

# Agent-Based Modeling and Simulation

## Analyzing and Understanding ABMs

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# Credits

- ▶ These slides are completely based on the book of Railsback and Grimm [2], chapter 22.
- ▶ Any difference with this source is my responsibility.

# Experimenting

- ▶ Once we have built a new ABM, even a preliminary version of one, it becomes another piece of software that we want to **understand**:
  - ▶ What **results** does the model produce, under what **conditions**?
  - ▶ How do results **change** when parameters, input data, or initial conditions change?
  - ▶ And most importantly, **why** does the model produce the results it does?
  - ▶ What is the model trying to tell us about **how** it works, and how the real system works?



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# Scientific Method

- ▶ To turn experimentation into a scientific method, we make our experiments **reproducible**:
  - ▶ by **completely describing** the model just as empirical scientists describe the materials and methods used in a laboratory or field study;
  - ▶ by **precisely documenting** all the parameter values, input data, and initial conditions we use;
  - ▶ and by **documenting and analyzing** the results of our experiments.



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# Controlled Experiments

- ▶ Controlled simulation experiments are also **key** to analyzing and understanding what models do.
- ▶ “Controlled” does not, however, imply that a simple protocol for analyzing models exists.
- ▶ Rather, how we analyze a model still depends on the model, the question and system it addresses, our **experience**, and the problem-solving **heuristics** we know and prefer.
- ▶ Heuristics, or rules of thumb, for problem solving are characterized by the fact that they are often useful, but **not always**: we simply have to try them.
- ▶ Using heuristics does not mean that modeling is unscientific: heuristics are the basis of any **creative** research.



# Objectives

1. Understand the **purpose** and **goals** of analyzing full ABMs, including both “finished” models and preliminary versions of models that you plan to develop further.
2. Learn and try ten **heuristics** –techniques or tricks that are often useful– for analyzing ABMs.
3. Become familiar with common ways that **statistical analysis** is used to understand ABMs.



# Introduction

- ▶ The Segregation model in the Social Science section of NetLogo's Models Library was inspired by a simple model by the Nobel laureate Thomas Schelling [3, 4].
- ▶ Following is an ODD description of this model.



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# Purpose

- ▶ The model addresses **segregation** of households in cities: why do members of different groups (e.g., racial, ethnic, religious) tend to occupy different neighborhoods?
- ▶ The model explores the relationship between segregation patterns and the **tolerance** of individuals for unlike neighbors.



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# Entities, State Variables, and Scales I

- ▶ The model entities include turtles that represent **households**, and patches that represent **houses**.
- ▶ Households are characterized by their **location** (which patch they occupy) and their **color**, which represents the group they belong to, either blue or red.
- ▶ Households also have a state variable **happy?**, a boolean variable set to false if the household has more unlike neighbors than it tolerates.
- ▶ The grid cells make up a **square** of  $51 \times 51$  cells, with no depiction of roads or other spaces between them. The space is **toroidal**.



# Entities, State Variables, and Scales II

- ▶ The length of a time step is unspecified but represents the time in which a household would decide **whether to move**. The number of time steps in a model run is an emergent outcome: the model runs **until all households are happy** and, therefore, stop moving.



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# Process Overview and Scheduling

- ▶ The following **actions** are executed, in this order, once per time step:
  - ▶ If all households are happy (happy? is true) then the model **stops**.
  - ▶ The households that are not happy (happy? is false) execute the submodel **move**. The order in which these households execute is randomly shuffled each time step.
  - ▶ All households **update** their happy? variable.
  - ▶ Outputs for system-level results are updated.



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# Design Concepts I

- ▶ The **basic principle** of segregation is the question of whether strong individual behaviors are necessary to produce striking system patterns –does the presence of strong segregation mean that households are highly intolerant– or can such strong patterns emerge in part from the system's structure?
- ▶ The key **outcomes** of the model are segregation patterns –especially, how strongly segregated the entire system is; these outcomes emerge from how households respond to unlike neighbors by moving.
- ▶ The households' **adaptive behavior** is to decide whether to move: they move when their objective –to live in a neighborhood with the fraction of unlike neighbors below their intolerance threshold– is not met.



