# Agent-Based Modeling and Simulation Collectives

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Dr. Alejandro Guerra-Hernández (UV) Agent-Based Modeling and Simulation

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- These slides are completely based on the book of Railsback and Grimm [2], chapter 16.
- Any difference with this source is my responsibility.



# Collectives

- Agent-based models represent a system by modeling its individual agents, but surprisingly many systems include intermediate levels of organization between the agents and the system.
- Agents of many kinds organize themselves into what we call collectives: groups that strongly affect both the agents and the overall system. This chapter is about how to model such intermediate levels of organization.



## Representations

- As a completely emergent characteristic of the agents: the agents have behaviors that allow or encourage them to organize with other agents, the collectives exist only when the agents organize themselves into collectives, and the collectives are given no behaviors or variables of their own.
- As a separate, explicit type of entity in an ABM with its own actions: the modeler specifies how the collectives behave and how agents interact with the collectives.



# Implementation

- NetLogo provides breeds: a way to model multiple kinds of turtles or links.
- Breeds let us do many things, including modelling collectives as explicit entities.



# Learning Objectives

- Learn what collectives are and several ways of modeling them.
- Become expert at using NetLogo breeds to represent kinds of turtles.
- Develop experience with stochastic modeling by programming and analyzing a new example model, of African wild dogs that live in packs.
- Become familiar with logistic functions, which are particularly useful for representing probabilities and functions that vary nonlinearly between zero and one.



# Cooperation

- Cooperation among individuals has important advantages, so it is not surprising that systems of all kinds include groups of cooperating individuals.
- Example: In biology, cells of different types form organs and other structures to perform functions that the whole organism depends on.
- Example: In many animal and human societies, family and social groups are very important to the behavior, survival, and reproduction of individuals.
- Example: Economic systems contain many networks and organizations through which individuals and firms cooperate.



# Explicit modelling

- A collective agent typically has its own state variables (especially, a list of the individuals in the collective) and traits (often including rules for how individuals are added and removed from the collective).
- The individuals often have a state variable for which collective they belong to and traits for how the collective affects them.
- This approach of modeling collectives explicitly often adds the least complexity: we can ignore all the individual-level detail that, in reality, produces the behavior of the collective.
- We can think of collectives and their traits as a simplified model of their individuals.



# Representing Collectives via Patches

- If the individual agents in a model are represented as turtles, sometimes it makes sense to represent collectives of turtles as patches.
- This approach can work, e.g., when a collective is a group of individuals that occupy the same space.
- Example: A population of animals or people that live in family or social groups that each occupy a territory (or business, or town).
- Patch variables can describe the collective's characteristics (e.g., the number of members, perhaps broken out by age, sex, etc.), and the primitive turtles-here provides an agentset of member turtles.
- The patch's traits could include rules for collective behaviors such as adding and removing individuals.



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# Representing Collectives via Links

- NetLogo's links might seem natural for modeling collectives because we can think of a collective as a group of individuals that are linked in some way.
- However, NetLogo does not have a built-in way to give variables or rules to a network of linked turtles –only to the turtles or links.
- So links are not easily used to represent collectives with their own behaviors.



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# Using Breeds to Model Multiple Agent Types

- NetLogo provides breeds for modeling different kinds of turtles or links.
- Breeds are equivalent to subclasses in an OOP language: they let you create several specialized kinds of turtles that have different characteristics from each other, while still sharing some common variables and traits.



#### Alternatives

# Key Points Using Breeds I

- Breeds must be defined at the top of the program.
- Turtles have a built-in state variable breed. If you create a turtle using create-turtles, its breed variable is set to "turtles." Using create-<breeds> creates a turtle of the specified breed: create-wolves creates a turtle with its value of breed equal to wolves.
- You can change what breed a turtle is by simply using set breed.
- Each breed can have its own unique state variables, defined via a <br/>
  <br/>
  ds>-own statement.
- Each breed still has the built-in variables (xcor, ycor, color, etc.) that NetLogo provides for all turtles.
- The statement turtles-own can still be used to define variables that all breeds use.

