Appendix A

TABLE OF GENERAL PROPERTIES OF LAPLACE TRANSFORMS

$$f(s) = \int_0^\infty e^{-st} F(t)\, dt$$

	$f(s)$	$F(t)$
1.	$a f_1(s) + b f_2(s)$	$a F_1(t) + b F_2(t)$
2.	$f(s/a)$	$a F(at)$
3.	$f(s-a)$	$e^{at} F(t)$
4.	$e^{-as} f(s)$	$u(t-a) = \begin{cases} F(t-a) & t > a \\ 0 & t < a \end{cases}$
5.	$s f(s) - F(0)$	$F'(t)$
6.	$s^2 f(s) - s F(0) - F'(0)$	$F''(t)$
7.	$s^n f(s) - s^{n-1} F(0) - s^{n-2} F'(0) - \cdots - F^{(n-1)}(0)$	$F^{(n)}(t)$
8.	$f'(s)$	$-t F(t)$
9.	$f''(s)$	$t^2 F(t)$
10.	$f^{(n)}(s)$	$(-1)^n t^n F(t)$
11.	$\frac{f(s)}{s}$	$\int_0^t F(u)\, du$
12.	$\frac{f(s)}{s^n}$	$\int_0^t \cdots \int_0^t F(u)\, du^n = \int_0^t \frac{(t-u)^{n-1}}{(n-1)!} F(u)\, du$
13.	$f(s)\, g(s)$	$\int_0^t F(u)\, G(t-u)\, du$

	$f(s)$	$F(t)$
14.	$\int_s^\infty f(u)\,du$	$\frac{F(t)}{t}$
15.	$\frac{1}{1-e^{-sT}}\int_0^T e^{-su}F(u)\,du$	$F(t) = F(t+T)$
16.	$\frac{f(\sqrt{s})}{s}$	$\frac{1}{\sqrt{\pi t}}\int_0^\infty e^{-u^2/4t}F(u)\,du$
17.	$\frac{1}{s}f(1/s)$	$\int_0^\infty J_0(2\sqrt{ut})\,F(u)\,du$
18.	$\frac{1}{s^{n+1}}f(1/s)$	$t^{n/2}\int_0^\infty u^{-n/2}J_n(2\sqrt{ut})\,F(u)\,du$
19.	$\frac{f(s+1/s)}{s^2+1}$	$\int_0^t J_0(2\sqrt{u(t-u)})\,F(u)\,du$
20.	$\frac{1}{2\sqrt{\pi}}\int_0^\infty u^{-3/2}e^{-s^2/4u}f(u)\,du$	$F(t^2)$
21.	$\frac{f(\ln s)}{s\ln s}$	$\int_0^\infty \frac{t^u F(u)}{\Gamma(u+1)}\,du$
22.	$\frac{P(s)}{Q(s)}$ $P(s)$ = polynomial of degree less than n, $Q(s) = (s-\alpha_1)(s-\alpha_2)\cdots(s-\alpha_n)$ where $\alpha_1, \alpha_2, \ldots, \alpha_n$ are all distinct.	$\sum_{k=1}^{n}\frac{P(\alpha_k)}{Q'(\alpha_k)}e^{\alpha_k t}$

Appendix B

TABLE OF SPECIAL LAPLACE TRANSFORMS

	$f(s)$	$F(t)$
1.	$\frac{1}{s}$	1
2.	$\frac{1}{s^2}$	t
3.	$\frac{1}{s^n}$ $\quad n = 1, 2, 3, \ldots$	$\frac{t^{n-1}}{(n-1)!}$, $\quad 0! = 1$
4.	$\frac{1}{s^n}$ $\quad n > 0$	$\frac{t^{n-1}}{\Gamma(n)}$
5.	$\frac{1}{s-a}$	e^{at}
6.	$\frac{1}{(s-a)^n}$ $\quad n = 1, 2, 3, \ldots$	$\frac{t^{n-1}\,e^{at}}{(n-1)!}$, $\quad 0! = 1$
7.	$\frac{1}{(s-a)^n}$ $\quad n > 0$	$\frac{t^{n-1}\,e^{at}}{\Gamma(n)}$
8.	$\frac{1}{s^2+a^2}$	$\frac{\sin at}{a}$
9.	$\frac{s}{s^2+a^2}$	$\cos at$
10.	$\frac{1}{(s-b)^2+a^2}$	$\frac{e^{bt}\sin at}{a}$
11.	$\frac{s-b}{(s-b)^2+a^2}$	$e^{bt}\cos at$
12.	$\frac{1}{s^2-a^2}$	$\frac{\sinh at}{a}$
13.	$\frac{s}{s^2-a^2}$	$\cosh at$
14.	$\frac{1}{(s-b)^2-a^2}$	$\frac{e^{bt}\sinh at}{a}$