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# Design Concepts II

- ▶ The behavior does not involve **learning**, or prediction other than the implicit prediction that moving might lead to a neighborhood where the tolerance objective is met.
- ▶ Households **sense** the color of households on the eight surrounding patches.
- ▶ **Stochasticity** is used in two ways:
  - ▶ to initialize the model so that it starts unsegregated, and
  - ▶ to determine the new location of households when they move, because modeling the details of movement is unnecessary for this model.
- ▶ **Observations** include a visual display of which color household is on each grid cell, and two numerical results: the mean percentage (over all households) of neighbors of similar color and the percentage of unhappy households.



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# Initialization

- ▶ A **user-chosen number of households** (typically 2000, out of the 2601 patches that represent houses) are initialized.
- ▶ They are each **placed** on a random empty grid cell and given a color randomly, with equal probability of red and blue.
- ▶ The variable `happy?` is then calculated for all households.



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# Input Data

- ▶ The model does not use input from external models or data files.

# Submodels I

**Move** is performed by individual households if they are unhappy. The household chooses a direction randomly from a uniform continuous distribution between 0 and 360 degrees, then moves forward a distance drawn randomly from a uniform continuous distribution of 0 to 10 grid cell widths. If there is already a household on the grid cell at this new location, the household moves again with a new random direction and distance. If the new grid cell is empty, the household moves to its center.



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# Submodels II

**Update** is conducted by all households to determine whether they tolerate their neighborhood. The tolerance of households is determined by a parameter `%-similar-wanted`, which can range from 0 to 100 and applies to all households. A household's neighbors are all households on the eight surrounding patches. The household's variable `happy?` is set to false unless the number of neighbors with the household's color is greater than or equal to `%-similar-wanted` divided by 100 and multiplied by the number of neighbors.



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# Experiment

- ▶ Start NetLogo and **open** the segregation model.
- ▶ Press the **setup** and **go** buttons.
- ▶ With the density of turtles set to 95% and %-similar-wanted set to 30%, it takes about 15 ticks until all households are happy. The average similarity of neighborhoods is about 70% –that is, turtles have on average 70% neighbors of the same kind. The View shows relatively small clusters of red and blue households, mixed with smaller clusters of empty patches.



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# Results

- ▶ The model thus demonstrates, in the current settings, the point Schelling (1971) wanted to make: even **relatively small** intolerance to unlike neighbors can lead, without any other mechanism, to segregated neighborhoods.
- ▶ Do we understand the results of our first experiment? It is not entirely clear why the small intolerance of 30% gives rise to an average similarity of 70% or more. Let us see how model output changes if we chose **extreme values** of %-similar-wanted.



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# H1. Try extreme values of parameters.

- ▶ When parameters are set to the extremes of their range, model outcome often is **simpler** to predict and understand.
- ▶ Set %-similar-wanted to small values (5%) and large values (90%). Before you press the go button, try to predict the outcome of these experiments!



# Low Intolerance

- ▶ If intolerance is very low, most households are happy in the initial random distribution, and it takes only a **few** time steps until they are all happy.
- ▶ The **average** similarity among neighbors is near 50%, which might seem surprisingly high until we remember that, with only two colors, average similarity should be near 50% even when households are randomly distributed.
- ▶ This is an important realization already: that the final average similarity is always **at least** 50% no matter how low %-similar-wanted is.



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# High Intolerance

- ▶ If intolerance is very high, almost all households are unhappy at the beginning, but movement **never stops** and average neighbor similarity never changes much from 50%.
- ▶ The first result is easy to understand; the second is less clear: why does the average similarity among neighbors not just increase with %-similar-wanted?
- ▶ We can only assume that there is a critical value of %-similar-wanted **above** which it becomes very unlikely that moving households ever will find a neighborhood that suits their tolerance. Let us test this hypothesis.



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## H2. Finding tipping points.