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# Key Points Using Breeds II

- If you have defined the breeds "wolves" and "sheep," you can use ask wolves [...] or sheep [...] to have only wolves or only sheep do something. But ask turtles [...] still causes all turtles of all breeds to do something. Likewise, you can use mean [age] of sheep and mean [age] of turtles to get the average age of just sheep and of all turtles.
- Several of the agentset primitives have breed versions. Example: sheep-here reports an agentset of sheep on a patch, while turtles-here reports all turtles on the patch.
- Breeds do not have their own contexts, all are treated as being in turtle context. Hence, a wolf can try to run a procedure that you wrote for sheep. However, if a wolf tries to use a variable that was defined only for sheep, an error is produced.

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# Similar Breeds

- Although breeds can be very different, e.g., wolves and sheep, this is not necessarily the case.
- They can represent slight variatons of the same agent to analyse the effect of the variation in the same simulation.
- Example: Growers versus savers in an economy, where the only difference between them is the parameter controlling how rapidly they expand.



# Individuals as Breeds

- We can represent individuals as one breed, while representing collectives as a separate breed with completely different variables and traits.
- For this, we must keep track of which individuals belong to each collective, and which collective each individual belongs to.
- The trick consists in giving each individual a variable that contains the collective he belongs to; and giving each collective a list or an agent set of their members.
- ► Updating: each time an individual leaves or joins a collective:
  - We need to remove or add the individual from/to the collective in question.
  - We need to update the individual's variable representing the collective in question.



# Social Groups

- Some of the clearest examples of collectives are populations of animals that form family or social groups.
- These animals typically cooperate within their collectives to obtain food and other resources.
- Example: Mating and reproduction can be tightly regulated within the collectives: a group's dominant α male and female mate, but suppress others' reproductive behavior.
- Yet the behavior of the individuals strongly affect their collective: the reproductive output of the α couple dependes on
  - How other members decide whether to remain or seek better opportunities elsewhere, and
  - whether individuals leave or die determines the size and structure of the group.



# African Wild Dogs

- Our example is a model of Lycaon pictus which are endangered and carefully managed in a number of nature reserves in South Africa [1].
- Detailed observations of the wild dogs have been collected for many years, so we have extensive empirical information to use:
  - The frequency with which packs produces pups decreases as the total population of dogs increases.
  - Subordinate adults (non α, two or more years old) often leave their pack as dispersers (50% of the time).
  - When two disperser groups of opposite sex and originating from different packs meet, they usually (64% of the time) join to form a new pack.
  - The annual mortality rate varies with dog age and is much higher for dispersing dogs than for those in a pack.



# Structure Of The Model

- Individual dogs need to be model, since many processes depend on individual variables, e.g., age, sex, and social status.
- It is clear that no one, but two kinds of collectives are important:
  - Packs
  - Disperser groups
- Therefore we need three breeds: dogs, packs, and disperser groups.



# Stochastic Processes

- Stochastic traits would be very useful.
- The behaviors of the dogs and the collectives they form are complex, but frequencies and rates observed in the field can be used as probability parameters for stochastic traits.
- Reproduction, dispersal decisions, and mortality can all be modeled as stochastic events with probabilities based on observed frequencies.



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# Management Actions

- 1. To increase the "carrying capacity" or the reserve, the maximum number of dogs that are resources to support. This can be achieve by increasing the size of the reserve or by reducing the competition for prey from other predator such as lions.
- 2. To reduce the mortality of dispersers, which is high by, e.g., favoring dispersers encounters.



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#### Purpose

#### Evaluating how persistence of a wild dog population depends on:

- The reserve's carrying capacity,
- The ability of dispersing dogs to find each other, and
- The mortality risk of dispersing dogs.
- Measures of persistence include the average number of years (over a number of replicate simulations) until the population is extinct, and the percentage of simulations in which the population survives for at least 100 years.