	$f(s)$	$F(t)$
15.	$\dfrac{s-b}{(s-b)^2-a^2}$	$e^{bt}\cosh at$
16.	$\dfrac{1}{(s-a)(s-b)}$ $a \neq b$	$\dfrac{e^{bt}-e^{at}}{b-a}$
17.	$\dfrac{s}{(s-a)(s-b)}$ $a \neq b$	$\dfrac{be^{bt}-ae^{at}}{b-a}$
18.	$\dfrac{1}{(s^2+a^2)^2}$	$\dfrac{\sin at - at\cos at}{2a^3}$
19.	$\dfrac{s}{(s^2+a^2)^2}$	$\dfrac{t\sin at}{2a}$
20.	$\dfrac{s^2}{(s^2+a^2)^2}$	$\dfrac{\sin at + at\cos at}{2a}$
21.	$\dfrac{s^3}{(s^2+a^2)^2}$	$\cos at - \frac{1}{2}at\sin at$
22.	$\dfrac{s^2-a^2}{(s^2+a^2)^2}$	$t\cos at$
23.	$\dfrac{1}{(s^2-a^2)^2}$	$\dfrac{at\cosh at - \sinh at}{2a^3}$
24.	$\dfrac{s}{(s^2-a^2)^2}$	$\dfrac{t\sinh at}{2a}$
25.	$\dfrac{s^2}{(s^2-a^2)^2}$	$\dfrac{\sinh at + at\cosh at}{2a}$
26.	$\dfrac{s^3}{(s^2-a^2)^2}$	$\cosh at + \frac{1}{2}at\sinh at$
27.	$\dfrac{s^2+a^2}{(s^2-a^2)^2}$	$t\cosh at$
28.	$\dfrac{1}{(s^2+a^2)^3}$	$\dfrac{(3-a^2t^2)\sin at - 3at\cos at}{8a^5}$
29.	$\dfrac{s}{(s^2+a^2)^3}$	$\dfrac{t\sin at - at^2\cos at}{8a^3}$
30.	$\dfrac{s^2}{(s^2+a^2)^3}$	$\dfrac{(1+a^2t^2)\sin at - at\cos at}{8a^3}$
31.	$\dfrac{s^3}{(s^2+a^2)^3}$	$\dfrac{3t\sin at + at^2\cos at}{8a}$

	$f(s)$	$F(t)$
32.	$\frac{s^4}{(s^2+a^2)^3}$	$\frac{(3-a^2t^2)\sin at + 5at\cos at}{8a}$
33.	$\frac{s^5}{(s^2+a^2)^3}$	$\frac{(8-a^2t^2)\cos at - 7at\sin at}{8}$
34.	$\frac{3s^2-a^2}{(s^2+a^2)^3}$	$\frac{t^2\sin at}{2a}$
35.	$\frac{s^3-3a^2s}{(s^2+a^2)^3}$	$\frac{1}{2}t^2\cos at$
36.	$\frac{s^4-6a^2s^2+a^4}{(s^2+a^2)^4}$	$\frac{1}{6}t^3\cos at$
37.	$\frac{s^3-a^2s}{(s^2+a^2)^4}$	$\frac{t^3\sin at}{24a}$
38.	$\frac{1}{(s^2-a^2)^3}$	$\frac{(3+a^2t^2)\sinh at - 3at\cosh at}{8a^5}$
39.	$\frac{s}{(s^2-a^2)^3}$	$\frac{at^2\cosh at - t\sinh at}{8a^3}$
40.	$\frac{s^2}{(s^2-a^2)^3}$	$\frac{at\cosh at + (a^2t^2-1)\sinh at}{8a^3}$
41.	$\frac{s^3}{(s^2-a^2)^3}$	$\frac{3t\sinh at + at^2\cosh at}{8a}$
42.	$\frac{s^4}{(s^2-a^2)^3}$	$\frac{(3+a^2t^2)\sinh at + 5at\cosh at}{8a}$
43.	$\frac{s^5}{(s^2-a^2)^3}$	$\frac{(8+a^2t^2)\cosh at + 7at\sinh at}{8}$
44.	$\frac{3s^2+a^2}{(s^2-a^2)^3}$	$\frac{t^2\sinh at}{2a}$
45.	$\frac{s^3+3a^2s}{(s^2-a^2)^3}$	$\frac{1}{2}t^2\cosh at$
46.	$\frac{s^4+6a^2s^2+a^4}{(s^2-a^2)^4}$	$\frac{1}{6}t^3\cosh at$
47.	$\frac{s^3+a^2s}{(s^2-a^2)^4}$	$\frac{t^3\sinh at}{24a}$
48.	$\frac{1}{s^3+a^3}$	$\frac{e^{at/2}}{3a^2}\left\{\sqrt{3}\sin\frac{\sqrt{3}\,at}{2} - \cos\frac{\sqrt{3}\,at}{2} + e^{-3at/2}\right\}$