- ▶ If a model shows qualitatively different behaviors at different extreme values of a parameter, vary the parameter to try to find a tipping point: a parameter range where the behavior **suddenly changes**.
- ▶ This is an extremely important heuristic because it helps us understanding different **regimes of control**: below a critical parameter value, process A may be dominant and control the behavior of the system, but above that tipping point control is taken over by process B.
- ▶ Identifying regimes of control and understanding how they emerge reveals a great deal about how a model works.



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# Observation

- ▶ To put this heuristic into practice, we would usually program some quantitative outputs, and then run sensitivity experiments.
- ▶ However, Segregation is such a simple model that it is sufficient to simply watch the View while we change the value of %-similar-wanted.



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# Experiment

- ▶ Set %-similar-wanted to 90%; click setup and then go. Then, use the slider to slowly decrease this parameter –slowly, because it might take some time until the system responds.
- ▶ Bang! Very abruptly, when %-similar-wanted reaches 75%, model behavior changes and segregation occurs.
- ▶ “Bang! Very abruptly, when %-similar-wanted reaches 75%, model behavior changes and segregation occurs.



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# Results

- ▶ Note that the system needs quite a long time for all households to become happy (about 150 ticks).
- ▶ Another difference is that the clusters of households of the same color are much larger.
- ▶ And a striking feature of the final distribution of households is that regions of different colors are separated by strips of empty patches. Why?
- ▶ One of the things we need to understand about a model is how it got to its current state as a consequence of what happened over time.
- ▶ How could we do this without visualizing the current state from many different perspectives?



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### H3. Try different visual representations.

- ▶ To better understand an ABM, we often look at different visual outputs to detect patterns and establish causal relationships.
- ▶ Kornhauser, Wilensky, and Rand [1] list many useful ways to change the View of NetLogo models to reveal hidden structures and correlations.
- ▶ With the Segregation model, the ability to tell happy from unhappy households would allow us to better understand why and where clusters of happy households form or dissolve.



# Experiment

- ▶ Save the segregation model under a new name, such as `my-segregation-model.nlogo`.
- ▶ In the procedure `update-turtles`, insert, at the end of the `ask turtles` block, code like this:

```
1  ifelse happy?  
2    [ set size 1.0 ]  
3    [ set size 0.6 ]
```

- ▶ Now, happy households are represented as large turtles and unhappy ones as small.
- ▶ Repeat the experiment described above, where you start with `%-similar-wanted = 90%` and slowly decrease it as the model runs.



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# Results

- ▶ With high intolerance (90%), we never see any clusters of happy households. With lower %-similar-wanted, more and more small clusters of happy blue or red households occur but never persist.
- ▶ At the very sharp transition point (75%) clusters start to persist and grow slowly, with a lot of dynamics going on at their boundaries.
- ▶ But still, the simulation runs too fast to really understand what happens.



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## H4. Run the model step-by-step.

- ▶ Stepping through a simulation, tick by tick, is crucial because it allows us to look at the **current state** of the model, predict what will **happen next**, and see whether we were right.
- ▶ Add a button step to the Interface and make it call the go procedure, but with the **forever** option off.



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# Experiment

- ▶ Now, by stepping through the simulation at the critical parameter value  $\% \text{-similar-wanted} = 75\%$ , we see that clusters of happy households tend to grow.
- ▶ We understand that inside such clusters there cannot be any dynamics: everybody is happy, so nobody moves. And the clusters do not contain any isolated empty patches for long.
- ▶ Why? (We leave answering this question to you.)



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# Observations

- ▶ What we see is that the driving dynamics are going on at the boundaries of clusters.
- ▶ We realize that a household's decision to move affects not only its own happiness but also that of its old and new neighbors. Movement of households can sometimes turn other happy households into unhappy ones.
- ▶ Now, how do the boundaries between blue and red clusters emerge?



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## H5. Look for striking patterns in the output.

- ▶ Patterns are key to discovering important **mechanisms** and the **internal organization** of complex systems.
- ▶ With %-similar-wanted set to 75%, run the model several times.



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# Observations

- ▶ The clusters of blue and red households are quite clear, but the pattern is **not yet clear enough** to indicate anything particular.



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## H6. Parameter experimentation.

- ▶ At an interesting point in parameter space, keep the controlling parameter **constant** and vary other parameters.
- ▶ A parameter space is the set of **all possible values** of model parameters.
- ▶ A point in this space corresponds to one specific set of parameter values.