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# Entities, State Variables, and Scales I

- ► Three kinds of agents: dogs, packs, and disperser groups.
- Dogs have state variables for age in years, sex, the pack or disperser group they belong to, and social status.
- Social status has one of the following values:

Status	Age	Observations	
pup	0		
yearling	1		
subordinate	$\geq 2$	if it is not an $lpha ext{-dog}$ .	
alfa	$\geq 2$	Choosen randomly, one male and one female.	
disperser	$\geq 2$	Member of a disperser group, in- stead of a pack.	
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# Entities, State Variables, and Scales II

- Dog packs have no state variable, except for a list (or agentset) of the dogs belonging to the pack, i.e., members.
- Disperser groups have a state variable for sex (all members are of the same sex) and the agentset of their members.
- The time step is one year.
- The model is non spatial: locations of packs and dogs are not represented.



# Process Overview and Scheduling I

#### 1. Age and social status update:

- The age of all dogs is incremented. Their social status is updated accordingly to their new age.
- Each pack updates its α male and female: If there is no α of a sex, a subordinate of that sex is randomly selected as alpha.
- 2. Reproduction:
  - If the pack does not include both an α female and male, no pups are produced.
  - Otherwise, the probability of a pack of producing pups P depends on the total of number dogs in the entire population at the current time step N, modeled as a logistic function of N. This function depends on the carrying capacity of the reserve, initially 60 dogs. P = 0.5 when N is half of the carrying capacity. P = 0.1 when N is the total carrying capacity.

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# Process Overview and Scheduling II

- If the pack reproduces, the number of pups is drawn from a Poisson distribution that has a mean birth rate (pups per pack per year) of 7.9. Sex is assigned to each pup randomly, with a 0.55 probability of being male. Pup age is set to 0.
- 3. Dispersal:
  - If a pack has no subordinates, then no disperser group is created.
  - If a pack has only one subordinate of it sex, it has a probability of 0.5 of forming a one-member dispersing group.
  - If a pack has 2 or more subordinates of the same sex, they always form a disperser group.
  - Dogs that join a disperser group no longer belong to their original pack, and their social status is set to disperser.



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# Process Overview and Scheduling III

4. Dog mortality is scheduled before pack formation, because mortality of dispersers is high. Whether or not each dog dies is determined stochastically:

Status	Probability
disperser	0.44
yearling	0.25
subordinate	0.20
alfa	0.20
pups	0.12

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 Mortality of collectives, if any pack or dispersal group has no members, it is removed from the model. If any pack contains only pups, the pups die and the pack is removed.

# Process Overview and Scheduling IV

- 6. Pack formation. Disperser groups may meet other disperser groups, and if they meet a group of the opposite sex and from a different pack, they may or may not form a new pack. Each disperser group execute the following steps in random order:
  - Determine how many times the group meets another disperser group. This is modeled as a Poisson process with the rate of meeting (mean number of times per year that another group is met) equal to the number of other disperser groups times a parameter controlling how often groups meet. This parameter can potentially be greater than 1, but is given a default value of 1.0. The following steps are repeated up to rate of meeting times, stopping if the disperser group selects another to join.
  - Randomly select other disperser group. It is possible to select the same other group more than once.



# Process Overview and Scheduling V

- If the other group is of the same sex, or originated from the same pack, then do nothing more.
- If the other group is of the opposite sex and a different pack, then there is a probability of 0.64 that the two groups join into a new pack. If they do not join, nothing else happens.
- If two disperser groups do join, a new pack is created immediately and all the dogs in the two groups become its members. The alpha male and female are chosen randomly; all other members are given a social status of "subordinate." The two disperser groups are no longer available to merge with remaining groups.



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# Initialization

- ► The model is initialized with 10 packs and no disperser groups.
- The number of dogs in each initial pack is drawn from a Poisson distribution with mean of 5 (the Poisson distribution is convenient even though its assumptions are not met in this application).
- The sex of each dog is set randomly with equal probabilities.
- The age of individuals is drawn from a uniform integer distribution between 0 and 6.
- Social status is set according to age.
- The alpha male and female of each pack are selected randomly from among its subordinates; if there are no subordinates of a sex, then the pack has no alpha of that sex.