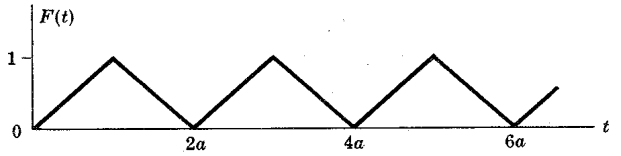
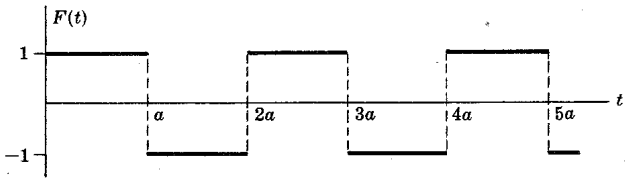
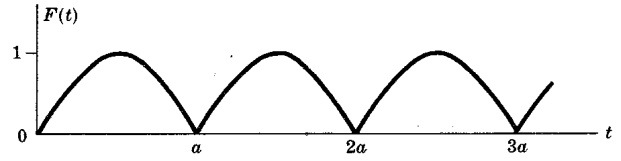
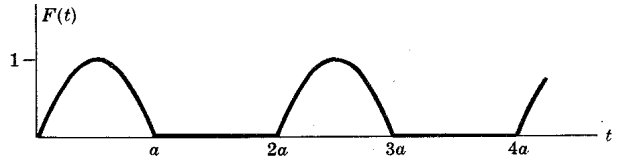
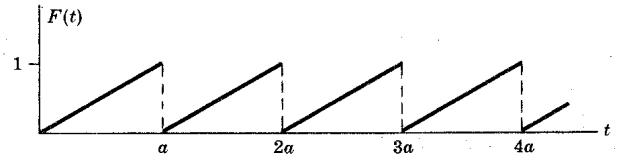
	$f(s)$	$F(t)$
49.	$\frac{s}{s^3+a^3}$	$\frac{e^{at/2}}{3a}\left\{\cos\frac{\sqrt{3}\,at}{2}+\sqrt{3}\sin\frac{\sqrt{3}\,at}{2}-e^{-3at/2}\right\}$
50.	$\frac{s^2}{s^3+a^3}$	$\frac{1}{3}\left(e^{-at}+2e^{at/2}\cos\frac{\sqrt{3}\,at}{2}\right)$
51.	$\frac{1}{s^3-a^3}$	$\frac{e^{-at/2}}{3a^2}\left\{e^{3at/2}-\cos\frac{\sqrt{3}\,at}{2}-\sqrt{3}\sin\frac{\sqrt{3}\,at}{2}\right\}$
52.	$\frac{s}{s^3-a^3}$	$\frac{e^{-at/2}}{3a}\left\{\sqrt{3}\sin\frac{\sqrt{3}\,at}{2}-\cos\frac{\sqrt{3}\,at}{2}+e^{3at/2}\right\}$
53.	$\frac{s^2}{s^3-a^3}$	$\frac{1}{3}\left(e^{at}+2e^{-at/2}\cos\frac{\sqrt{3}\,at}{2}\right)$
54.	$\frac{1}{s^4+4a^4}$	$\frac{1}{4a^3}(\sin at\cosh at-\cos at\sinh at)$
55.	$\frac{s}{s^4+4a^4}$	$\frac{\sin at\sinh at}{2a^2}$
56.	$\frac{s^2}{s^4+4a^4}$	$\frac{1}{2a}(\sin at\cosh at+\cos at\sinh at)$
57.	$\frac{s^3}{s^4+4a^4}$	$\cos at\cosh at$
58.	$\frac{1}{s^4-a^4}$	$\frac{1}{2a^3}(\sinh at-\sin at)$
59.	$\frac{s}{s^4-a^4}$	$\frac{1}{2a^2}(\cosh at-\cos at)$
60.	$\frac{s^2}{s^4-a^4}$	$\frac{1}{2a}(\sinh at+\sin at)$
61.	$\frac{s^3}{s^4-a^4}$	$\frac{1}{2}(\cosh at+\cos at)$
62.	$\frac{1}{\sqrt{s+a}+\sqrt{s+b}}$	$\frac{e^{-bt}-e^{-at}}{2(b-a)\sqrt{\pi t^3}}$
63.	$\frac{1}{s\sqrt{s+a}}$	$\frac{\operatorname{erf}\sqrt{at}}{\sqrt{a}}$
64.	$\frac{1}{\sqrt{s}\,(s-a)}$	$\frac{e^{at}\operatorname{erf}\sqrt{at}}{\sqrt{a}}$
65.	$\frac{1}{\sqrt{s-a}+b}$	$e^{at}\left\{\frac{1}{\sqrt{\pi t}}-b\,e^{b^2t}\operatorname{erfc}(b\sqrt{t})\right\}$