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# Observations

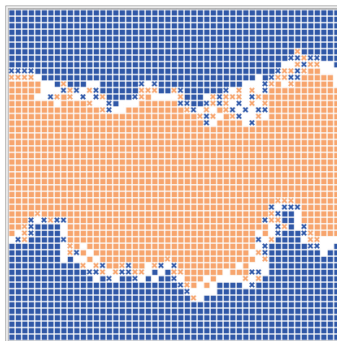
- ▶ This heuristic allows us to explore how our controlling parameter, or the process represented by it, is affected by other processes in the model.
- ▶ Trying to understand a model includes trying to understand the interactions of mutual controls.



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# Experiment

- ▶ With %-similar-wanted set to 75%, decrease the number of households, for example to 1500, then increase it to, say, 2400.
- ▶ Now you can see that the pattern becomes much clearer and more striking if we increase the number of households so there are very few free patches.



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# Observations

- ▶ It is also striking that the system now usually segregates into only one cluster of each color (remember, the boundaries are toroidal).
- ▶ What might be the reason for this? When you see this pattern emerge, note that small clusters, with their strongly curved boundaries, usually do not survive unless they manage, by chance, to develop longer sections of their boundary that are more or less straight.
- ▶ This suggests a hypothesis: that straight boundaries create spatial configurations that make the nearby households less sensitive to the influence of single neighbors of the opposite color; there will always be enough of one's own kind to keep the proportion of unlike neighbors below the intolerance level.



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# Testing the Hypothesis

- ▶ Measure (and visualize), in detail, the similarity of neighbors as perceived by households at the boundary; this would be easier if we set up specific configurations in a much smaller system of, say,  $5 \times 5$  patches. Often, this is the only and best way to test our hypothesis.
- ▶ By making a prediction from it and testing the prediction. If our hypothesis that straight boundaries lead to stable results is correct, then straight boundaries should become less important when intolerance is less, because then curved boundaries can also “survive” the influence of single unlike neighbors.
- ▶ Hence, we can predict that when we slowly decrease %-similar-wanted, clusters should increase in numbers, decrease in size, and have more curved boundaries.



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# Experiment

- ▶ Study segregation patterns for %-similar-wanted smaller than 75%, keeping the number of households at 2400.
- ▶ Our prediction turns out to be correct!
- ▶ We still did not explain why for %-similar-wanted = 75% (and also down to about 58%), empty patches are only found between the boundaries of the red and blue clusters. We leave explaining this to you.



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# Final comments

- ▶ The Segregation model is admittedly very simple and not that hard to understand.
- ▶ Nevertheless, we applied heuristics that are useful for analyzing any model.
- ▶ Note that Schelling's model of segregation –which, ironically, was only verbally formulated and never implemented by Schelling himself– was and still is highly influential.
- ▶ Myriad modifications of this model have been made because real cities and households are of course much more complex; these models have explored things like larger neighborhoods, social networks, spatial barriers, housing prices, and variable intolerance levels.



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# H7. Currencies I

- ▶ Use several “currencies” for evaluating your simulation experiments. ABMs are rich in structure, including many agents and spatial units, all in different states. It is impossible to take all this information into account.
- ▶ We therefore need to find **currencies**, usually summary statistics or observations, that capture an important property of the model system in a single number.
- ▶ Currencies correspond to the things empirical researchers **measure** (or wish they could measure) in the real system.
- ▶ **Example:** In population models an obvious currency is population size: the total number (or weight, or wealth, etc.) of all agents.



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## H7. Currencies II

- ▶ We therefore could analyze the time series of population size produced by the model. But even time series can contain too much detailed information, so we often look at even **coarser** currencies such as the mean and range of values over time.
- ▶ Often, finding useful currencies is a nontrivial and important task of the modeling process. This is why **Observation** is included in the Design Concepts section of the ODD protocol.
- ▶ Usually, we try several currencies, or observations, and see how sensitive they are and how much they help us **understanding** the model. In most cases, one currency is not sufficient, and we need to view the model from different perspectives.



