies appeared, so that the goal of setting a problematic situation that serve these teachers to confront their different solutions and help the debate and reflection was achieved. We notice that a few participants did not perceive the independence of trials, as they used the previous outcomes before predicting the colour. Results were very similar in the different settings (Mexico, Portugal and Spain) in spite of the participants' different background and teaching experience (secondary education or University) and agreed with Batanero et al. results

Since teachers were asked to write and justify their solutions as a part of the workshop, they therefore had some time to reflect in the problem, and, as a consequence of the debate, in the final step the majority of teachers changed to the correct strategy. Consequently the data suggest a general positive change in these teachers' conceptions about the concepts involved in the activity. The activity also served to reach another aim, consisting in making teachers aware of their own probability misconceptions and prepare them to assess these misconceptions later in their own students. Another remark is that, at the end of the activity, still about a third of participants were still convinced that their own (incorrect) strategies were better than that of their classmates. Probably more teaching time would have been needed with some teachers in order that they would completely overcome their incorrect initial misconceptions.

In the phase of debate, teachers were able to provide different correct justifications, such as using only a simple experiment (there are two cards with same colour in both faces) or a composed experiment (considering first the selection of the card and then one of the two possible faces for each card). More sophisticates proofs included using the Bayes theorem or a Chi-square statistical test to compare empirical results with an equiprobability model. But still some teachers provided wrong justifications, such as relying only on the empirical results to "prove" the best strategy.

FINAL REFLECTIONS

Teachers need support and adequate training to succeed in achieving an adequate equilibrium of intuition and rigour when teaching probability. Unfortunately, due to time pressure teachers do not always receive a good preparation to teach probability in their initial training. However, activities such as the one analysed in this presentation can serve to simultaneously increase the teacher probability and professional knowledge. Moreover, despite the acknowledged fact that probability is distinct and different from other areas of mathematics and the implied need to provide mathematics teachers with a special preparation to teach this topic, teachers can work many mathematical concepts (such as numbers, proportions, ratios, combinatorics, proofs, etc.) when working with this type of activity. Much more research is still needed to clarify the essential components in the preparation of teachers to teach probability as the adequate method in which each component should be taught. The significant research efforts focusing on mathematics teacher education and professional development in the past decade (e.g., Ponte & Chapman, 2006; Hill, Sleep, Lewis & Ball, 2007; Wood, 2008) have not been reflected in statistics education. This is an important research area that can contribute to improve statistics education at school level.

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