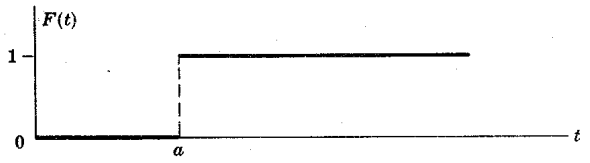
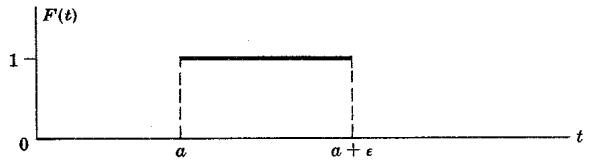
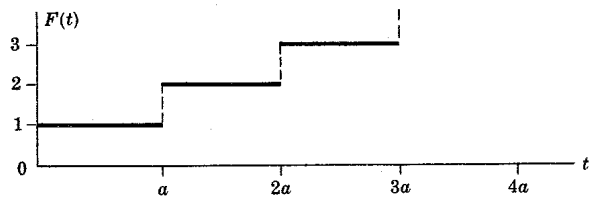
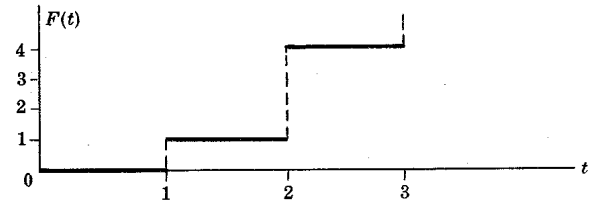
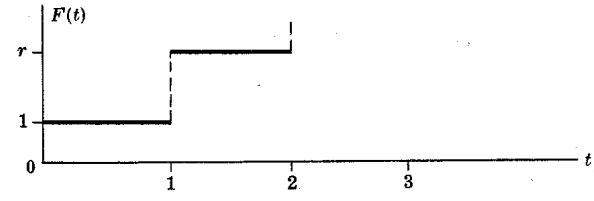
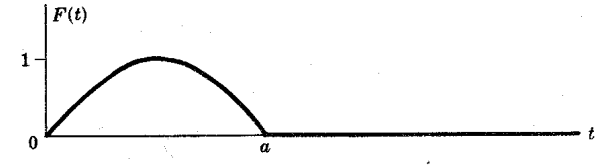
	$f(s)$	$F(t)$
66.	$\dfrac{1}{\sqrt{s^2+a^2}}$	$J_0(at)$
67.	$\dfrac{1}{\sqrt{s^2-a^2}}$	$I_0(at)$
68.	$\dfrac{(\sqrt{s^2+a^2}-s)^n}{\sqrt{s^2+a^2}} \quad n>-1$	$a^n J_n(at)$
69.	$\dfrac{(s-\sqrt{s^2-a^2})^n}{\sqrt{s^2-a^2}} \quad n>-1$	$a^n I_n(at)$
70.	$\dfrac{e^{b(s-\sqrt{s^2+a^2})}}{\sqrt{s^2+a^2}}$	$J_0(a\sqrt{t(t+2b)})$
71.	$\dfrac{e^{-b\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}}$	$\begin{cases} J_0(a\sqrt{t^2-b^2}) & t>b \\ 0 & t<b \end{cases}$
72.	$\dfrac{1}{(s^2+a^2)^{3/2}}$	$\dfrac{tJ_1(at)}{a}$
73.	$\dfrac{s}{(s^2+a^2)^{3/2}}$	$tJ_0(at)$
74.	$\dfrac{s^2}{(s^2+a^2)^{3/2}}$	$J_0(at)-atJ_1(at)$
75.	$\dfrac{1}{(s^2-a^2)^{3/2}}$	$\dfrac{tI_1(at)}{a}$
76.	$\dfrac{s}{(s^2-a^2)^{3/2}}$	$tI_0(at)$
77.	$\dfrac{s^2}{(s^2-a^2)^{3/2}}$	$I_0(at)+atI_1(at)$
78.	$\dfrac{1}{s(e^s-1)}=\dfrac{e^{-s}}{s(1-e^{-s})}$ See also entry 141, Page 254.	$F(t)=n, \quad n\leqq t<n+1, \quad n=0,1,2,\ldots$
79.	$\dfrac{1}{s(e^s-r)}=\dfrac{e^{-s}}{s(1-re^{-s})}$	$F(t)=\sum_{k=1}^{[t]} r^k$ where $[t]$ = greatest integer $\leqq t$
80.	$\dfrac{e^s-1}{s(e^s-r)}=\dfrac{1-e^{-s}}{s(1-re^{-s})}$ See also entry 143, Page 254.	$F(t)=r^n, \quad n\leqq t<n+1, \quad n=0,1,2,\ldots$
81.	$\dfrac{e^{-a/s}}{\sqrt{s}}$	$\dfrac{\cos 2\sqrt{at}}{\sqrt{\pi t}}$