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# Kinds of Currencies I

- ▶ Standard measures of the statistical distribution of results, whether distributions among agents of their state variables or distributions over time of system-level results, including:
  - ▶ The mean and median,
  - ▶ The standard deviation or variance,
  - ▶ The minimum and maximum,
  - ▶ And whether they fit theoretical shapes (normal, exponential, power law, etc.) and, if so, the distribution coefficients.
- ▶ Characteristics of time series such as positive or negative trends, autocorrelation, or the time until the system reaches some state such as having no more agents or becoming static, e.g., until movement stops in the segregation model.
- ▶ Measures of spatial distributions and patterns such as spatial autocorrelation, fractal dimensions, and point pattern metrics.



# Kinds of Currencies II

- ▶ Measures of difference among agents, such as whether the distribution among agents of some state variable is uni- or multi-modal, e.g., is wealth distributed normally among agents or are there distinct groups of rich and poor? and whether different kinds of agents are segregated over time or space.
- ▶ Stability properties (Grimm and Wissel 1997), which can provide insight into a system's internal mechanisms and be important for management problems. Currencies can evaluate stability properties such as how quickly the system returns to some “normal” state after a disturbance, how large the “domain of attraction” spanned by two or more state variables is (outside this domain, the system loses important characteristics, e.g., spatial segregation, in the Segregation model), and what buffer mechanisms damp the system's dynamics.



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## H8. Simplified Models

- ▶ Analyze simplified versions of your model.
- ▶ Once we have chosen our first currencies, we proceed to the most important approach for analyzing models: Simplify! ABMs (and other models) can be hard to understand because so many different factors affect model behavior.
- ▶ Often, it is relatively easy to reduce this complexity and make understanding what mechanisms cause what results much more feasible.



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# Kinds of Simplifications

- ▶ Make environment constant.
- ▶ Make space homogenous (all patches are the same, and constant over time).
- ▶ Reduce stochasticity, for example by initializing all agents identically or by replacing random draws with the mean value of their distribution (e.g., replace set size  $2 + \text{random-float } 10$  with set size 6).
- ▶ Reduce the system's size by using fewer turtles and patches.
- ▶ Turn off some actions in the model's schedule (just comment them out in the go procedure).
- ▶ Manually create simplified initial configurations and states that allow you to check whether a certain mechanism works as you assume.



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# Observations

- ▶ When you make these kinds of simplifications, the model version will no longer be a good representation of the system you want to understand.
- ▶ You create, so to say, simplified worlds that are unrealistic but easier to understand.
- ▶ Grimm found that many developers of ABMs did not systematically simplify their models for analysis, perhaps because of a psychological barrier: if you spent so much time developing a model that seems to represent a real system reasonably well, should you then make this model extremely unrealistic?
- ▶ Yes, definitely! Modeling includes as much deconstruction as construction!



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## H9. Bottom-up Analysis

- ▶ Any system that requires an agent-based modeling approach is very difficult to understand without first understanding the behavior of its parts: the agents and their behavior.
- ▶ It is important that we test and understand these behaviors first, before we turn to the full model.
- ▶ That is why we emphasize analyzing submodels independently before putting them in an ABM and systematically developing theory for agent behaviors.
- ▶ Even more of these kinds of low-level analysis may be needed to understand the full model, especially if unexpected dynamics seem to emerge.



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# H10. Unrealistic Scenarios

- ▶ The idea of this heuristic is to simulate scenarios that could never occur in reality.
- ▶ Why? Because a simple and direct way to see the effect of a certain process or structure on overall model behavior is to just remove it.
- ▶ **Example:** The analyses of how investor behavior affects double-auction markets provides an interesting contrast: models that deliberately used unrealistically simple investment behaviors were shown to produce system-level results that were not so unrealistic. The conclusion in that case was that complex agent behavior might not be the mechanism generating complex market dynamics after all; the market rules themselves might be more important than anticipated.