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## The exectution tab



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# Global variables

Two global variables are required:

```
1 globals
2 [
3 years-to-simulate ;; set to one houndred by default
4 prob-reproduction ;; set by a logistic func in reproduce
5 ]
```



## Breeds

Kinds of agents are implemented as breeds, both individuals (dogs) and collectives (packs and disperser-groups):

```
1 ; Breeds represent collectives in the model
2
3 breed [dogs dog]
4 breed [packs pack]
5 breed [disperser-groups disperser-group]
```



# Other Variables I

> You can define variables for the declared dogs breed:

```
dogs-own
1
2
   L
     age
3
     sex
4
     status
5
     my-pack
6
     my-disperser-group
7
   ]
8
```

#### As well as packs:

```
1 packs-own
2 [
3 pack-members
4 ]
```



# Other Variables II

And disperser groups:

```
1 disperser-groups-own
2 [
3 group-sex
4 group-members
5 ]
```



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## Setup

```
to setup
1
     ca
2
з
     set years-to-simulate 100 ; number of years to be simulated
4
5
     create-packs initial-number-of-packs ; ten by default
6
     Γ
7
       initial-setup ; configure a new pack
8
     ٦
9
10
     reset-ticks
11
   end
12
```

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# Initial setup I

```
to initial-setup ; a pack procedure
1
     ; set a location and shape just for display
2
     setxy random-xcor random-ycor
3
     set shape "box"
4
5
     ; create a pack of dogs
6
     let num-dogs random-poisson initial-mean-pack-size
7
8
     hatch-dogs num-dogs
9
10
       ; first, set display variables
11
       set shape "dog"
12
       set heading random 360
13
       fd 2
14
15
       ; now assign dogs variables
16
```

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# Initial setup II

```
ifelse random-bernoulli 0.5
17
       [set sex "male"]
18
       [set sex "female"]
19
20
       set age random 7
21
       update-status
22
       set my-pack myself
23
     ]; end of hatch-dogs
24
25
     ; initialize the pack's agentset of members
26
     set pack-members dogs with [my-pack = myself]
27
28
     ; now select the alpha dogs in the pack
29
     update-alphas
30
   end ; end of initial-setup
31
```



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Case Study: A Wild Dog Model with Packs

Implementation

# Random bernoulli does not exist

```
1 to-report random-bernoulli [ p ]
2 report random-float 1
```



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# Updating the status of a dog

```
to update-status ; a dog procedure
1
     ifelse age = 0
2
     [set status "pup"] ; a pup's age is younger than one year
3
     [ifelse age = 1
4
       [set status "yearling"]; a yearling's age is one
5
       [if status != "alpha"
6
         [set status "subordinate"]
7
8
       : older than one
9
    end ; set-my-status end
10
```



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#### Implementation

# Updating alphas I

```
to update-alphas ; a pack procedure
1
     if not any? pack-members with [sex = "female" and status =
2
          "alpha"]
3
       if any? pack-members with [sex = "female" and status =
4
            "subordinate"]
        Γ
5
         ask one-of pack-members with [sex = "female" and status
6
              = "subordinate"]
7
           set status "alpha"
8
           set size 1.5
9
10
11
12
13
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```

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# Updating alphas II

```
if not any? pack-members with [sex = "male" and status =
14
          "alpha"]
15
       if any? pack-members with [sex = "male" and status =
16
            "subordinate"]
        Ľ
17
         ask one-of pack-members with [sex = "male" and status =
18
              "subordinate"]
19
           set status "alpha"
20
           set size 1.5
21
22
23
24
   end
          end update-alphas
25
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```

# Testing the set up





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# The Schedule I

```
to go
1
     tick
2
     if ticks >= years-to-simulate [stop]
3
4
     set prob-reproduction logFunc count dogs ; a logistic
5
          function of the current number of dogs
6
     ; age and social status update
7
     ask dogs
8
9
       set age age + 1 ;; dogs become one year older
10
       update-status ;; updated accordingly to their age
11
12
13
     ask packs [update-alphas]
14
     ask packs [reproduce]
15
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                                                          . . . . . . .
```