	$f(s)$	$F(t)$
82.	$\dfrac{e^{-a/s}}{s^{3/2}}$	$\dfrac{\sin 2\sqrt{at}}{\sqrt{\pi a}}$
83.	$\dfrac{e^{-a/s}}{s^{n+1}} \qquad n > -1$	$\left(\dfrac{t}{a}\right)^{n/2} J_n(2\sqrt{at})$
84.	$\dfrac{e^{-a\sqrt{s}}}{\sqrt{s}}$	$\dfrac{e^{-a^2/4t}}{\sqrt{\pi t}}$
85.	$e^{-a\sqrt{s}}$	$\dfrac{a}{2\sqrt{\pi t^3}}\, e^{-a^2/4t}$
86.	$\dfrac{1 - e^{-a\sqrt{s}}}{s}$	$\operatorname{erf}(a/2\sqrt{t})$
87.	$\dfrac{e^{-a\sqrt{s}}}{s}$	$\operatorname{erfc}(a/2\sqrt{t})$
88.	$\dfrac{e^{-a\sqrt{s}}}{\sqrt{s}\,(\sqrt{s}+b)}$	$e^{b(bt+a)} \operatorname{erfc}\left(b\sqrt{t} + \dfrac{a}{2\sqrt{t}}\right)$
89.	$\dfrac{e^{-a/\sqrt{s}}}{s^{n+1}} \qquad n > -1$	$\dfrac{1}{\sqrt{\pi t}\, a^{2n+1}} \displaystyle\int_0^\infty u^n\, e^{-u^2/4a^2t} J_{2n}(2\sqrt{u})\, du$
90.	$\ln\left(\dfrac{s+a}{s+b}\right)$	$\dfrac{e^{-bt} - e^{-at}}{t}$
91.	$\dfrac{\ln[(s^2 + a^2)/a^2]}{2s}$	$\operatorname{Ci}(at)$
92.	$\dfrac{\ln[(s + a)/a]}{s}$	$\operatorname{Ei}(at)$
93.	$-\dfrac{(\gamma + \ln s)}{s}$ γ = Euler's constant = .5772156...	$\ln t$
94.	$\ln\left(\dfrac{s^2 + a^2}{s^2 + b^2}\right)$	$\dfrac{2(\cos at - \cos bt)}{t}$
95.	$\dfrac{\pi^2}{6s} + \dfrac{(\gamma + \ln s)^2}{s}$ γ = Euler's constant = .5772156...	$\ln^2 t$
96.	$\dfrac{\ln s}{s}$	$-(\ln t + \gamma)$ γ = Euler's constant = .5772156...
97.	$\dfrac{\ln^2 s}{s}$	$(\ln t + \gamma)^2 - \frac{1}{6}\pi^2$ γ = Euler's constant = .5772156...