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# Goal

- ▶ Many scientists automatically think of **statistics** when they think about analysis and understanding.
- ▶ The goal of statistics is to extract understanding, and perhaps infer causal relations, from a **fixed and usually limited set of data**.
- ▶ Agent-based modeling, in contrast, can produce **as many data as we want**, and offers additional ways to develop understanding and deduce mechanisms.
- ▶ If a simulation experiment does not provide a clear enough answer, we **manipulate the model** world further and run new experiments until we get clear results.
- ▶ The purposes, and underlying mind-sets, of statistics and simulation modeling are thus **quite different**.



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# Summary statistics.

- ▶ Aggregating model output into statistics such as the mean and standard deviation is useful, but remember that in model results, **extreme** or **unusual values** are often important clues to understanding.
- ▶ They should not be thrown away as **outliers** as they often are in statistical analysis of empirical data.



# Contrasting Scenarios.

- ▶ Statistics can be used to detect and quantify **differences** between simulation scenarios, much like testing for differences among laboratory treatments.
- ▶ This analysis often requires arbitrary assumptions that affect results –especially, the **Number of replicates** for each treatment.
- ▶ While the same is true for laboratory and field experiments, it is **easier** to change these assumptions when analyzing model results.



# Quantifying correlative relationships. I

- ▶ Regression methods such as analysis of variance and general linearized modeling can be used to see which inputs have **strongest effects** on which model outputs, and whether there are interactions among factors.
- ▶ The approach is simply to look for statistical relationships among inputs (here, including parameter values and initial conditions as well as time-series inputs) and outputs.
- ▶ This approach can be very useful, in particular if a model is complex and we initially have **no idea** where to expect causal relationships.
- ▶ Of course, these regression methods do not directly identify **causal relationships**, but they can provide important clues by revealing the relative importance of different factors.



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# Quantifying correlative relationships. II

- ▶ As detectives trying to understand why an ABM behaves as it does, we should consider such statistical **meta-models** as a starting point, not the end, of our analysis.
- ▶ The meta-model points to factors that we should **explore** in more detail using simulation experiments.

# Comparing model output to empirical patterns.

- ▶ During **calibration** and other times when we want to compare model results to observed patterns quantitatively, there is a wide variety of statistical approaches that can be useful.
- ▶ The extensive literature on model calibration, **fitting of models to data**, and **model selection** is applicable.



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# Detectives

- ▶ To understand **what** an ABM does and **why**, we must again be detectives—as we were when searching for programming errors.
- ▶ Good detectives combine reasoning and strong inference, systematic analysis, intuition, and creativity.
- ▶ Obviously, we cannot teach you how to reason logically, be creative, and have good intuition. You have to develop your intuition through **experience**.
- ▶ Thus, the key messages of this session are that once we build an ABM, or even the first time we freeze the design, we need to try to understand what it does; and **controlled simulation experiments** are how to do it. Now try it!



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# Publications

- ▶ In addition to the analysis heuristics presented here, you should try to learn from **published analyses**.
- ▶ If you read a publication that presents some exciting **new insights** using an ABM, ask yourself: how did they do that?
- ▶ The **key experiments** presented in a publication can give you a good idea what techniques and heuristics the authors used to obtain their insights.



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# Heuristics

- ▶ You can try them into the following sequence:
  - ▶ Analyzing from the bottom up.
  - ▶ Define and test several currencies for evaluating your simulation experiments.
  - ▶ Try different views (visual outputs).
  - ▶ Run the model step-by-step.
  - ▶ Analyzed simplified versions of your model.
  - ▶ Try extreme values of parameters.
  - ▶ Look for striking, or strange, patterns in the model output.
  - ▶ Find tipping points in model behavior.
  - ▶ From an interesting point in parameter space, keep the controlling parameter constant and vary other parameters.
  - ▶ Explore unrealistic scenarios.



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# Suggestion

- ▶ Add your **own heuristics** to ours.
- ▶ Intuition is important, but **critical reflection** on how your intuition, and heuristic approaches, work is no less important.
- ▶ Good science always is based on a mixture of **creativity**, which can include quite chaotic, unsystematic things, and **rigorous methods**, which can be pedantic and boring.
- ▶ Switch between these two attitudes, try looking at your model from different perspectives, and you will quickly learn—how to learn from ABMs!

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