# The Schedule II

```
16 ask packs [disperse]
17 ask dogs [dog-die]
18 ask packs [pack-die]
19 ask disperser-groups [if count group-members = 0 [die]]
20 ask disperser-groups [pack-formation]
21 end ; go ends
```



# The logistic function

- They are useful for modeling how the probability of some event depends on one or several variables, because:
  - They produce values in the probability range;
  - They reproduce how probabilities can be very low over a wide range of conditions, very low over a wide range, very high over another range, and change sharply between these two ranges.





# Computation

#### Identify two points in the logistic curve:

- P1 = 30, 0.5, i.e., when the population of dogs is half the capacity of the reserve, the probability of reproduction is 0.5.
- P2 = 60, 0.1, i.e., when the population of dots equals the capacity of the reserve, the probability of reproduction is 0.1.
- Calculate the following variables:

• 
$$D = ln(P1/(1 - P1))$$
  
•  $C = ln(P2/(1 - P2))$   
•  $B = (D - C)/(X1 - X2)$   
•  $A = D - (B \times X1)$ 

• Return P = Z/(1+Z) where  $Z = exp(A + (B \times X))$ .



# Coding the logistic function

```
to-report logFunc [x]
1
     let x1 30
2
     let p1 0.5
3
     let x2 60
4
     let p2 0.1
5
6
     let d ln (p1 / (1 - p1))
7
     let c ln (p2 / (1 - p2))
8
     let b (d - c) / (x1 - x2)
9
     let a (d - (b * x1))
10
11
     let z exp(a + (b * x))
12
13
     report z / (1 + z)
14
   end ; logFunc ends
15
```



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# Testing the function

#### You can test the function using the command center:

A T	
Command Center	
observer> show logFunc 30 observer: 0.5 observer: 0.1000000000000000 observer: 0.1000000000000000 observer> show logfunc 10 observer: 0.812268223212867 observer> show logfunc 5 observer: 0.8618832502903921	
observer>	



# Reproducing I

```
to reproduce ; a pack procedure
1
     if count pack-members with [status = "alpha"] = 2 ; there
2
         is a male and female alphas
3
       let number-pups random-poisson 7.9;; number of born pups
4
5
       if random-bernoulli prob-reproduction
6
       Γ
7
         hatch-dogs number-pups
8
9
           set shape "dog"
           set age 0 ; new borns are 0 years old
11
           set status "pup"
12
13
           ifelse random-bernoulli 0.55 ; sex is randomly set
14
           [set sex "male"]
15
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                                                        → Ξ →
```

# Reproducing II





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# Dispersing I

```
to disperse ; a pack procedure
1
     let male-subordinates pack-members with [status =
2
         "subordinate" and sex = "male"]
     let female-subordinates pack-members with [status =
3
         "subordinate" and sex = "female"]
4
     if any? female-subordinates
5
6
       ifelse count female-subordinates = 1 ; creates female
7
           disperser groups
       Γ
8
         if random-bernoulli 0.5 [ create-disperser-group-from
9
             female-subordinates ]
       ]
10
11
         create-disperser-group-from female-subordinates
12
```

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# Dispersing II

```
13
14
15
     if any? male-subordinates
16
17
       ifelse count male-subordinates >= 1 ; creates male
18
            disperser groups
        Γ
19
          if random-bernoulli 0.5 [ create-disperser-group-from
20
              male-subordinates ]
21
22
          create-disperser-group-from male-subordinates
23
24
25
   end
          disperse ends
26
                                                                    Universidad Veracruzan
```

# Dispersing groups I

```
to create-disperser-group-from [some-dogs] ; a pack procedure
1
     hatch-disperser-groups 1; create one disperser group
2
3
       set group-members some-dogs ; with some-dogs as members
4
       set group-sex [sex] of one-of some-dogs ; all members
5
           have same sex
6
       set shape "car"
7
       set heading random 360
8
       fd_2
9
       ask some-dogs
11
12
         set my-disperser-group myself
13
         set status "disperser"
14
         set color green
15
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```

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# Dispersing groups II