	$f(s)$	$F(t)$
98.	$\dfrac{\Gamma'(n+1) - \Gamma(n+1)\ln s}{s^{n+1}} \qquad n > -1$	$t^n \ln t$
99.	$\tan^{-1}(a/s)$	$\dfrac{\sin at}{t}$
100.	$\dfrac{\tan^{-1}(a/s)}{s}$	$\mathrm{Si}(at)$
101.	$\dfrac{e^{a/s}}{\sqrt{s}}\,\mathrm{erfc}\,(\sqrt{a/s}\,)$	$\dfrac{e^{-2\sqrt{at}}}{\sqrt{\pi t}}$
102.	$e^{s^2/4a^2}\,\mathrm{erfc}\,(s/2a)$	$\dfrac{2a}{\sqrt{\pi}}\,e^{-a^2t^2}$
103.	$\dfrac{e^{s^2/4a^2}\,\mathrm{erfc}\,(s/2a)}{s}$	$\mathrm{erf}\,(at)$
104.	$\dfrac{e^{as}\,\mathrm{erfc}\,\sqrt{as}}{\sqrt{s}}$	$\dfrac{1}{\sqrt{\pi(t+a)}}$
105.	$e^{as}\,\mathrm{Ei}\,(as)$	$\dfrac{1}{t+a}$
106.	$\dfrac{1}{a}\left[\cos as\left\{\dfrac{\pi}{2} - \mathrm{Si}\,(as)\right\} - \sin as\,\mathrm{Ci}\,(as)\right]$	$\dfrac{1}{t^2+a^2}$
107.	$\sin as\left\{\dfrac{\pi}{2} - \mathrm{Si}\,(as)\right\} + \cos as\,\mathrm{Ci}\,(as)$	$\dfrac{t}{t^2+a^2}$
108.	$\dfrac{\cos as\left\{\dfrac{\pi}{2} - \mathrm{Si}\,(as)\right\} - \sin as\,\mathrm{Ci}\,(as)}{s}$	$\tan^{-1}(t/a)$
109.	$\dfrac{\sin as\left\{\dfrac{\pi}{2} - \mathrm{Si}\,(as)\right\} + \cos as\,\mathrm{Ci}\,(as)}{s}$	$\dfrac{1}{2}\ln\left(\dfrac{t^2+a^2}{a^2}\right)$
110.	$\left[\dfrac{\pi}{2} - \mathrm{Si}\,(as)\right]^2 + \mathrm{Ci}^2\,(as)$	$\dfrac{1}{t}\ln\left(\dfrac{t^2+a^2}{a^2}\right)$
111.	0	$\mathcal{N}(t)$
112.	1	$\delta(t)$
113.	e^{-as}	$\delta(t-a)$
114.	$\dfrac{e^{-as}}{s}$ See also entry 139, Page 254.	$u(t-a)$

	$f(s)$	$F(t)$
115.	$\frac{\sinh sx}{s \sinh sa}$	$\frac{x}{a} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi x}{a} \cos \frac{n\pi t}{a}$
116.	$\frac{\sinh sx}{s \cosh sa}$	$\frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} \sin \frac{(2n-1)\pi x}{2a} \sin \frac{(2n-1)\pi t}{2a}$
117.	$\frac{\cosh sx}{s \sinh sa}$	$\frac{t}{a} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \cos \frac{n\pi x}{a} \sin \frac{n\pi t}{a}$
118.	$\frac{\cosh sx}{s \cosh sa}$	$1 + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} \cos \frac{(2n-1)\pi x}{2a} \cos \frac{(2n-1)\pi t}{2a}$
119.	$\frac{\sinh sx}{s^2 \sinh sa}$	$\frac{xt}{a} + \frac{2a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \sin \frac{n\pi x}{a} \sin \frac{n\pi t}{a}$
120.	$\frac{\sinh sx}{s^2 \cosh sa}$	$x + \frac{8a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^2} \sin \frac{(2n-1)\pi x}{2a} \cos \frac{(2n-1)\pi t}{2a}$
121.	$\frac{\cosh sx}{s^2 \sinh sa}$	$\frac{t^2}{2a} + \frac{2a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos \frac{n\pi x}{a} \left(1 - \cos \frac{n\pi t}{a}\right)$
122.	$\frac{\cosh sx}{s^2 \cosh sa}$	$t + \frac{8a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^2} \cos \frac{(2n-1)\pi x}{2a} \sin \frac{(2n-1)\pi t}{2a}$
123.	$\frac{\cosh sx}{s^3 \cosh sa}$	$\frac{1}{2}(t^2 + x^2 - a^2) - \frac{16a^2}{\pi^3} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^3} \cos \frac{(2n-1)\pi x}{2a} \cos \frac{(2n-1)\pi t}{2a}$
124.	$\frac{\sinh x\sqrt{s}}{\sinh a\sqrt{s}}$	$\frac{2\pi}{a^2} \sum_{n=1}^{\infty} (-1)^n n\, e^{-n^2\pi^2 t/a^2} \sin \frac{n\pi x}{a}$
125.	$\frac{\cosh x\sqrt{s}}{\cosh a\sqrt{s}}$	$\frac{\pi}{a^2} \sum_{n=1}^{\infty} (-1)^{n-1} (2n-1)\, e^{-(2n-1)^2\pi^2 t/4a^2} \cos \frac{(2n-1)\pi x}{2a}$
126.	$\frac{\sinh x\sqrt{s}}{\sqrt{s} \cosh a\sqrt{s}}$	$\frac{2}{a} \sum_{n=1}^{\infty} (-1)^{n-1} e^{-(2n-1)^2\pi^2 t/4a^2} \sin \frac{(2n-1)\pi x}{2a}$
127.	$\frac{\cosh x\sqrt{s}}{\sqrt{s} \sinh a\sqrt{s}}$	$\frac{1}{a} + \frac{2}{a} \sum_{n=1}^{\infty} (-1)^n e^{-n^2\pi^2 t/a^2} \cos \frac{n\pi x}{a}$
128.	$\frac{\sinh x\sqrt{s}}{s \sinh a\sqrt{s}}$	$\frac{x}{a} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} e^{-n^2\pi^2 t/a^2} \sin \frac{n\pi x}{a}$
129.	$\frac{\cosh x\sqrt{s}}{s \cosh a\sqrt{s}}$	$1 + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} e^{-(2n-1)^2\pi^2 t/4a^2} \cos \frac{(2n-1)\pi x}{2a}$
130.	$\frac{\sinh x\sqrt{s}}{s^2 \sinh a\sqrt{s}}$	$\frac{xt}{a} + \frac{2a^2}{\pi^3} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} (1 - e^{-n^2\pi^2 t/a^2}) \sin \frac{n\pi x}{a}$
131.	$\frac{\cosh x\sqrt{s}}{s^2 \cosh a\sqrt{s}}$	$\frac{1}{2}(x^2 - a^2) + t - \frac{16a^2}{\pi^3} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^3} e^{-(2n-1)^2\pi^2 t/4a^2} \cos \frac{(2n-1)\pi x}{2a}$