```
16
         move-to my-disperser-group
17
         set heading [heading] of my-disperser-group
18
         fd 1 + random-float 2
19
       ; end of ask some-dogs
20
     : end of hatch
21
22
     let dogs-former-pack [my-pack] of one-of some-dogs
23
     ask dogs-former-pack
24
25
       set pack-members pack-members with
26
         [status != "disperser"]
27
28
   end
29
```

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#### Implementation

# Dogs' death

```
to dog-die
1
     ifelse status = "disperser"
2
      [ if random-bernoulli 0.44 [die] ]
3
      [ifelse status = "yearling"
4
        [ if random-bernoulli 0.25 [die] ]
5
6
         ifelse status = "subordinate" or status = "alpha"
7
          [ if random-bernoulli 0.20 [die] ]
8
9
            if random-bernoulli 0.12 [die] ; pups probability of
10
11
12
13
   end
14
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```

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# Packs' death

```
to pack-die ; a pack procedure
ifelse count pack-members = 0
[die] ; the pack dies if it has no members
[if all? pack-members [status = "pup"]
[die] ; the pack dies if all its members are pups
[die] ; the pack dies if all its members are pups
[]
7 end
```



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# Packs' formation I

```
to pack-formation ; a disperser-group procedure
1
     let num-groups-met random-poisson count disperser-groups *
2
         group-meeting
     let our-sex group-sex
3
     let our-pack [my-pack] of one-of group-members
4
     let our-members group-members
5
6
     let new-pack-formed false ; true if a new pack is formed
7
8
     repeat num-groups-met
9
10
       ask one-of disperser-groups
11
12
         let their-members group-members
13
        let their-sex group-sex
14
         let their-pack [my-pack] of one-of group-members
15
```



#### Implementation

# Packs' formation II

```
16
          if their-sex != our-sex and their-pack != our-pack
17
18
            if random-bernoulli 0.64
19
            Γ
20
              set new-pack-formed true
              hatch-packs 1
22
              Γ
23
                set shape "box"
24
                setxy random-xcor random-ycor
25
                set pack-members (turtle-set their-members
26
                    our-members)
                ask pack-members
27
28
                  set shape "dog"
29
                  set heading random 360
30
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                  fd 2
31
```

# Packs' formation III

```
set status "subordinate"
32
                  set my-pack myself
33
34
                set pack-members dogs with [my-pack = myself]
35
                update-alphas
36
                ; show-pack ; to debug
37
              ٦
38
              die ; the other disperser-group dies
39
40
41
42
43
     if new-pack-formed [die] ; the former disperser-group dies
44
   end
45
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```

# Debugging

```
1 to show-pack ; a debug procedure for pack-formation
2 show ticks
3 show pack-members
4 ask pack-members
5 [
6 show status
7 ]
8 end ; show-pack ends
```



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# Output



#### Representations I

- Collectives can be modeled as an emergent trait of the individuals: individuals are modeled in such a way that they can form collectives; and the collectives a strictly an outcome of the model. Example. Flocking and the butterflies.
- When collectives have important and even moderately complex behavior, it is typically necessary to model them explicitly as a separate kind of entity with its own state variables and traits. In NetLogo, one way to model collectives explicitly is to assign the properties and traits of collectives to patches. This approach can work well when the collectives each occupy a piece of some landscape, and in models that are nonspatial so patches can represent only the collectives.

#### Representations II

In other models, though, the breeds feature of NetLogo is especially useful for representing collectives because it lets us create multiple kinds of turtle (or link).



# Cost

- Collectives are often essential in ABMs, but their use has a cost in addition to programming.
- Because collectives are an additional level of organization, we now need to model and analyze how the properties and behavior of collectives and individuals affect each other, as well as how the system affects both collectives and individuals.



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# Referencias I

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