	$f(s)$	$F(t)$
132.	$\dfrac{J_0(ix\sqrt{s})}{s\,J_0(ia\sqrt{s})}$	$1 - 2\sum_{n=1}^{\infty} \dfrac{e^{-\lambda_n^2 t/a^2} J_0(\lambda_n x/a)}{\lambda_n J_1(\lambda_n)}$ where $\lambda_1, \lambda_2, \ldots$ are the positive roots of $J_0(\lambda) = 0$
133.	$\dfrac{J_0(ix\sqrt{s})}{s^2 J_0(ia\sqrt{s})}$	$\frac{1}{4}(x^2 - a^2) + t + 2a^2 \sum_{n=1}^{\infty} \dfrac{e^{-\lambda_n^2 t/a^2} J_0(\lambda_n x/a)}{\lambda_n^3 J_1(\lambda_n)}$ where $\lambda_1, \lambda_2, \ldots$ are the positive roots of $J_0(\lambda) = 0$
134.	$\dfrac{1}{as^2} \tanh\left(\dfrac{as}{2}\right)$	**Triangular wave function** 
135.	$\dfrac{1}{s} \tanh\left(\dfrac{as}{2}\right)$	**Square wave function** 
136.	$\dfrac{\pi a}{a^2 s^2 + \pi^2} \coth\left(\dfrac{as}{2}\right)$	**Rectified sine wave function** 
137.	$\dfrac{\pi a}{(a^2 s^2 + \pi^2)(1 - e^{-as})}$	**Half rectified sine wave function** 
138.	$\dfrac{1}{as^2} - \dfrac{e^{-as}}{s(1 - e^{-as})}$	**Saw tooth wave function** 

	$f(s)$	$F(t)$
139.	$\frac{e^{-as}}{s}$ See also entry 114, Page 251.	**Heaviside's unit function** $u(t-a)$ 
140.	$\frac{e^{-as}(1-e^{-\epsilon s})}{s}$	**Pulse function** 
141.	$\frac{1}{s(1-e^{-as})}$ See also entry 78, Page 249.	**Step function** 
142.	$\frac{e^{-s}+e^{-2s}}{s(1-e^{-s})^2}$	$F(t) = n^2, \quad n \leqq t < n+1, \quad n = 0, 1, 2, \ldots$ 
143.	$\frac{1-e^{-s}}{s(1-re^{-s})}$ See also entry 80, Page 249.	$F(t) = r^n, \quad n \leqq t < n+1, \quad n = 0, 1, 2, \ldots$ 
144.	$\frac{\pi a(1+e^{-as})}{a^2s^2+\pi^2}$	$F(t) = \begin{cases} \sin(\pi t/a) & 0 \leqq t \leqq a \\ 0 & t > a \end{cases}$ 

Appendix C

TABLE OF SPECIAL FUNCTIONS

1. Gamma function

$$\Gamma(n) = \int_0^\infty u^{n-1} e^{-u}\, du, \qquad n > 0$$

2. Beta function

$$B(m, n) = \int_0^1 u^{m-1} (1-u)^{n-1}\, du = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}, \qquad m, n > 0$$

3. Bessel function

$$J_n(x) = \frac{x^n}{2^n\,\Gamma(n+1)} \left\{ 1 - \frac{x^2}{2(2n+2)} + \frac{x^4}{2 \cdot 4(2n+2)(2n+4)} - \cdots \right\}$$

4. Modified Bessel function

$$I_n(x) = i^{-n} J_n(ix) = \frac{x^n}{2^n\,\Gamma(n+1)} \left\{ 1 + \frac{x^2}{2(2n+2)} + \frac{x^4}{2 \cdot 4(2n+2)(2n+4)} + \cdots \right\}$$

5. Error function

$$\operatorname{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-u^2}\, du$$

6. Complementary error function

$$\operatorname{erfc}(t) = 1 - \operatorname{erf}(t) = \frac{2}{\sqrt{\pi}} \int_t^\infty e^{-u^2}\, du$$

7. Exponential integral

$$\operatorname{Ei}(t) = \int_t^\infty \frac{e^{-u}}{u}\, du$$

8. Sine integral

$$\operatorname{Si}(t) = \int_0^t \frac{\sin u}{u}\, du$$

9. Cosine integral

$$\operatorname{Ci}(t) = \int_t^\infty \frac{\cos u}{u}\, du$$

10. Fresnel sine integral

$$S(t) = \int_0^t \sin u^2\, du$$

11. Fresnel cosine integral

$$C(t) = \int_0^t \cos u^2\, du$$

12. Laguerre polynomials

$$L_n(t) = \frac{e^t}{n!} \frac{d^n}{dt^n} (t^n e^{-t}), \qquad n = 0, 1, 2, \ldots$$